MODELLING IMPROVED WORKING INTEREST IN OIL AND GAS INVESTMENTS

BY

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ABSTRACT

Risk pervades all aspects of the oil and gas Industry. Of all the risks, technical risk in the exploration phase is the most challenging, in which the chance of drilling a dry hole is quite significant. Investors usually increase their odds of success by investing in a fraction of a risky prospect, the Working Interest, (WI). The commonly used analytical expressions for estimating working interest suffer from the "Paradox of Aversion to Incremental Reward" (PAIR) – decreasing WI recommendation when returns are better than expected, contrary to what investors actually do in practice. This study was designed to develop a Risk Adjusted Value (RAV) model that corrects for PAIR and predict realistic estimates of WI in oil and gas assets.

Analytical models were developed using a 2-outcome risky prospect, with specified parameters of chance factor (Ps), success value (V), and failure cost (C). Relationships were derived for Expected Value (EV), RAV for specified levels of Risk Tolerance, (RT) and WI. Two hybrid Expected Utility/Expected Value (EU/EV) models were then constructed in order to correct for PAIR – one combining exponential utility function with EV, and the other hyperbolic utility function with EV. The relative impact of the significant variables on RAV was investigated through sensitivity analysis and Monte Carlo simulation. Analysis was then extended to two risky ventures. Optimizations of portfolio of selected risky ventures reflecting the risk characteristics in different phases of upstream oil and gas business were performed for unlimited and limited capital allocation conditions.

For particular values of C, V and Ps, WI varied linearly with RT for the two hybrid models in the expected utility maximization region. Also, the higher the RT value, the higher the recommended WI lies in the EV maximization region. Cost has the highest impact on asset's RAV regardless of the preference function employed. Relative impacts of V and Ps depended on the individual project under consideration. The Hyperbolic/EV model consistently recommended higher WI than the Exponential/EV model. Optimization of the portfolio of selected ventures with RAV as the objective function resulted in 257% higher investment recommendation by the hyperbolic model than the exponential model for the unlimited capital situation. For limited capital, the risk premium for the exponential model was 611% of that required for the hyperbolic model which is an indication of the latter models' risk tolerance.

A method for the determination of RAV and WI was developed which corrected for PAIR. The method can be employed for risky asset valuations, especially in budget constrained environments where different investment options compete for limited capital.

Key Words: Working Interest, Preference functions, Aversion to Reward, Risk Adjusted Value
Word Count: 438

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CERTIFICATION

I certify that this work was carried out by Engr. L. O. Akinpelu in the Department of Petroleum Engineering, University of Ibadan

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ABBREVIATIONS

BBL	-	Barrel
CAPEX	-	Capital Expenditure
OPEX	-	Operating Expense
CF	-	Cash flow
NPV	-	Net Present Value, $
DCFROR	-	Discounted Rate of Return, %
DPI	-	Discounted Profit to Investment
GROR	-	Growth Rate of Return, %
EV	-	Expected Value or EPV (Expected Present Value)
DCF	-	Discounted Cash Flow
PI	-	Performance Index
PAIR	-	Paradox of Aversion to Incremental Reward

NOMENCLATURE

i	-	Interest (Discount) Rate
P	-	Oil Price ($/bbl)
C	-	Production Costs ($/bbl)
D_i	-	Instantaneous Decline, 1/time
N_p	-	Recoverable Barrels, bbls
V_i	-	In-situ value of a Barrel of Reserves, $/bbl
W	-	Working Interest, %
V	-	Present value of Success outcome, $
C	-	Cost of Failure Outcome, $
μ	-	Mean or Expected value of distribution of Outcome Values, $
σ^2	-	Variance of Distribution of Outcome Values
v	-	Volatility –uncertainty of mean or expected value
R	-	Risk Tolerance
$U(x)$	-	Utility of variable, x
$E(x)$	-	Expectation of x

Subscript

sup	-	Upside
ds	-	Downside
s	-	Success
f	-	Failure

CHAPTER ONE

INTRODUCTION

1.1 Oil and Gas Business, Risk and Uncertainty

The oil and gas business is inherently risky. There are considerable uncertainties in the several variables that go into the determination of asset value. Because of the size of investments required for developing petroleum assets, decision makers are understandably very sensitive to the level of risk that they bear in developing a particular prospect or asset. They are anxious about whether a prospect will pay off, result in a loss or even total bankruptcy of the enterprise. It is therefore not surprising that the Petroleum Industry has been in the fore front of research into how to incorporate risk into investment decision making.

Petroleum property valuation is the procedure by which the commercial value of an oil and gas asset is assessed. It provides the "Fair Market Value" of deposits to prospective buyers, sellers, lenders and tax assessors. The value of a petroleum asset is subject to various risks including oil price risk, reserves uncertainty, production rate uncertainty, drilling and well costs, fiscal and political risks amongst many other numerous risk and uncertainties. While risk and uncertainty are usually used interchangeably, they are technically different descriptions of situations in which we have difficulty determining their outcomes. Risk means the potential or chance of a loss in a chance situation in which we know, in advance, all the potential outcomes, but for which we do not know which outcome will come to pass. For example, in the toss of a coin, there are two outcomes that can occur, a head or a tail. It is also known that for a fair coin, the probability of each outcome is 0.5. However, before the toss, it is not known whether a Head or a Tail will be the outcome.

Uncertainty, on the other hand, applies to a situation in which the outcomes that can occur and the probabilities of their occurrence cannot be fully determined. When drilling an exploratory or wild cat well, there is the risk of coming up with a dry hole or making a discovery. These are two distinct events that can occur in that exploration venture. The chance probabilities of each of those two events can also be estimated, for instance, using historical success rate probabilities in a region that has been fairly explored. However, in the event of a discovery, the size of the discovery just made is uncertain. Will it be commercial? What level of profitability can reasonably be expected from such a venture? For the duration of the project, how will the oil price “behave”? How about the fiscal regime applicable in the region in which the discovery has just been made? Will the taxes and/or royalty be changed or left alone? The oil and gas business is fraught with so many situations that qualify as risky or uncertain. Investors in the business understand and accept this as part of the business. This understanding and acceptance of the risky nature of the business have spurred efforts at quantifying the relevant risks, so the individual investor or Company can make informed investment decisions.

1.2 Discounted Cash Flow Analysis and Other Valuation Methodologies

Petroleum property valuation provides a deterministic estimate of the worth of an oil field asset or accumulation to prospective buyers, sellers, lenders and tax assessors. It provides a basis for taking investment decisions and ranking assets. The value of a petroleum asset is subject to various risks amongst which are the following:

i. Oil and Gas price risk
ii. Uncertainty in Reserve accumulations
iii. Production rate levels
iv. Decline rate and profile
v. Drilling costs, Well and Pipeline costs
vi. Operating and Maintenance Costs
vii. Fiscal Regimes stability
viii. Geopolitical risk

Discounted cash flow (DCF) analysis considers the asset as an opportunity in which an initial investment is made in anticipation of the generation of future cash flows. By taking time value of money into account, the sum of the discounted cash flows less the initial investments or any other investment made is deemed the value of the asset.

Other measures of asset worth commonly used include rate of return (ROR), discounted profit to investment (DPI), discounted payback and growth rate of return (GROR). As the NPV relationship shows, the discount rate used has a strong influence on asset value, especially for long duration projects, like exploration and development projects. The selection of the appropriate discount rate therefore is critical to the evaluation process.

The issue of what discount rate to select is one of the major flaws of DCF analysis. What should the discount rate reflect? Should it be cost of borrowed capital or should it be the reinvestment rate, since every project embarked upon ought to match at least the historical rate of return for the firm? To complicate matters, early practitioners of DCF analysis intuitively saw a convenient and simple means to account for risk by adjusting the discount rates. A project perceived to have a high risk attracted higher discount rates. However, the adjustment process was very subjective, mostly arbitrary and entirely dependent on the analyst. Attempts at a more formal process of selecting discount rates include the Capital Asset Pricing Model (CAPM). Sensitivity analysis and Monte Carlo Simulation techniques extend DCF analysis beyond providing single point estimates of value but still do not account for the individual's decision maker's perception of and attitude towards risk.

1.3 Special Characteristics of Petroleum Assets

Reserves

Most upstream oil and gas projects share certain properties that strongly influence the pattern and behavior of cash flows and hence asset value. An initial investment is spent on exploration activities including surveys, geological and geophysical interpretations, wild cat and appraisal drilling. Considerable time is spent by geologists and engineers trying to quantify exploration and development risks viz:

i. Dry Hole Risk
ii. Discovery Size -Commercial or Non-Commercial
iii. Recovery drive mechanisms that have significant impact on production and decline rate
iv. Resource Classification – Proved, Probable, Possible

The industry standard classification of reserves is the Society of Petroleum Engineers (SPE) Resource Classification Framework developed in collaboration with the World Petroleum Council (WPC). Reserves are grouped into three categories, Proved, Probable and Possible categories according to the degree of uncertainty surrounding the discovery. Proved reserves are accumulations that can be produced with in-place facilities and at current economic conditions, and hence have the least uncertainty about them. The possible reserves category has the most uncertainty, while probable reserves require more work (or uncertainty resolution) to move them into the proved reserves category.

Production Rate

The rate of production and hence the eventual value of an asset is strongly influenced by the development strategy adopted. A field development strategy is determined from knowledge of the reservoir engineering principles and mechanisms identified as being predominant in the discovered field. There are serious investment costs implications arising from a choice of development strategy – primary recovery versus enhanced recovery, solution gas drive and water injection or water flooding. Recently, Deep Water assets have benefited from horizontal well technology enabling high production levels and minimizing development well costs. Used in conjunction with pressure maintenance and artificial lift from onset of production, horizontal well technology have enhanced the value of these assets by mitigating the serious negative impact (in terms of time value of money) of the large upfront capital outlays deployed long before any revenue generation from production . Rate acceleration projects, applicable to newly discovered or already producing fields and achieved mostly through infill drilling, create economic value by shifting future cash flows forward in time. In combination with price, production strategy can have a significant impact on eventual asset value

Price Volatility

Commodity prices exhibit two common types of price fluctuations– "Random Walk" and "Mean Reversion". A "Random Walk" price fluctuation tends to wander off rather than return to its starting point, any chance departure from existing price levels tends to become permanent (Smith). Mean reversion, on the other hand, describes a process whereby the commodity price is self-correcting largely through the mechanism of demand and supply. Oil and gas prices tend to be mean-reverting, the fundamental forces of supply and demand tend to outweigh and outlast the other variables in the price equation.

The impact of development strategy on value of a barrel in place is easily demonstrated by a slight modification of the analytical representation of NPV. By assuming exponential production and reasonably long field life, the in situ value of a barrel, Vi is shown to be (**Smith 2003**):

$$V_i = {}^{NPV}/_{N_p} = \frac{D_i}{(D_i+i)}(P - C_p) \quad \ldots\ldots (1.1)$$

Where D_i is the production decline rate, P is the Price ($/barrel), and C_p is the production costs ($/barrel)

Though highly simplified, Equation 1.1 shows the value of reserves is a function of not only of the market price and production costs, but also of the decline rate, D_i which is determined by the rate of exploitation of discovered reserves, itself a function of the development strategy adopted. Using typical values of 10% per annum for Di and i respectively, the in situ value is of a barrel of reserve is given by

$$V_i = \frac{1}{3}P \quad \ldots\ldots (1.2)$$

This is the prevalent rule of thumb in reserves evaluation generally expressed as the "value of oil in the ground is approximately 1/3rd the well head price. An alternative development strategy that doubles the rate of extraction or decline while leaving the other variables in Equation 1 unchanged leads to a new unit value of a barrel ($/bbl):

$$V_i = \frac{4P}{9} \quad \ldots\ldots (1.3)$$

Value has thus been increased by an additional *P/9* just by the choice of development strategy which must of course; factor in the additional cost to achieve the incremental production. Value therefore depends on management's willingness to identify alternative development concepts and production strategies, and the ability to adapt flexibly to changes in the economic environment (**Smith 2003)**. This particular instance of aggressive exploitation is typically referred to as production rate acceleration in the petroleum Industry.

The problems with DCF analysis (difficulty in choice of discount rates, inflexibility in timing of investments, etc.) therefore led to the quest for other valuation procedures and particularly, for a formal treatment of risk in investment analysis. The first formal procedure entailed "playing" "what if" with the most significant variables in the discounted cash flow model. Sensitivity analysis (of the important project variables) helps to define a "band" for expected profitability expected from the investment under consideration. Where the levels of the assumptions for the important variables can be specified and grouped (as cases or scenarios), the relevant profit measures for particular scenarios can be determined – this is called (scenario analysis). In complex projects with very many variables that can change quite drastically, sensitivity analysis has serious limitations since a lot of data is generated with little insight into the actual risk level of the investment.

1.4 Expected Value (EV) Approach

Developments in the application of probability theory to financial problems offered new insights and broadened the analysis of investment risk. In particular, the data generated from sensitivity analysis of the deterministic model will possess a mean and a variance. The mean is a measure of central tendency of a group of data, while variance is a measure of dispersion of project worth measure. The Mean-Variance or Expected Value (EV) approach was therefore a logical sequel to the deterministic DCF and sensitivity/scenario analysis. The Mean-Variance approach not only takes into account the value created from the investment, but also the distribution of those values. The (E-V) approach stipulates that if we can specify the various discrete outcomes of a chance event, their payoff values and probabilities, the expected value (expectation or mean) is the probability weighted average of the payoffs contingent on each of the outcomes. For continuously distributed random variables, the means and variances can be estimated by integration of the appropriate probability density functions.

In statistical terms, the expected value (or mean) is the first moment of the distribution of outcomes of the investment, while the second moment is the variance. These are the two most important parameters of project worth and convey to the investment analyst, the measure of central tendency of the distribution of project worth, as well as the variability about the mean. Higher moments, such as the skewness or third moment (of the distribution) can be estimated, provide their own insights but are rarely used in most investment analysis of risky prospects.

The “Mean Variance” approach stipulates that where two projects or investments have the same expected value, the one with the lower variance should be preferred. Conversely, if two projects have the same measure of variability or variance, then the investment with the higher Expected Value should be preferred. This leads to the Markowitz Efficiency criterion which states that in a portfolio of projects, there is an efficiency frontier which satisfies the two rules: expected value and variance.

A glaring pitfall of the expected value approach is that it does not address individual preferences for risk. All decision makers, regardless of their individual preferences, must choose according to the Mean-Variance criterion, it stipulates. Secondly, expected value predicts that 100% of a project should be invested in since its premise is the maximization of (expected) value. This ignores the capacity of the individual investor to accommodate risk. The assumption of risk neutrality is a major flaw of the Mean-variance approach. Investors whether they be individuals or firms, ***have different capacities for risk***, which is a function of their wealth positions. Risk neutrality is only one class of investor risk behavior. The Risk Adjusted value (or Certainty equivalent) approach helps to correct for this flaw in the expected value approach.

Thirdly, the EV approach fails to differentiate between upside and downside deviation in measuring investment performance. For instance, the commonly used Performance Index (PI) metric is used to rank projects based on the estimation of the mean and standard deviation. PI is expected value per unit of standard deviation. However, PI is based on total variance and the failure to distinguish between upside and downside deviation sometimes leads to a paradox in which high value creating prospects are ranked low because of their higher upside variances.

A precursor to the risk adjusted value approach is the concept of risk sharing or simply taking a fractional part of a prospect particularly in exploration economics where dry hole risk is a major consideration. Arps (1974) pioneered the “Gambler’s Ruin” concept as the probability that the entire exploration budget is exhausted through a series of dry holes before the next discovery is made that will recoup earlier losses. Risk control, then takes the form of increasing the odds of a discovery while minimizing Gambler’s Ruin. From Arp’s pioneering study, taking a fractional working interest became accepted as a viable risk control strategy. This strategy has assumed the utmost significance with exploration moving into harsher and more challenging (riskier) Deep water environments requiring substantial upfront investments. Risk sharing arrangements and partnerships therefore proliferate in all phases of the industry. The Gambler’s Ruin concept however, ignores individual investor risk preferences, which is the major focus of this research study. In the expected utility maximization approach, the concept of optimum working interest determination not only addresses risk sharing, but also ensures that the working interest chosen results in the maximum risk adjusted value (RAV). RAV thus guarantees all of the benefits of Arp's Gamblers Ruin concept as well as investment value creation of expected value/expected utility maximization approaches.

1.5 Risk Adjusted Value (RAV) Approach

Individual risk preferences should be factored into investment risk analysis since it has long been recognized in the field of economic behavioral research that individuals have different "appetites" for risk. While some individuals actively seek risk, most are risk averse. Risk preference theory, first enunciated by Jon Neumann and Von Morgenstern (1944) in their seminal publication, Theory of Games and Economic Behavior is the most consistent and comprehensive theory of risk which has seen some application particularly in the oil and gas Industry. The Von Newman-Morgenstern's frame work begins with the premise that the individual's fundamental measure of wellbeing is quantity of wealth which can be represented by an analytic (utility) function representing that individual's preference for risk. A utility function must have a unique inverse; hence it must be a monotonically increasing function of the independent variable (wealth). Wealth is converted into utiles for each outcome in a risky investment and expected utility (the probability weighted average of the utilities of each outcome) is estimated. The fundamental premise of the RAV approach therefore is expected utility maximization. Expected utility is converted back into monetary values or its certainty equivalent (CE). The certainty equivalent (CE) for risk-averse investors is less than the expected profit (or expected value) by an amount equal to the risk premium. The risk premium is the amount by which the expected profit is discounted by the decision maker.

Utility functions employed to model investor risk behavior generally fall into three categories: Risk seeking, Risk averse and Risk neutral. Most investors are risk averse or risk avoiding. In rare cases, hybrid utility functions are used to model the dynamic nature of investor risk behavior. Cozzolino extensively used the axiomatic approach of the Von Newman framework to conclude that the exponential utility function was the appropriate form of utility to use when analyzing financial risk. He showed that exponential utility functions can be used to model a wide range of investor utility function forms and demonstrated its equivalence to the mean variance efficiency criterion of Markowitz. A particularly desirable characteristic of the exponential utility function is the ease with which it can be converted back into actual monetary values or certainty equivalents. However, other researchers have demonstrated the use of utility function forms different from the exponential. A particularly worrying aspect of the exponential form is the "exaggeration effect" in high loss situations. The hyperbolic utility function corrects for this – by a more gradual sloping profile in high loss situations – the practical interpretation being that investors are willing to accept some level of risk in particular types of investments, such as in the oil and gas business, and their decisions ought to be based on informed risk taking.

Various forms of the exponential and the hyperbolic utility function are used to model investor risk behavior. Other forms are also used including hybrid utility functions, but the two are the most common because they describe to a large extent commonly observed investor risk behavior. Figures 1a and 1b show comparison of a form of the exponential model with the Hyperbolic Model. For the risk neutral (or expected value) investor, the utility function is a straight line through the origin. Wealth is plotted on the x-axis while the utility values corresponding to the payoff outcomes are plotted on the value axis for different risk tolerance values, R. Expectedly, the higher the value of R, the less concave the utility profile. The focus on this study is on the risk avoiding investor, since most investors fall into that class.

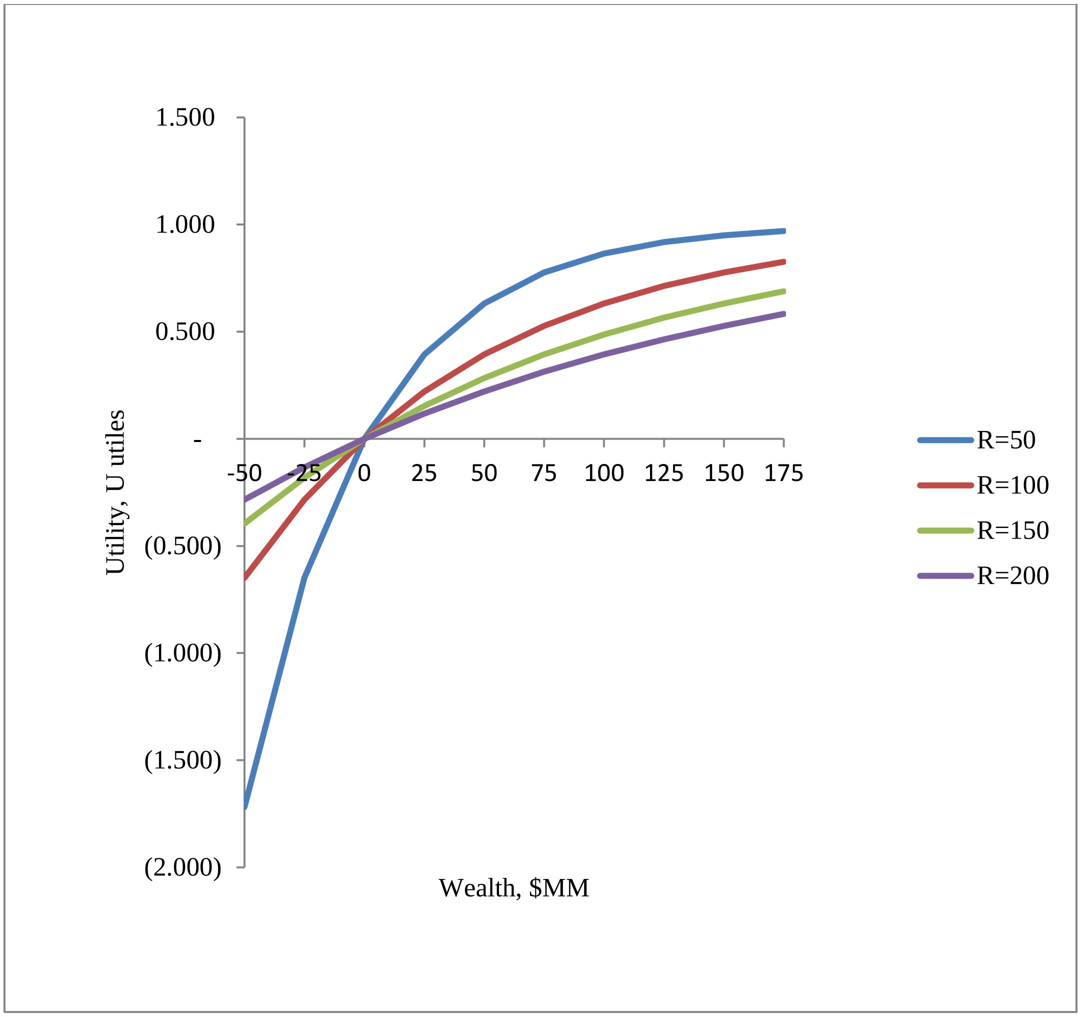


Figure 1.1a Exponential utility function - $1\text{-}e^{-x/R}$ for different Risk Tolerances, R

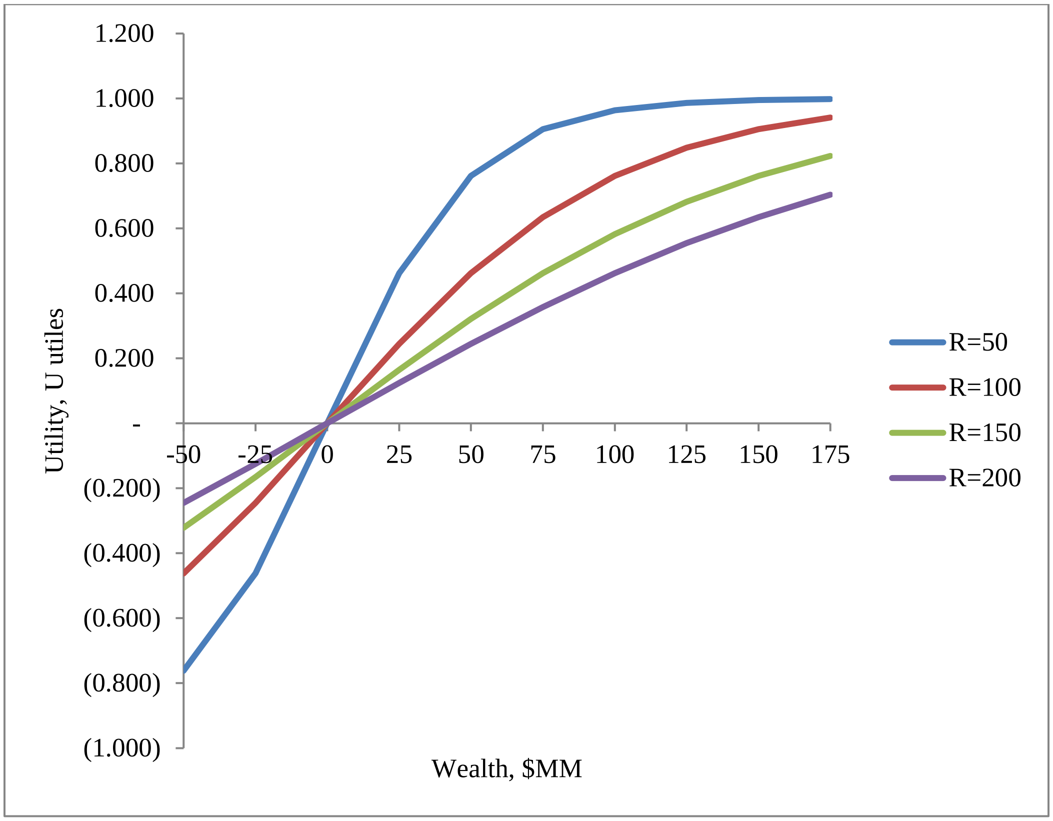
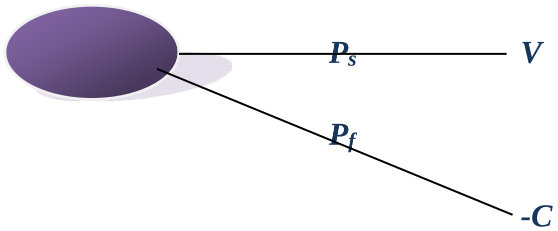


Figure 1.1b Hyperbolic utility function, Tanh(x/R) for different Risk Tolerances, R

For high loss situations, the hyperbolic model is more stable and likely more reflective of oil and gas investor behavior that accepts some measure of risk is inherent in the business and are willing to accommodate such in their investment decision making.

Consider a simple two outcome prospect



Specifying the fractional working interest as, W, the expected present value of the prospect is

$$EPV = W\left[P_s V - P_f C\right] \quad \ldots\ldots (1.4)$$

Using the exponential utility function to model risk, estimation of the certainty equivalent (CE) or the risk adjusted value (RAV) is a simple inversion also of the exponential form and is given:

$$RAV = -Rln\left\{P_s e^{-\left(\frac{WV}{R}\right)} + P_f e^{\left(\frac{WV}{R}\right)}\right\} \quad \ldots\ldots (1.5)$$

The risk adjusted value, RAV is a nonlinear function of Working Interest (W) – thus there is an optimum working Interest:

$$W_{opt} = \frac{R}{V+C}\ln\frac{VP_s}{CP_f} \quad \ldots\ldots (1.6)$$

However, optimum working interest, W_{opt} is a linear function of risk tolerance, R for specific V, C, and P_s

The risk adjusted value at optimum working interest is:

$$RAV_{Wopt} = -R \ \ln\left[P_s K^{-V/(V+C)} + P_f K^{-C/(V+C)}\right] \quad \ldots\ldots (1.7)$$

Where $K = \frac{VP_s}{CP_f}$

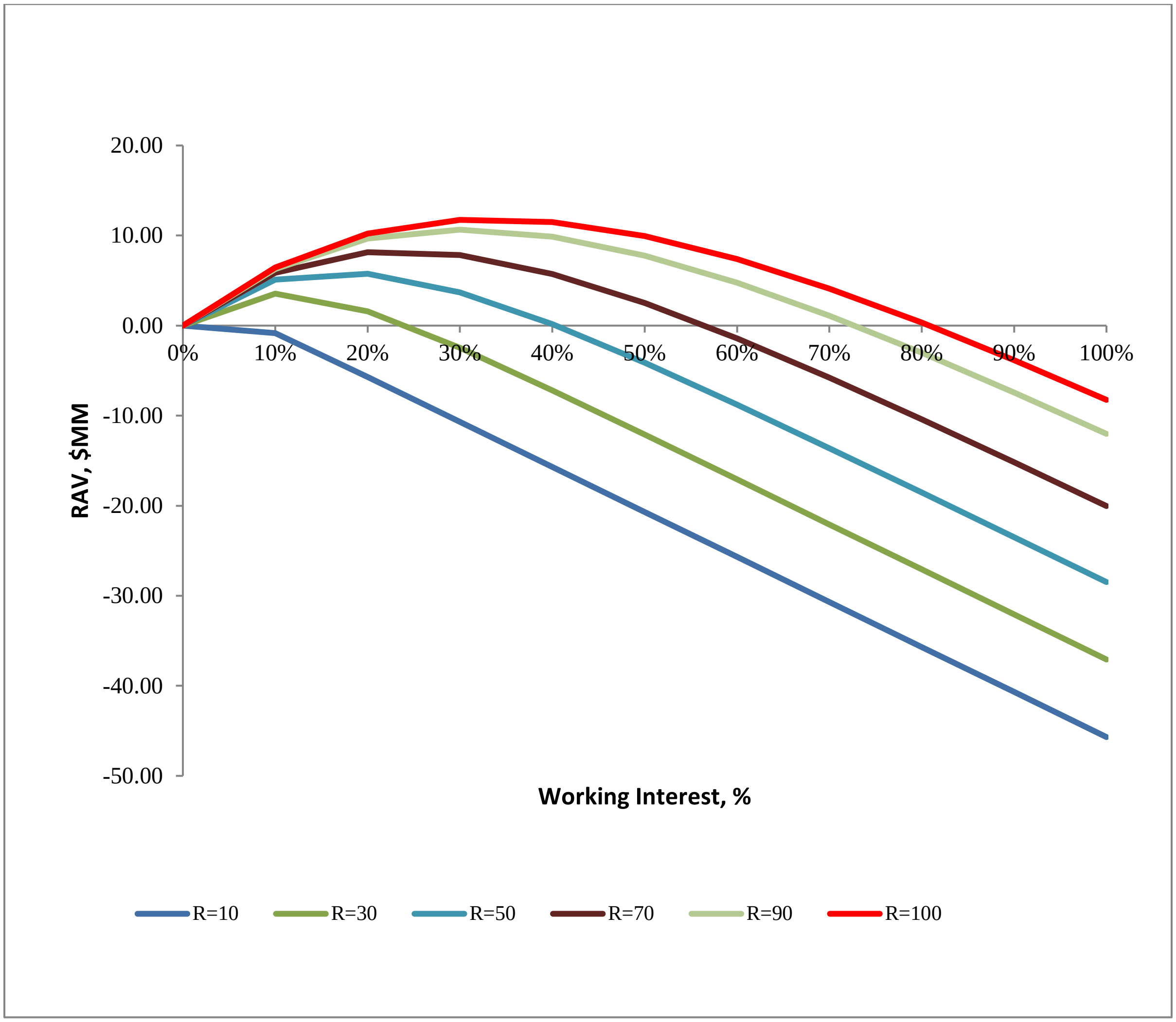


Figure 1.2 RAV versus Working Interest, WI- Exponential Model

With Equation 1.5, the risk adjusted value (and hence the certainty equivalent) can be estimated for different working interests and the optimum working interest determined easily. The apparent risk tolerance, break even working interest, and maximum risk tolerance can all be determined from the RAV relationship shown in Equation 1.5. A plot of risk adjusted value (RAV) versus working interest for this prospect is shown in Figure 1.2 for different risk tolerance levels.

Lerche and Mackay (**1993**), in their investigation of the use of the hyperbolic model, used the hyperbolic function to model risk but exponential form for inversion into real monetary values and derived the following relationship

$$RAV = -Rln\left\{1 + P_f \tanh\left(\frac{WC}{R}\right) - P_s \tanh\left(\frac{WV}{R}\right)\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.8)$$

The plot of Risk Adjusted value (RAV) versus working interest for this prospect using the hyperbolic model is shown in Figure 1.3.

The magnitude of the difference between calculated risk adjusted values (RAV) using the exponential and hyperbolic models is plotted in Figure 4. The figure shows that for most working interest values, the hyperbolic model gives higher RAV values than the exponential model. This is expected since the exponential utility function exaggerates risk aversion, particularly in high loss situations and the useful attribute of the hyperbolic model is its greater stability in the management of high loss scenarios.

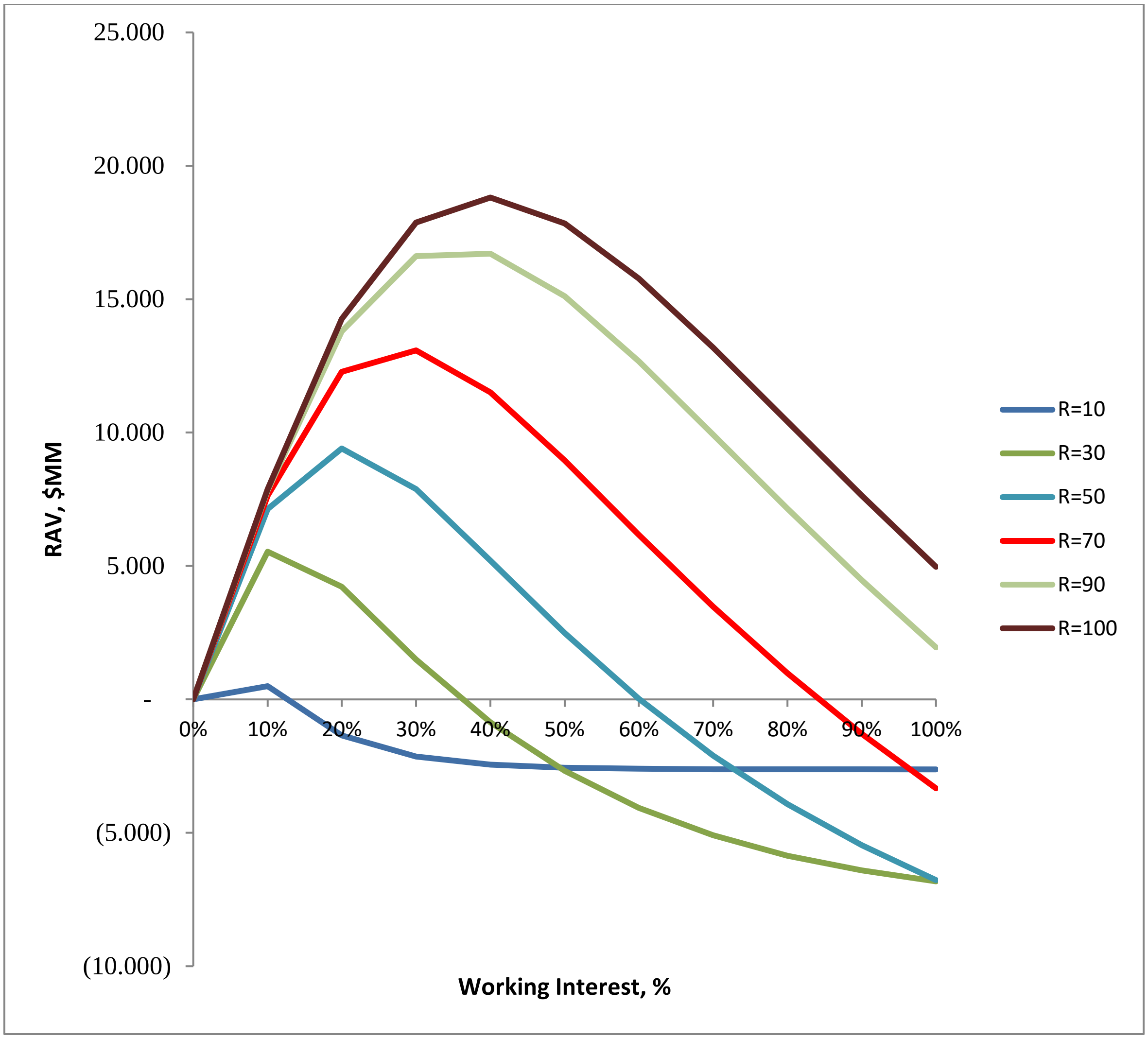


Figure 1.3 RAV versus Working Interest – Hyperbolic Model

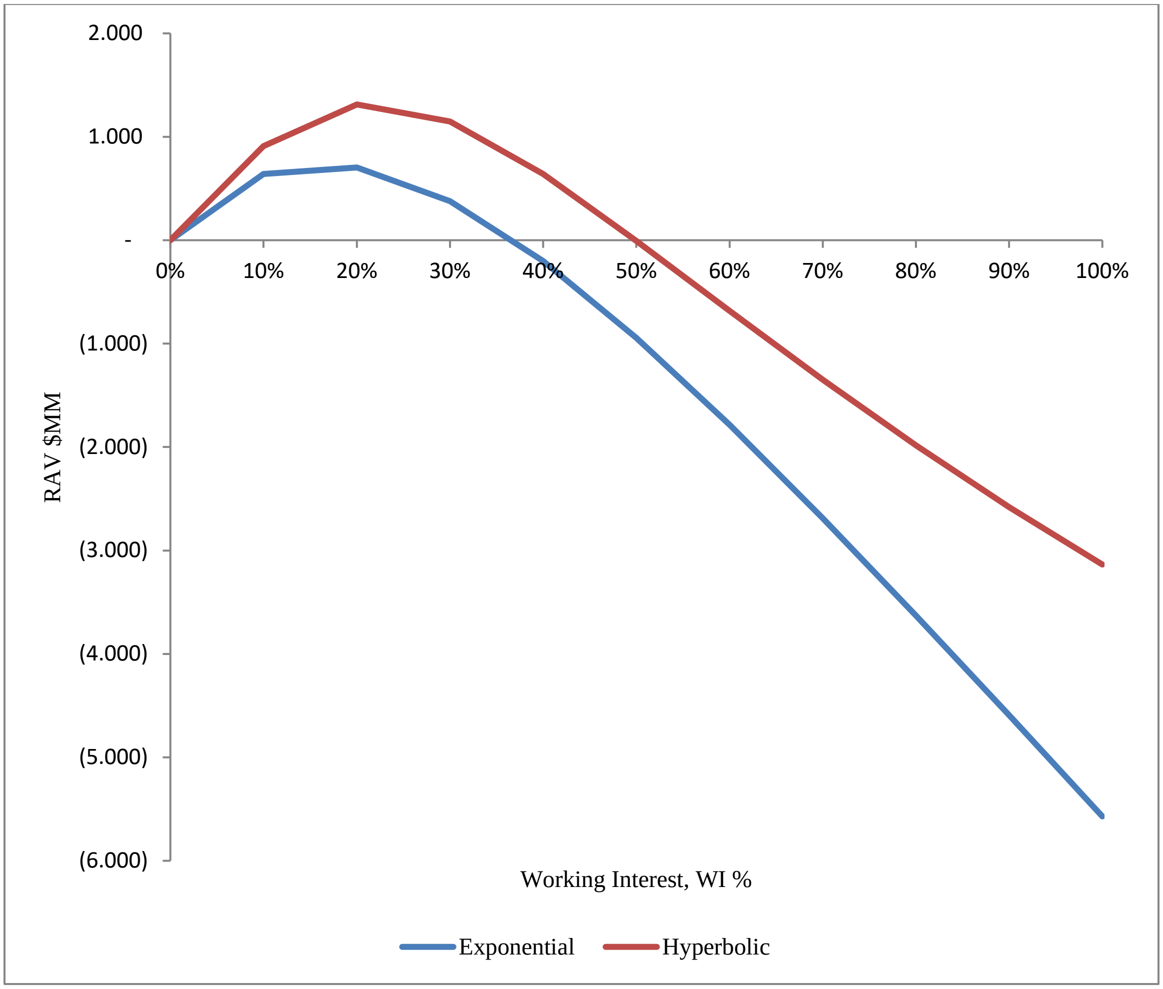


Figure 1.4 Comparing Exponential and Hyperbolic RAVs versus WI

As in the risk neutral situation, the expected utility maximization approach suffers from the Paradox of aversion to incremental reward (PAIR) because the working interest relationship is also an inverse function of total variance, hence it also does not distinguish between upside and downside variance. Intuitively, reward ought to be desirable to the investor, but the resultant increase in variance informs taking not more than some optimum working interest in RAV analysis. Both the exponential and the hyperbolic RAV models exhibit PAIR and hence do not properly reflect investor behavior in High Gain situations. To correct for PAIR, the RAV models developed using the utility functions will have to be modified. This is the motivation for this research – exploring analytical Risk adjusted value models that can address the "gap" or observed Paradox of Aversion to Incremental Reward in determining the appropriate working interest in the evaluation of risky petroleum investments.

1.6 Proposed Study

The preceding analysis highlight the issues that are worthy of study in the application of the specific form of the utility function to model risk in oil exploration and development investment analysis and in particular, the determination of the appropriate working interest (WI) to "take" in these ventures. The hyperbolic utility model corrects for the exaggeration of risk aversion consequent upon the use of the exponential model in high loss situations, because it is more stable in such situations. In high gain situations, both the exponential and hyperbolic models show increasing working interest (WI) participation until a particular value and then, recommend lower participation as gain increases – a situation not consistent with rational investment behavior. A rational investor actually should take more of the assets with increasing level of returns and avoid the Paradox of aversion to incremental reward (PAIR). This paradox is also evident in the risk neutral or expected value maximization context through the use of a measure like the Performance Index (PI)

The following are the specific Research questions that this study will address:

I. The proposed study will demonstrate analytically the Paradox of aversion to incremental reward (PAIR) both in expected value (E-V) and expected utility (E-U) maximization contexts and its impact in investment decision making in risky prospects. The study will show why ignoring PAIR will lead to incorrect investment decisions.

II. The study will explore ways to correct for PAIR in EV and EU contexts. In an E-V context, The Performance Index measure will be modified such that it distinguishes between upside and downside deviations. For expected utility, the Risk Adjusted Value (models) will be modified to correct for PAIR and applied to determine working interests for individual prospects as well as for a portfolio of prospects. The modified RAV models will also be applied for ranking prospects in unlimited and limited capital allocation situations

III. The investigation will also focus on how the hyperbolic model compares with the more popular exponential model in modeling investor risk behavior. The hyperbolic model is frequently cited for its greater stability in management of high loss situations indicating an appreciation of the inherent risk in oil and gas exploration by the investor.

IV. The application of Risk Adjusted Value Analysis to the design of Bids in the Licensing of exploration blocks will be investigated to explore optimum acreage size. From the point of view of Government- the optimum acreage size should seek to maximize returns (signature bonuses) to Government.

1.6.1 Justification for Study

Oil and gas continues to be the bedrock of modern economic development supplying the energy for manufacturing and the transportation of goods and services. It is not a coincidence that the most advanced developed economies have the most energy intensive economies. While the negative impact of fossil fuel use will continue to spur research into cleaner forms of energy, cheap oil for energy, transportation and manufacturing will ensure that demand for petroleum will continue for several decades to come especially as most Governments try to lift their populations from poverty through economic development. Additional reserves will therefore be needed to satisfy World energy demand which will come from exploration and development of new fields. New investments required will be based on sound investment analysis judgment: a careful balance (trade off) between the amount of value created and the attendant risk involved. Current commonly used measures like the Performance Index (PI) and working interest determination through the use of preference models suffer from the Paradox of aversion to Incremental Reward (PAIR). The Risk adjusted value approach, as a formalized and systematic approach to handling risks so pervasive in the oil and gas industry, especially in the exploration and development phase combines the advantages of value creation and incorporation of individual preference towards risk. The capital intensive nature of the oil business makes the consideration of bankruptcy or Gambler's Ruin imperative and firms always have to explore the various risk mitigation or risk dilution strategies at their disposal: from taking fractional working interests to Farm-out and Joint ventures deals, and other contractual agreements including Production sharing and sole risks contracts.

In the Niger Delta, the current trend towards Production sharing contracts (PSCs), especially in the development of high risk deep water prospects was not only informed by the rapidly increasing Cash call arrears owed operators by Government, but also a realization that Government does not have the expertise to deal with the multifarious risks involved in exploration and development phases of the business. More and more of these risks will have to be addressed as the larger and larger investments increasingly are deployed to much more challenging environments (deep water areas). Research is therefore needed to improve the appropriateness of commonly used investment performance measures and correct their limitations. PAIR is one of these limitations and this study directly focuses on correcting for PAIR both in the EV and EU maximization contexts.

1.6.2 Relevance/Merits of Proposal

A formal treatment of risk will always be of relevance to prudent investment analysis especially in the oil and gas business requiring large upfront capital investments that might significantly impact the long term survival of a firm (Gambler's Ruin considerations). The use of hurdle rate is too subjective and arbitrary in the context of today's investment environment and the disadvantages of DCF analysis have been well documented. The requirement for computing resources that hitherto precluded the use of probabilities and simulation techniques have been virtually eliminated with the availability of powerful and cheap computing resources in the employ of most investment analysts.

Risk Adjusted Value Analysis also has very interesting potential applications beyond correcting for the Paradox of Incremental Reward. RAV analysis can form the basis for bid and licensing round design, and the allocation of working interests to the different partners in a joint venture. On a Country level, a risk adjusted value analysis of prior project decisions can potentially yield insights into the risk tolerances of firms involved in these projects – information useful to

Governments in the event that a particular class of (Risk) firms is desired to be targeted for new acreage development, fiscal regime design and overall long term management of a Country's petroleum industry.

CHAPTER TWO

LITERATURE REVIEW

2.1 Asset Valuation

The Literature on Petroleum Asset Valuation is quite extensive. Since the inception of the Oil and Gas Industry around the 1850s, there have always been questions on how much to pay for the rights to drill on a land containing possible deposits, incidental compensations to the owner of the land, or the exchange fair value of an asset offered for exchange. Petroleum Property Valuation is the systematic procedure by which the commercial value of oil and gas fields is assessed and provides the "Fair Market Value" of deposits to prospective buyers, sellers, lenders and tax assessors. What makes the valuation complex and challenging is the realization that a petroleum asset is a "long life" asset whose productive life will span several years in future and as such, the impact of possible future events have to be factored somehow into the valuation process. Since the future is largely uncertain, the valuation process is therefore *inherently* risky. It is not surprising therefore that the ultimate value derived from an evaluation process consists of two parts: a deterministic valuation (that encompasses one point values of the major variables in the economic model) and a probabilistic (risked) which takes care of the deviations from assumed values of these critical variables.

The value of a Petroleum asset is subject to various risks including

i. Oil Price Risk
ii. Uncertainty in Reserves
iii. Production and Decline Rates
iv. Drilling, Well Costs and other Capital Costs
v. Operating and Maintenance Expenses
vi. Fiscal Regimes
vii. Country or Region Political Risk

This literature review will therefore explore the evolution and practice of evaluating petroleum assets along two paths – the ***Deterministic*** approaches (Fair Market Value and Income approach or Discounted cash flow analysis (DCF) and the more recent ***Probabilistic*** approaches – Expected Value, Expected Utility and Mean-Variance approaches that include a formal assessment of the risks involved in a project through the use of statistical decision theory (probabilities).

While various approaches have been employed over the years to arrive at the value of a petroleum asset, the most popular deterministic valuation methods have been the "Fair Market Value" (FM) approach and the Income approach or the Discounted cash flow (DCF) analysis. "Fair Market Value" is defined as "the most probable price, in cash, or terms equivalent to cash, or in other precisely revealed terms, for which the specified property rights should sell after reasonable exposure in a competitive market under all conditions requisite to fair sale, with the buyer and seller each acting prudently, knowledgeably, and for self-interest, and assuming that neither is under undue duress (**Smith2003**)

Some of the salient features of the "Fair Market Value approach includes but not limited to the following:

i. It represents an arm's length transaction between buyer and seller, who are both deemed knowledgeable enough to conduct such transaction
ii. Neither the seller nor the buyer are under any obligation to sell or buy
iii. It is assumed that the asset is placed in a competitive market – freely accessible to all potential buyers for a reasonable period of time
iv. Market value is only that value transferable from one typical owner to another or private market value
v. The price paid for a similar property in an arm's-length transaction is accepted as the best evidence of fair market value lacking which, a capitalization of the property's likely net earning power may be used to estimate its market value, in accordance with a competitive market concept.
vi. Royalty streams, where applicable are included as part of Market value

Discounted cash flow analysis involves the estimation of annual net income from the expected annual costs and revenues associated with the development of the oil and gas rights or property under realistic conditions. Annual net cash incomes are then discounted to their present value. Thus, the rights or property's net income potential, discounted to the present, provides an estimate of current tract sale value if similar tract sales data is not available

Thus from the definition given; the net present value of an asset can be shown mathematically thus:

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+i)^t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.1)$$

where the annual net incomes are cash flows in discrete time intervals, t and i is the discount rate If the cash flows are received and discounted on a continuous basis, the net present value will be shown thus:

$$NPV = \int_0^T CF_t e^{-it} dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.2)$$

From the functional relationships, the net present value is impacted not only by the size of the cash flows and their timing, but also by the discount rate used. The choice of what discount rate to use has always been a contentious issue. Should the discount rate reflect the cost of capital for field development or should it be the historical reinvestment rate for the investing firm or individual, since one investment will cost one the opportunity of investing in another asset, especially in a limited capital environment. A much more fundamental problem with the DCF method has to do with investors using the discount rate to reflect the risk of the investment. An investment that is perceived as risky has the streams of future incomes discounted at a higher or "hurdle rate". The choice of discount rate is thus wholly dependent on the perception of who is conducting the analysis –completely subjective and arbitrary, instead of a formal systematic assessment of the project's risk. In spite of these problems, the discounted cash flow method continues to endure as a valuation process in industry because of its simplicity, and ease of estimation of several popular project metrics like net present value (NPV), the discounted profit to investment (DPI) and rate of return (ROR).

Various methodologies have been proposed to formally estimate the appropriate discount rate to be used in DCF analysis, since the selection of discount rate is critical to the valuation process. Some of these formal methods include the Capital Asset Pricing Model (CAPM), which makes the distinction between diversifiable risk and non-diversifiable risk. Diversifiable risks do not contribute to the risk premium. A substantial portion of exploration risks are diversifiable. It is also worth noting that the single discount rate is a composite of all the interacting risk factors – a situation that does not lend itself to useful risk control and risk mitigation.

2.2 Unique Characteristics of Petroleum Properties

The pattern and profile of cash flows of most petroleum assets and hence subsequent value are determined by a set of unique characteristics. There is usually a long lead time between the commencement of the exploration process, discovery and first oil production, making time value of money a major consideration in asset valuation. In addition, most assets have fairly long production lives, exposing investments through several cycles of oil price highs and lows. Generally, the major risks can be categorized broadly as technical, commercial and political risks.

i. **Exploration and Development Risks** – these are the major technical risks that fundamentally define the oil and gas business. These risks demand that highly skilled specialists are committed to conducting detailed geological and geophysical work to arrive at a formal assessment of the risks involved in a particular asset. These risks can be further classified as:

    a. **Dry Hole Risks** – the risk that in spite of the best effort of the specialists, the drilling and prospecting outcome can still be a dry hole or a succession of dry holes, putting the entire exploration budget at risk. If all the wells turn out to be dry without a single successful well helping to recover the exploration investment, this phenomenon is referred to as "Gambler's Ruin". It is not surprising therefore those early efforts at formal risk assessment of petroleum assets focused mainly on assessing "Gambler's Ruin" probability and designing the exploration program to preventing that this occurs – wealth preservation.

    b. **Discovery Size-**This is the next major technical risk. Petroleum asset economics are invariable production volume dominated, so it is crucial that even when a discovery is made, the size of the discovery has to be deemed commercial at prevailing economic conditions. A Non-Commercial discovery is not worth much to any investing firm.

    c. **Resource Classification** – The Classification of the discovery is also important. Petroleum reserves are classified as Proved, Probable or Possible – each of these categories reflecting the degree of confidence in the reserve estimates. Proved reserves carry highest degree of confidence (95%) while the Possible Reserves represent the least – 10%. Proved reserves are usually used in most economic evaluations.

ii. **Commercial Risk– Oil Price Risk**. Like any other commodity, oil prices are very volatile. The volatility of oil prices, when looked at over a long period of time can take two paths - "Mean Reverting" or "Random Walk" path. Short term fluctuations in the prices of oil and gas at the wellhead are likely to be reversed in due course once the factors of supply and demand leading to the price fluctuations stabilize – this is called reversion to the mean. Random Walk process, on the hand, represents a situation in which the Oil price wanders off completely from the historical path. This reflects a fundamental shift in the market dynamics such as breakthrough in technology (for example the discovery of the electric light bulb at the turn of the 20th displaced oil as the primary means of household lighting).

iii. **Rate of Production** and hence the eventual value of an asset is strongly influenced by the development strategy adopted. The strategy is determined from knowledge of the reservoir engineering principles and mechanisms identified as being predominant in the field. There are serious investment costs implications arising from a choice of strategy – primary recovery versus enhanced recovery, combination recovery – solution gas drive and water inject or water flooding. Recently, Deep Water Assets have benefited from Horizontal well technology that have led to high rate production wells, as well as pressure maintenance and artificial lift from the onset as a result of the Capital intensity of these projects. The rate of production and Oil price determine the revenue streams of the petroleum asset from which the annual costs will be deducted, the appropriate taxes paid to arrive at the cash flows which determine the value of the asset.

iv. **Capital Costs –** Oil and Gas Field developments are very capital intensive. To develop a small size field of 20 million barrels of reserves requires investments that may run into several million dollars. These costs are mainly drilling, well costs, pipeline and other facilities costs. In order not to overrun initial project costs and impair asset value, Oil and Gas Assets development and execution demand the highest standards of project management from conception, execution and operation and project look back to harness lessons learned.

v. **Project Duration –** Most petroleum assets can be classified as "long life" assets meaning the productive life of the field usually will span several years exposing the cash flow streams of the project to various risks – mostly market, technology and political risks. Time value of money thus plays a major impact in the valuation process.

vi. **Fiscal Regimes –** Petroleum assets are located in very many diverse areas of the World with different Governments, political and economic systems. In most parts of the world, subsurface rights such rights to minerals deposits are vested in the Government rather than the Land Owner, with the notable exception being the United States, where the Land Owner has rights to subsurface mineral deposits. Fiscal regimes are combination of royalties and taxes that are applicable in the petroleum industry in any Country. The Fiscal regime usually will be designed to extract the most value from the asset for the host Government. However, Governments also realize that Companies usually multinational companies that invest in these assets must realize some reasonable returns on their investments, otherwise they will not invest. The multinationals explore several investing opportunities around the world, so each petroleum asset is competing for the same investment dollars with assets in other areas of the World.

vii. **Political Risks –** risks pertain to the risk of appropriation of a petroleum asset by a Government (outright nationalization by government fiat) or an agent of Government, such as through a judicial determination in the event of a contractual dispute. Political risks will also include the risk of war or violet conflict breaking out in a region, foreclosing the normal operations of an oil asset.

The major risks in a deterministic evaluation of an Oil and Gas asset are as outlined: Technical Risks, Commercial Risks and Political Risks respectively. These risks must be taken into account, each on its own and in the complex manner in which they interact, in the evaluation process in a logical and systematic manner rather than in the arbitrary and subjective way of using the discount rate as a hurdle rate to account for risk. This is the approach and thrust of this work.

2.3 Real Options Approach

The Real Options approach of evaluation is a recent methodology that was developed in the Finance industry, and in particular, the derivatives or financial options in the equity market. Financial options are options that allow the investor to "bet" the direction that an equity price will follow rather than investing in the actual asset itself. If the "bet" comes good, the investor will exercise the option to invest otherwise, he will not. In the latter case, the loss is limited to the value of the option, so the advantage over the DCF approach is the managerial flexibility and value of information derived from "waiting". By updating information on the uncertain variables, for example Oil Price and acting accordingly, management adds some value to the valuation. The evaluation process is called "Real Options" valuation because the process evolved from financial options and the methodology carried over into "real assets" like oil and gas. The options approach provides some explicit treatment of the relationship between the Lease life (or Lease duration), degree of price volatility and asset value. Various studies have sought to demonstrate the applicability of the Real Options approach to the particular case of petroleum asset valuation. However, this methodology is still very new and the literature is limited but growing. It is expected that, as more research is conducted in this area, more insights will gained into the still vexing questions that DCF analysis has failed to answer thus far.

2.4 **Sensitivity Analysis**

Discounted cash flow analysis (DCF) has the implicit assumption that all the variables in the DCF model will come to pass as estimated. The outcome of the model or output measures of project worth – net present value (NPV), discounted profit to investment (DPI) or internal rate of return (IRR) are one point measures reflecting the assumptions regarding each of the important variable in the model. Because investments are made in the expectation of future returns, the futuristic element in valuations informs that there will deviations from the expected in some or all of the variables, regardless of the best efforts in project conception and implementation. Project costs will vary either through inaccurate data estimation or poor project implementation. Oil and gas prices are subject to the vagaries of the international market that is dependent on a complex combination of fundamental market considerations and geopolitical risks. The technical risks are especially problematic resulting in variation in the estimation of actual reserves and production volumes which are the key inputs in deriving the revenue generation levels of the asset under consideration. Ultimate recoverable reserves and (hence production) are dependent on several geological and engineering factors that are difficult to determine precisely even with current technologies. Reserves figures are continuously being updated and refined by petroleum engineers as field data become available and extensively analyzed. Proved reserves, used in economic evaluations have the least degree of uncertainty, while possible reserves have the largest suggesting a lot more geological and engineering work still need to be done to make them useful for asset valuation.

The usual procedure of ensuring that estimates of the key variable are incorporated in asset valuation is by identifying the key variables in the DCF model and deliberately varying them from their expected or mean values to determine their relative impact on project outcome parameters. This process is called sensitivity analysis. The analyst determines the upper and lower bounds of the key variables in the model and runs the DCF model several times with the changing variables. The project worth parameters are also therefore determined on three point basis – High, Medium and Low Cases or Scenarios. These sometimes are classified as Optimistic, Most likely or Pessimistic – reflecting the expectations of the analyst or project sponsors.

Sensitivity analysis is limited in usefulness, especially in situations in which there are more than three variables that vary on more than three levels of optimism or pessimism. Results proliferate rapidly and have to be organized in order for the investment decision maker to make any sense of it. A DCF model with four important variables varying along three levels of High, Low, Medium results in 4^3 (64) project outcomes. Many Oil and Gas assets usually have more than four important variables that must be accounted for – considerably stretching the utility of sensitivity analysis and suggesting that some other more sophisticated approaches to handle uncertainty need to be employed.

Sensitivity analysis is also limited by the very nature of the process – the impact of each variable of project NPV is considered in isolation of the other important variables. It ignores the impact of the variables acting in concert. It also ignores the interdependencies amongst the variables – in which a change in a variable changes the estimation of another variable. Where the variables are multiplicative in nature, the potential for under or overestimating the project performance metric variation from the most likely estimate will be high. A significant improvement in capturing the effect of different combinations of the key project variables changing from their expected values, either in isolation or acting in concert is through Monte Carlo simulation.

2.5 Probabilistic Approaches in Economic Valuation

The type of risks inherent in the oil business has been previously classified into three: Technical, Commercial and Political risks. Risk is actually quite pervasive in all phases of the industry and especially has a large impact in the Exploration phase where the very survival of the investing firm may be at risk. It is a difficult subject – since it is the study of human decision making in the presence of incomplete or uncertain information. However, a lot of progress has been made in three areas which provide us significant insights into decision making under certainty. These are:

1. Probability theory – this is the cornerstone of risk assessment and the language of uncertainty is probability.
2. Risk or Utility preferences are now better understood. They provide useful insight into the how individuals make decisions under uncertainty
3. Recent advances in the value of information helps us to better understand the role of information in decision making.

2.5.1 Expected Value Maximization

The first formal evaluation of risk is the expected value approach (EV). Expected value is a probability weighted average of the value of the outcomes of a risky event. The value of the outcomes that goes into the estimation of expected value are the same deterministic values derived from either the "Fair Market Value" approach or from DCF analysis. In essence, expected value builds on earlier valuation done using either of the deterministic approaches and therefore extends our valuation analysis. Under a limited capital constraint, it is useful to use the ratio of expected value divided by expected investment, EV/EI. Expected investment is pertinent in a situation where a successful discovery will attract completion costs, whereas dry holes will not be completed (**Newendorp1975**).

The utility of statistical decision theory in economic evaluations of Oil and Gas ventures was recognized as far back as the early part of the 1960s. One of the first papers to highlight this value was **Prudent Risk taking** by Arps (**1974**) who approached the valuation problem using the Gambler's Ruin estimation. Estimation of Gambler's Ruin in a venture signals to the investor the level of risk that the entire exploration budget is exposed to. Since wealth preservation ought to be the first prerequisite of any rational investor, Gambler's Ruin probability estimation invites further action from the investor in terms acceptability or otherwise of the attendant level of risk. At once, it is a first level measure of individual risk preference – some individuals will accept while others will reject. It also invites further action in what might be done to reduce the probability of Gambler's Ruin (risk mitigation and control).

Grayson Jackson (1962) was one of the first to introduce inferential statistics to drilling problems. He illustrated the differences and agreements between classical and inferential statistics using a numerical drilling example. In the classical approach – the problem is posed as a decision between two competing hypothesis viz:

Null Hypothesis: **H_0; Ps > 0.2** and

Alternate Hypothesis: **H_1; Ps < 0.20**.

Rejection of the Null Hypothesis when it is true (that is the firm abandons the drilling program when it is actually profitable) results in a ***Type 1 error*** – missing out on a productive venture. If the firm accepts H_0 when it is false (i.e. Ps < 0.2), that results in a ***Type II error.*** Hence the outcome of a Type II error in this case is loss of drilling well cost. In order to choose between the two hypotheses, the classical statistician will sample to determine the true value of the success probability, Ps and formulate a decision rule to choose, focusing his attention on the choice between rejection or acceptance of the supposedly more serious null Hypothesis, H_0. Acceptance or rejection of H_0 will translate to a decision to "drill" or "not drill" by the investor.

The Bayesian approach will admit into the analysis, the intuition, experience and judgment of the statistician. The approach thus makes use of both objective and subjective information, which the classical approach rejects. The analysis is formalized by jointly considering the probability and the cost of error in an estimation of conditional and unconditional expected losses. The decision rule with the smallest unconditional expected losses is then selected.

Bayes analysis also permits the revision of prior probabilities with new information from sampling or drilling, to estimate new or posterior probabilities. Thus, the decision maker can continue to seek for new information as long as the value of this new information exceeds the cost. The full impact of new information is perhaps best harnessed in the more recent approach of Options valuation that captures the value of waiting for new information or managerial flexibility in investment decision making.

Greenwalt (1981) extended the concept of Gambler's Ruin probability estimation to develop a practical method to determine venture participation in a risky venture. By taking a fractional part of a venture, and participating in many more of such ventures, the investor is reducing the chance of Gambler's Ruin or increasing the odds that at least there will be one success that will recover the losses from prior dry holes. Greenwalt proposed that venture participation is essentially determined through a consideration of the relationship between *venture profitability*, *total risk investment*, *level of aversion to risk*, *probability of success* and *available risk investment funds*.

He began by equating the Probability of financial failure to the probability of all dry holes or $1 - S = (1 - P_s)^N$ where S is decision maker's aversion to risk, P_s is probability of success and N is the number of ventures (Gambler's Ruin) and solved for N, the number of ventures required for a particular level of Gambler's Ruin probability:

$$N = \frac{\log(1-S)}{\log(1-P_s)} \quad \text{(2.3)}$$

Since participation in a venture, F is dependent on available risk investment funds, M, before tax risk investment per venture, I_b and number of ventures, N, thus

$$F = \frac{M \log(1-P_s)}{I_b \log(1-S)} \quad \text{(2.4)}$$

While the approach was simple enough and straight forward, a practical difficulty arose in how to determine quantitatively the level of aversion to risk, S. Greenwalt's solution to this problem was to define a risk capacity, R thus:

$$R = \frac{One\ \ Success}{One\ Success + failures} \quad \text{(2.5)}$$

where the number of failures is determined by venture profitability, P_v divided by the risk investment per venture or after tax cost of failure, I_a.

$$R = \frac{1}{1+\frac{P_v}{I_a}} \quad \text{(2.6)}$$

So if $Ps > R$, the investment will return a profit, whereas if $Ps = R$, then the investment will return only a discounted payout or break even.

If the risk venture anticipates only a payout, then $Ps{=}R$ and $N = \frac{\log(1-S)}{\log(1-R)}$

$$H = \frac{M \log(1-R)}{I_b \log(1-S)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.7)$$

where H is desired working interest

$$\log(1-S) = \frac{M \log(1-R)}{I_b H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.8)$$

Finally, venture participation is given by:

$$F = \frac{H \log(1-P_s)}{\log(1-R)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.9)$$

The determination of venture participation by Equation 2.9 eliminates the necessity of quantifying a level of confidence or risk aversion in terms of S. Decision makers determine their fundamental aversion to risk in this equation by the selection of ***H (desired working interest)*** for any venture that anticipates only a discounted payout. The level of risk aversion changes according to log (1 – R), which is determined by the profitability and risk investment of each venture. An internal review of historical decisions ensures that we have insight into whether there is a consistent level of working interest that reflect the Company's level of risk aversion through a choice of desired working interest. Decision makers using this approach must determine the appropriate value of working interest, H for any venture that anticipates a discounted payout. The approach accommodates changing level of risk aversion through the change of working interest levels.

Greenwalt also demonstrated in his approach how to impose of capital constraint when determining venture participation especially for high cost ventures requiring significant capital outlays and hence higher probability of bankruptcy for the Company. The maximum desired risk investment (CM) is set by using a factor, C (obviously less than 1) on total capital M and working interest H thus becomes CM/I_b.

Thus venture participation in a Capital constrained environment becomes

$$F = \frac{CM \log(1-P_s)}{I_b \log(1-R)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.10)$$

McCray (1969) introduced the concept of repeated trials or simulation into the analyses of the outcomes of exploration economics. He assumed that the probability of complete failure in one trial is high, the probability of success is low and magnitude of occasional success is variable. Just like Greenwalt, he posited that when an exploration program consists of several wells, there is high probability that some discoveries will cover the costs of prior failures and yield some profit. He also assumed that the outcome of an individual well is probabilistic in nature raging from complete failure to success. The objective was to relate the probabilistic character of individual wells outcomes to the character of the entire exploration program through simulation modeling. He in essence, pioneered simulation modeling in exploration economic evaluations.
He used rectangular (uniform) probability distributions to describe reservoir and rock properties like areal extent, net pay thickness and fluid properties (viscosity) since these distribution types only require the specification of upper and lower values. All the other values between these upper and lower bounds are equally probable. The triangular distribution was used to model the financial outcomes of the successes. The expectancy or expected value is thus a probability weighted average of the well costs and the mean of a triangular distribution of the financial value of the successful discovery.

McCray developed mathematical relations describing the expected value of a number of ventures and the standard deviations above and below the mean. There were two types of analysis – firstly, he plotted the number of standard deviations versus the number of ventures on a normal probability graph.

Using a numerical example, he employed the triangular distribution to model resource size, operating income and present values to generate distributions of expected rates of return (RORs) and net present values (NPVs). He then ran several simulations to study the interactions of the various risk variables.

Working on the premise that wealth preservation or the prevention of economic ruin is the primary concern of the individual enterprise, he proceeded to develop analytical equations from the probability distributions generated from his simulation runs that will ensure a predetermined acceptable level of risk, say 5% or conversely 95% degree of success. He then calculated the number of ventures n that will assure that 95% degree of success. It then is quite straight forward to estimate the fractional participation in a venture since "the amount of money invested in any one enterprise multiplied by the number of ventures required for assurance of success should not exceed the total risk capital available" Thus fractional participation is given by:

$$f = \frac{M}{n_{0.95}C} \quad \text{.. (2.10a)}$$

where M is the total risk capital (or exploration budget) and f is the fractional participation, C is the investment in a venture. This is equivalent essentially to the approach by Greenwalt.

2.5.2 Expected Utility Maximization

The EMV concept implies the decision maker is totally impartial to money and the magnitudes of money involved in a gamble – implication he has a large amount of investment capital and can afford any of the potential losses. But we know people are not impartial to money, they have specific attitudes and feelings about money that depend on the amount of money they have, their personal risk preferences and their immediate or long term objectives. These attitudes may change from day to day, and may even be influenced by such factors as their immediate business environment or climate.

Preference theory is an attempt to quantify the individual's feelings and attitudes about money into a preference function that will be used in economic evaluations and decision analysis. The function will have all the attributes of the EMV concept plus the additional value of incorporating individual's attitudes towards risk in economic evaluations

Von Newman and Morgenstern (1944) developed the mathematical basis for preference (utility) theory. Their theory was based on eight (8) axioms which are generally regarded by most researchers in preference theory for their completeness and validity. The fall out of their mathematical treatment of decision maker behavior, however, was the fact that if a decision maker had a value system described by the eight axioms, then their attitudes towards money can be modeled by a mathematical or a utility function.

Utility theory has since found its way into many aspects of economic evaluations and decision making. Several preference curves have been postulated describing investor behavior which can generally be classified into risk averse, risk seeking and risk neutral behaviors. Most people are generally exhibit risk aversion in behavior.

The utility function describes an individual's behavior, so it is entirely descriptive. It does not prescribe an optimal behavior nor does it imply comparison with other individuals' functions. Utility curves are usually monotonically increasing functions suggesting increasing likeness for more wealth but marginal utility decreases (with more wealth). Preference theory has the property of expectation just like expected value, meaning the individual should maximize their expected utility – the probability weighted average of the utilities of the different outcomes in a decision situation.

Preference curves are generally developed through interviews and assessment test of the individual. He is confronted with several decision situations and asked to choose between the alternatives. From his choices, a function can generally be developed. Over the years assessment tests and procedures have improved to the point of standardization. Questions still remain as to how truly the function derived represents true investor behavior and whether there is a dynamic element to this behavior in which case the function may change over time.

Cozzolino (1977) postulated that the preference theory framework, first enunciated by Jon Neumann and Von Morgenstern in their Theory of Games and Economic Behavior is the most consistent and comprehensive theory of risk and wondered why it is not as frequently used as the Mean-Variance Efficiency criterion of Markowitz and other special purpose methods of classifying risk such as Modigliani-Miller – "Risk Class" structure, Probability of Bankruptcy Model and Capital Asset Pricing Model (CAPM) popular in the Finance literature.

He proceeded to use the axiomatic approach to rigorously show that the expected utility model is not only equivalent to the Mean-variance efficiency criterion, but much more comprehensive. He proposed a restriction of the expected utility model in order to gain ease of solution to practical risk problems. Cozzolino's goal was to develop a simple utility framework using three different axioms-all of them uniquely implying the exponential utility form of risk preference must be the appropriate individual risk function. His starting point was: an individual's fundamental measure of wellbeing is his quantity of wealth-that is the argument of the individual's utility function is wealth. Consistent with Von Newman and Morgenstern, he postulated that a utility function must have a unique inverse function; hence it must be monotonically increasing. He then proposed four fundamental definitions thus:

I At initial time profit opportunity is presented, the individual's utility position is the expected utility of terminal wealth

$E\{U(x+\tilde{z})\} = \int U(x+\tilde{z})f(z)dz$ …………………………………………….…. (2.11)

II The Certainty equivalent (CE) of a profit opportunity is the amount of money at which a decision maker would be indifferent between keeping and selling the profit opportunity.

$U(x+\pi_a) = E\{U(x+\tilde{z})\}$ ……………………………………………..…. (2.12)

III The risk premium is the amount by which the expected profit is discounted by the decision maker or the expected profit less the certainty equivalent.

$\pi(x,\tilde{z}) = E(\tilde{z}) - \pi_a(x,z)$ ……………………………………………..…. (2.13)

IV. Local risk aversion $r(x) = -\frac{U''(x)}{U'(x)}$ ……………………………………..……. (2.14)

These four definitions represent a well-known Von Newman – Morgenstern structure for quantifying risk but not frequently used as well as the Mean-variance efficiency criterion. Using the axiomatic approach, Cozzolino proposed a restriction of the general structure to arrive at utility theory representation that possesses the breadth and consistency of utility theory but simple enough for general application for most financial problems. By generalizing a global property of the Mean-Variance criterion– the additivity of the means, μ in a portfolio and showing that exponential form of Utility functions possesses that property when considering certainty equivalents, Cozzolino sought to justify that the preference function to use to model risky situations should be the exponential function. He proved this additivity property (of certainty equivalents) mathematically for the exponential function and also showed the invariance of conditional utility (of the exponential function). With these two restrictions in place, it was straight forward and easy to use the various forms of the exponential function to model risk behavior. However, developing a preference curve for an individual remains problematic, since most assessment methods are subjective and fail to take into account the dynamic nature of risk behavior.

Continuing his analysis, Cozzolino showed the Certainty Equivalent (CE) for the Exponential Utility function can be represented by:

$\pi_a(z,r) = -\frac{1}{r}\ln E(e^{-rz})$ ……………………………………………………..………….. (2.15)

The risk profile of this lottery possesses the following properties:

i. Limit $\pi_a(z,r) = E(z)$ – the intercept of the risk profile is the expected value as risk aversion, r tends to zero

ii. $\pi_a(z,r) = E(z)$ – the Certainty equivalent is a strictly decreasing function of risk aversion, r

iii. Limit $\pi_a(z,r) = E(z)$ – as risk aversion level increases and tends towards infinity, the Certainty equivalent approaches the worst possible outcome

iv. For a profit lottery, E(z) that is greater than zero and some chance of a negative outcome, there exists a critical level of risk aversion, r_c that makes the Certainty equivalent equal to zero

Cozzolino demonstrated his rigorous justification of the use of the exponential utility function with an oil field example of an exploratory well having different risk profiles:

i. A ***Full Interest*** Venture in which the probability of success is 0.2, the present value of a successful well is $1 million dollars and costs consist of $0.02 million for lease bonus and a dry hole drilling cost of $0.17 million.

ii. A ***Dry hole contribution*** offer of $0.0058 million (in the event of a dry hole)

iii. A ***50-50 partnership*** in which all costs and all revenues are shared equally between both partners

iv. A Farm Out with 1/8th Overriding Royalty Interest

v. A Promotion Deal in which an outside investor is willing to contribute 1/3rd of all costs in exchange for ¼ of all revenues

Cozzolino developed risk profiles for all five "Deals" and estimated the critical risk aversion values, r_c to demonstrate his 2-stage analysis procedure citing the advantage that this form of analysis can help maintain consistency of risk preferences when making some very difficult comparison.

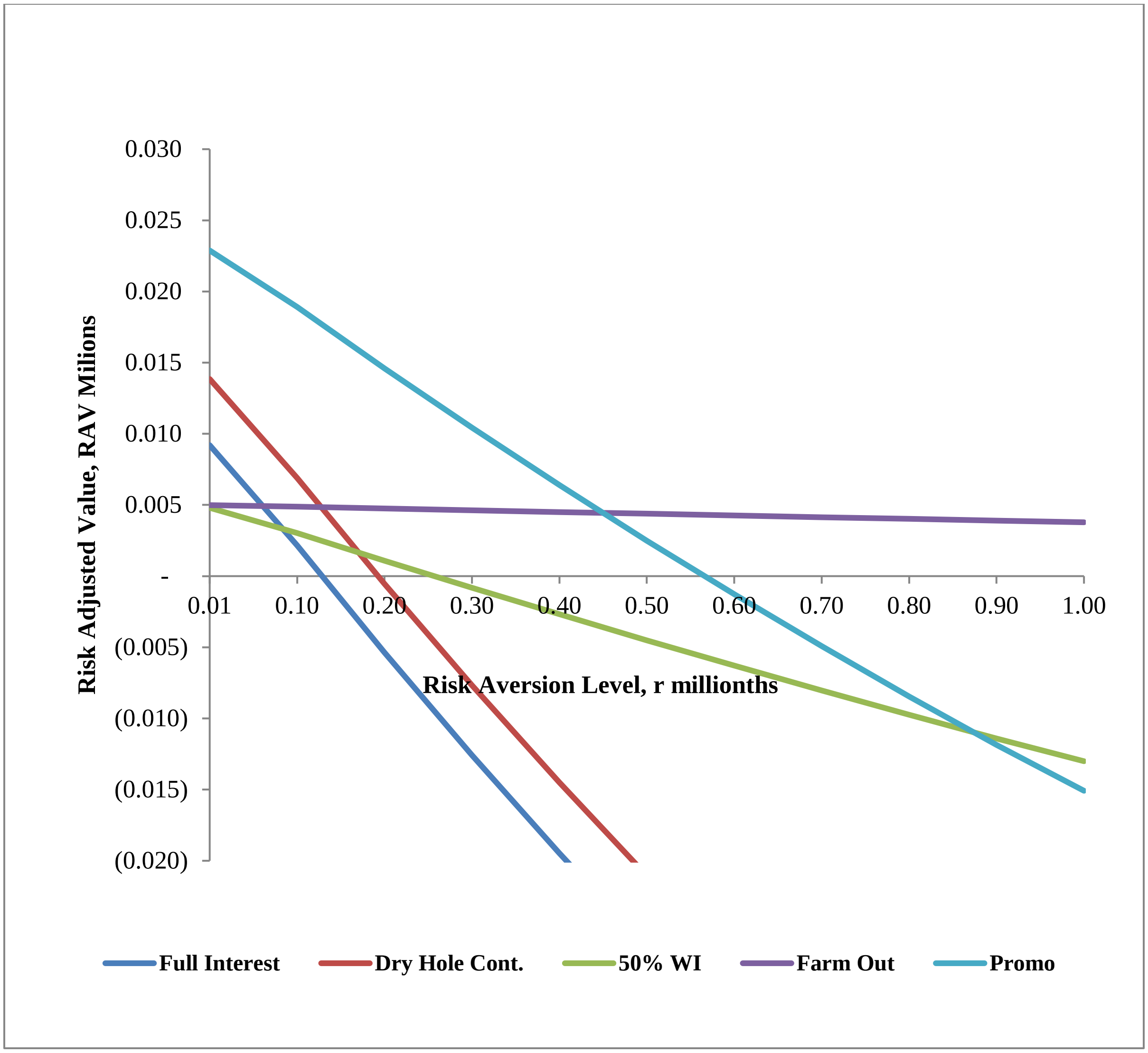


Figure 2.1 RAV Analyses of Five Options of a Lottery (Cozzolino, SPE 1977)

The Risk profiles for the Cozzolino example are shown in Figure 2.1. The critical level of risk aversion for each of the five options of the venture, r_c occurs where the RAV profile crosses the x-axis (RAV is zero). The profiles shows the sensitivity of the lottery to the risk aversion level ranging from the very sensitive (Full Interest option) to the almost insensitive (Farm out option).

Mackay (1995) described a process of estimating the risk adjusted value (RAV) or Certainty equivalent using a spreadsheet based program and a numerical example. He was one of the first investigators in the application of utility theory to financial problems to focus exclusively on determining the optimum working interest from the RAV – risk tolerance relationship. Using a risk tolerance level from prior ventures, he determined the optimum working interest for the single venture from a plot of RAV versus working interest and showed conclusively that the optimum working interest is less than 100% and hence the Risk Adjusted Value of a venture for a risk averse investor will necessarily be less than the expected value (EV). Mackay repeated the process for four other ventures and extended the analysis to a portfolio that included all the five ventures. He used linear programming to determine the optimum working interest for each venture in the portfolio constraining the working interests to between 0% and 100%. The latter part of the analysis concerned determining the optimum working interest in a limited capital environment. Mackay was the first to show that the RAV value was much more sensitive to the probability of success than the Risk Tolerance (RT) level when modeled over a reasonable range of uncertainty. He concluded that different risk tolerance values could be assigned to divisions in a Company as part of an overall risk management strategy.

Moore et al (2005) just like previous authors, stated that firms do not base their decisions on expected values alone. Maximizing expected present values implies that firms will seek 100% working Interest in attractive ventures, since expected value is a linear function of working interest. Due to risk aversion, firms will value ventures less than the expected present value.
However, firms do not routinely estimate Risk adjusted values of projects because of the perceived difficulty in estimating the levels of risk aversion, r or Risk tolerance (RT). Risk tolerance is usually defined as the reciprocal of risk aversion. However, Risk adjusted value or Certainty equivalent analysis is finding increasing use in management decisions making especially for large capital intensive projects, in which significant portion of total Company investments are exposed. Using the Cozzolino Exponential utility model, Moore et al estimated RAV, optimum working

interest (WI_{opt}) and Risk tolerance. In the Cozzolino scheme, the Risk adjusted value is generally given by the following:

$$RAV = -\left(1/r\right)\ln\left(\sum P_i e^{-rV_i}\right) \quad \ldots\ldots\ldots\ldots (2.16)$$

Risk Tolerance

$$RT = 1/r \quad \ldots\ldots\ldots\ldots (2.17)$$

For a two outcome project with a chance of a success P_s, success value V_s and failure value V_f

$$RAV = -RT\ln\left[P_s e^{-WI*PV_{s/RT}} + P_f e^{WI*PV_{f/RT}}\right] \ldots\ldots\ldots\ldots (2.18)$$

The Risk adjusted value (RAV) is a nonlinear function of working interest (WI) – thus there is an optimum working Interest:

$$WI_{opt} = \frac{RT}{PV_s+PV_f}\ln\frac{PV_sP_s}{PV_fP_f} \quad \ldots\ldots\ldots\ldots (2.19)$$

Optimum working Interest, WI is a linear function of Risk tolerance for specific PV_s, PV_f, and Ps Risk adjusted value at optimum working interest can be estimated thus:

$$RAV_{WIopt} = -RT\ \ln\left[P_s K^{-PV_s/(PV_s+PV_f)} + P_f K^{-PV_f/(PV_s+PV_f)}\right] \ldots\ldots\ldots\ldots (2.20)$$

Where $K = \frac{PV_sP_s}{PV_fP_f}$

Moore et al defined a "Grossed Up" Risk adjusted value as Risk adjusted value at optimum working interest divided by the optimum working interest, WI_{opt}:

Apparent Risk Tolerance,

$$ART = \frac{WI_{opt}*(PV_s+PV_f)}{\ln^{PV_sP_s}/_{PV_fP_f}} \quad \ldots\ldots\ldots\ldots (2.21)$$

The significant new insights from Moore et al are the following:

I. From their investigation and practice all over the World, the assumption that Risk Tolerance (RT) is constant ought to be relaxed. Risk Tolerance RT, they observed can vary from project to project, Country to Country, and by type of projects

II. Currently, recognition of uncertainty in the success and failure "legs" is not included in most RAV Analysis. Instead the use of mean success and failure values and chances are prevalent in Industry. These values are not single value estimates. Their investigation therefore explored the impact of changes in these values on the Risk Adjusted Value (RAV).

Moore et al ran sensitivity analysis on the key uncertain parameters of chance factor Ps, Value of Success, PVs and Cost of Failure, C using an example prospect with the following base assumptions:

1. Chance Factor, Ps=20%
2. Success Value, PVs = $200 Million
3. Cost of Failure, PVf = $16 Million
4. Risk Tolerance , RT = $75 Million with

The optimum working interest for this particular prospect was 40%. The following were their observations from the numerous sensitivity analyses done:

i. The Risk adjusted value analysis and results obtained are quite robust unless the chance factor Ps is significantly overestimated. For chance factors less than 15%, the optimum working interest is expectedly smaller, but significantly, the range of acceptable working interests is narrower, suggesting that care should be taken when evaluating prospects with higher risks

ii. The results are also robust unless the cost of failure, PV_f is significantly underestimated. As cost of failure increases, the optimum working interest decreases, but maximum acceptable working interest decreases significantly. Given the tendency to underestimate PV_f in the Industry, this suggests that it is prudent to err on the low side in choosing working interest for risky prospects.

iii. The sensitivity of WI to Value of success shows the optimum working interest increasing with value of success to a maximum and then decreases slowly, which intuitively is wrong, since one should take an increasing share with increasing reward. It has been shown elsewhere in this study that optimum working interest varies inversely as the variance and does not differentiate between downside variance (returns lower than expected) and upside variance (returns greater than expected)

Overall, Moore et al concluded that optimum working interest determination is not very sensitive to the accuracy of Ps, PV_s and PV_f except where the inputs are overly optimistic, hence RAV offers a robust approach provided Risk tolerance, RT can be determined . Several rules of thumb have been suggested for determining the Risk tolerance level, RT of a firm. These include some percentage of Gross Sales (6%), 100% - 150% of Net Income or 1/6th of Equity. Equating the risk tolerance to a fractional part of the E and P's Company exploration budget is quite popular (ARCO). Moore et al suggest RT should be allowed to vary by business unit, by country or grouping of similar ventures reflecting each Company's core areas, focus or niches in which it wishes to concentrate its investment.

Moore et al introduced some novel ideas into RAV applications. In their global practice they observed that prices paid in corporate transactions appear to carry a premium compared to the sum of the values of the individual assets in the Portfolio. This is the so called "Portfolio Effect" which they explained in terms of RAV. The overall chance of economic success in the portfolio becomes large for large portfolios, and hence dominates the estimation of the RAV of the portfolio. Consequently, the RAV of the portfolio may be greater than the sum of the RAVs of the individual prospects. They developed rules of thumbs for choosing values of Risk Tolerances for Exploration programs thus:

i.	Major/Large Independent, Low Country Risk, Core Area	\$500 million
ii.	Super Major, Moderate Country Risk, Non-Core	\$200 million
iii.	Large Independent, International Core Business Unit	\$100 million
iv.	Large Independent, Frontier and /or High Country Risk	\$50 million
v.	Medium Independent, Frontier and/or High Country Risk	\$20 million

Another novel application to which they employed RAV analysis is in Farm out arrangements. Farm out arrangements sometimes entails proposing a specific level for working interest for the Farminee. Utilizing the observation that at the optimum working interest, the gross value of the RAV is independent of the Risk Tolerance, theoretically all firms regardless of Risk tolerance would accept identical prorated farm out terms. Limiting the working interest offered therefore to the perceived optimum working interest, should maximize value to the Farmor. The same arguments will apply to Bidding Groups. RAV considerations suggest that alignment is achieved when all participants in the Bid process hold their optimum working interests, otherwise participants holding below their perceived optimum working interest may wish to overbid, while participants holding more than the perceived optimum may wish to underbid and may even see the risk exposure as too much to even want to bid at all. To achieve alignment by allocating the appropriate working interests in bidding groups, they suggest that:

1. Pre-evaluation estimates of success values, failure costs and chance of success, as well as historical estimates RT be used to optimize the number of participants
2. After agreeing actual valuation parameters, use Risk adjusted value analysis in the evaluation process

The same logic of maximizing RAV in allocating working interests in Farm out arrangements and Bidding Group alignment, was also extended to Licensing round design by Governments (Host Governments especially). By offering large blocks, host Governments may collect higher signature bonuses and target large work programs, but they may be unwittingly limiting the derivable RAVs of the blocks. Allocation of smaller blocks reduces the failure “leg”, increases the number of participants and/or increases their Optimum Working Interests. RAV analysis can therefore be used by Governments to tailor their exploration offerings to maximize the benefit to the Government. Of course all of these analyses assume that each firm (and the Government) is using the same evaluation process and accept the utility of risk adjusted value analysis.

Ourderni and Sullivan (1991) first espoused the semi-variance analysis as against the mainstream Mean-Variance Model. They postulated that Semi-Variance rather than Variance may be a more realistic measure of the riskiness an investment and proposed a hybrid utility preference model. The fundamental theoretical basis of their work is still Von Newman-Morgenstern Utility framework and classified utility functions into three broad categories:

i. Quadratic Utility functions (Expected Value-Variance Model)
ii. Linear Utility Functions and
iii. Hybrid Utility Functions

Their investigation started from the premise that the optimal investment policy under conditions of uncertainty maximizes expected utility. Implicit in maximization of expected utility is the operation of trading off expectation with variation or riskiness of a project. The preference curve should therefore reflect this trade off in its mathematical form. Their analysis showed that most forms of the exponential utility function can be represented by a quadratic function of the form

$U(R) = R - AR^2$where A is a constant that approximates a risk aversion attitude (or coefficient of risk aversion).

The first derivative of this function represents the marginal utility of the project for the investor and should be positive in order for the incremental amount of invested capital to be justified. Hence:

$1 - 2AR \geq 0$ for all values of R. The second derivative of the utility function is:

$$\frac{d^2U(R)}{dR^2} = -2A \quad \text{……………………………………………………………………………… (2.22)}$$

By the principle of marginal diminishing utility, this second derivative is negative. By taking the expectations of both sides of the quadratic Utility function, the following results:

$$E\{U(R)\} = \mu - A(\mu^2 + \sigma^2) \quad \text{………………………………………………………..…. (2.23)}$$

The quadratic utility function does confirm that an investor who seeks to maximize expected utility is trading off the expectancy versus the variability of the project's worth since the expected utility is function of the mean, μ and variance, σ only. The quadratic utility function thus implies investor behavior that prefers more wealth to less while avoiding the variability of the return, R. In fact this investor dislikes the variations in the random return which he describes as the riskiness of the project and/or investment. However, the flaw in the quadratic model is the same as in all Mean-variance models in that variability may be as a result of positive returns, which is desirable. The quadratic model fails to distinguish between variability on the upside (opportunity) and downside variability which is what most investors intuitively describe as risk. Secondly, the larger the values of A (the coefficient of risk aversion), the smaller is the range of validity of the quadratic model. For the linear utility function, the decision maker is neutral to risk and therefore bases his decisions on expected values, instead of expected utility. The risk aversion coefficient in this case is equal to zero and the Utility function is given by $U(R) = R$. This model ignores widely varying consequences including losses big enough to cause bankruptcy of the firm-most investors simply do not behave that way and hence the rarity of the use of the linear model in most evaluations.

Ourderni and Sullivan's (1991) work asserts that variance measures the risk of an investment by the scatter of the distribution of the measure of worth around the mean, μ, which might be of no critical relevance to the decision maker. In many situations, "*it is realistic to minimize under-attainment of an objective in terms of a certain measure of economic attractiveness, h.* It becomes more appropriate to focus on the distribution scatter around this critical value rather than around the mean. The decision maker might more specifically be interested only in negative deviations about that fixed point, expressing his /her downside risk aversion and relative indifference to positive deviations." The Semi-variance model is thus an appropriate measure of risk that offers the flexibility of ascribing to each investment dollar, differing utilities. In essence, risk is defined as the possible loss of investment (downside outcome), while unexpected high returns (pleasant surprise) is an opportunity (upside outcome) rather than a risk. This is a fundamental difference from the traditional Mean-Variance model that does not discriminate between downside risk and upside opportunity. This is the premise of their proposal for a hybrid model-a risk averse utility function to address the variation about the downside critical value and a linear utility model (beyond the critical value). For the linear model on the upside distribution of returns, the higher the unexpected high return, the more the fractional share that the investor should take since, expected value is a linear function of the working interest. In their particular case, the risk averting utility function is represented by a quadratic form for values of worth less R (a particular value).

The Mean-semi-variance model accounting for downside risk is also the main theme of **Estrada** (**2008**). He pointed out that though Markowitz pioneered Mean-variance optimization to choose efficient portfolios, in his seminal work, he (Markowitz) had pointed out that semi-variance is a more plausible measure of risk than total variance because *"an investor worries more about underperformance of an asset rather than over-performance"*. However, practitioners and academics have been using the Mean-Variance model to optimize their portfolio because of "cost, convenience and familiarity". The difference in cost occurs because efficient sets based on semi-variance took as much as four times computing time than those based on the Mean-Variance model. The difference in convenience stems from the fact that efficient sets based on variance require as inputs only the means, variances and co-variances whereas those based on semi-variance required the entire joint distribution of returns. However, the recognition of downside risk is gaining increasing attention amongst practitioners and academics.

Kim and Wallace (1998) attribute the gap between expectation based decisions and final realization after all uncertainty have been eliminated to be due to the fact that while expectation is based on an event being repeated infinitely, most decision events are single events and are not repeated. They argue that the source of discrepancy between initial expectation and final result can be caused by two factors:

i. Unreliability of the subjective probability distributions on which most expected value analysis is based
ii. Objective or frequency based probabilities represent and quantify uncertainties of events that are infinitely repeatable (for example the toss of a coin)

In decision making, Kim et al distinguishes between "good" and "bad" surprises and define the different kinds of uncertainties thus:

1. *Risk is the possibility of a single event outcome being lower than expected*
2. *Opportunity is the possibility of a single event outcome being higher than expected*

With the clear distinction between risk and opportunity, it is to be expected that a symmetric distribution will have the same risk and opportunity. Kim and Wallace(1998) were thus one of the first authors to explicitly make a distinction between variability on the upside and downside and significantly, they have defined risk as the downside variation rather than the total variation around the mean, which is largely responsible for the "paradox" of aversion to reward so often seen in most mean-variance analysis, including expected utility maximization. Kim and Wallace thus concluded (just like Estrada (2008) and Ourderni and Sullivan (1991)) that *semi variance analysis is a more realistic model of quantifying risk.*

Kim and Wallace (1998) extended the appropriateness of the semi-variance model by defining an opportunity to risk ratio, ή a metric that will explicitly indicate to the decision maker what variation dominates the uncertainty structure of an event. The opportunity to risk ratio is defined thus:

$$ή = \frac{\sigma_{sup}}{\sigma_{inf}} \quad \ldots\ldots (2.24)$$

$\sigma_{sup}, \sigma_{inf}$ represent the standard deviations on the upside and downside (of the mean) respectively.

A two dimensional plot of (μ, ή) might thus be used to understand the opportunity and risk of different decision alternatives rather than just the mean and variance. Kim and Wallace, with this metric have extended the realm of analysis of uncertain investments. When $ή = 1$, the uncertainty structure is symmetric in terms of risk and opportunity, while $ή > 1$ suggest an uncertainty structure with more opportunity than risk. Another metric that offers possible insight into the uncertainty structure of a venture whose outcome can be described with a probability distribution would be the third momentum of a random variable or measure of skewness. Any odd momentum of the distribution will have a positive or a negative sign- a positive sign indicating opportunity dominates the uncertainty structure of that distribution. In this scheme, we ignore the first momentum, which is expectation or the expected value, whose decision containing value we are trying to complement with additional insights. In terms of additional impact of new information, a ratio greater than one implies that additional positive information will increase the expectation more than additional negative information will decrease the expected value.

CHAPTER THREE

METHODOLOGY AND THEORETICAL FRAMEWORK

3.1 DCF, Mean-Variance; Mean-Semi-Variance and Risk Adjusted Value Analysis

The analysis of petroleum prospect investment evaluation and risk analysis, including quantitative individual preference for risks will begin with the deterministic or DCF analysis, proceed through

Mean-variance and Mean-semi-variance analysis and conclude with Risk adjusted value (RAV) or Certainty equivalent (CE) analysis. The application of Risk adjusted values will focus mainly on the risk averse forms of individual utility functions, especially the exponential and hyperbolic utility functions and their application in the determination of optimum working interest in oil and gas ventures.

3.11 Discounted Cash Flow (DCF) Analysis

The net present value (NPV) of the cash flows resulting from an investment can be analytically represented thus:

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+i)^t} \quad \text{………………………………………………………..………… (3.1)}$$

where CF_t are the cash flows in time t, *i* the interest (discount) rate.

When the investment is shown explicitly in the NPV relationship:

$$NPV = -I + \sum_{t=0}^{T} \frac{CF_t}{(1+i)^t}$$

On a continuous basis,

$$NPV = \int_0^T CF_t e^{-it} dt \quad \text{…………………………………………………………….……. (3.2)}$$

Another popular measure of project worth, the internal rate of return (ROR) is the discount rate that reduces NPV to zero or

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+ROR)^t} = 0$$

$$NPV = -I + \sum_{t=0}^{T} \frac{CF_t}{(1+ROR)^t} = 0 \quad \text{…………………………………………………………. (3.3)}$$

$$I = \sum_{t=0}^{T} \frac{CF_t}{(1+ROR)^t}.$$

Continuous cash flow streams and discounting are fairly straight forward and easy to analyze. Showing the investment, ***I,*** explicitly in present value terms:

$$NPV = -I + \int_0^t CF_t e^{-it} dt \quad \text{……………………………………………………...…… (3.4)}$$

Assuming constant cash flow per year,

$$NPV = -I + \frac{CF(1-e^{-it})}{i} \quad \text{……………………………………………………..……… (3.5)}$$

For the project life, T years

$$NPV = -I + \frac{CF(1-e^{-iT})}{i} \quad \text{………………………………………………………………….………… (3.6)}$$

If T is long, like most Oil and Gas assets are long life projects lasting more than 15-20 years, the exponential term tends to zero and

$$NPV = -I + \frac{CF}{i} \quad \text{……………………………………………………………………………… (3.7)}$$

The Internal rate of return, (ROR) also reduces the NPV to zero. Thus:

$$NPV = -I + \frac{CF\left(1-e^{-RORt}\right)}{i} = 0 \quad \text{………………………………………………………………… (3.8)}$$

We can show that $ROR = \frac{1}{t} ln\left(\frac{CF}{CF-iI}\right)$ ……………………………………………………….. (3.9)

In addition, for oil and gas assets, the cash flows are generated by production q and the net value of the barrel (*Price, P less Costs,* C_p), the preceding relationships can be represented thus

$$NPV = -I + \int_0^t q_i e^{-(D_i+i)t}(P - C_p)dt \quad \text{.. (3.10)}$$

$$NPV = -I + \frac{q_i(P-C_p)}{D_i+i}\left[1 - e^{-(D_i+i)t}\right] \quad \text{……………………………………………..….. (3.11)}$$

At payout, t = t_p

$$I = \frac{q_i(P-C_p)}{D_i+i}\left[1 - e^{-(D_i+i)t_p}\right] \quad \text{………………………………………………………... (3.12)}$$

$$\left[1 - e^{-(D_i+i)t_p}\right] = \frac{I(D_i+i)}{q_i(P-C_p)} \quad \text{…………………………………………………………… (3.13)}$$

Time to payout, *tp* is given by

$$t_p = \frac{1}{(D_i+i)} \ln\left[1 - \frac{I(D_i+i)}{q_i(P-C_p)}\right]^{-1} \quad \text{……………………………………………………...… (3.14)}$$

Further recall

$$NPV = -I + \frac{q_i(P-C_p)}{D_i+i}\left[1 - e^{-(D_i+i)t}\right] \quad \text{…………………………………………..…… (3.15)}$$

For the duration of the project or project life, T years, which is usually long (anywhere from 15 to 20 years typically), the net present value relationship is simplified since the exponential term reduces to zero. Thus

$$NPV = -I + \frac{q_i(P-C_p)}{D_i+i} \quad \ldots \ldots (3.16)$$

The discounted to profit to investment ratio, a measure of project worth that includes efficiency of investment money, DPI is given by

$$DPI = \frac{NPV}{I} = -1 + \frac{q_i(P-C_p)}{I(D_i+i)} \quad \ldots \ldots (3.17)$$

For the discrete case,

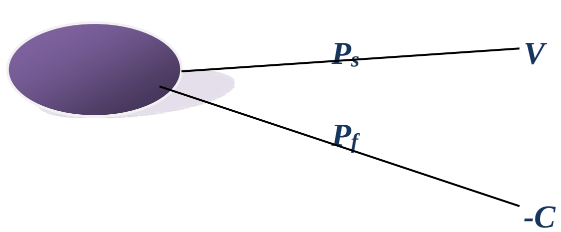
$$DPI = \frac{NPV}{I} = -1 + \frac{\sum_{t=0}^{T} \frac{CF_t}{(1+i)^t}}{I} \quad \ldots \ldots (3.18)$$

The preceding analytical relationships for net present value (NPV), discounted profit to investment (DPI), Internal rate of return (IRR or ROR) and payout, t_p capture the most important deterministic project worth characteristics which will inform the decision maker on whether to invest or not and also serve as a basis for project comparison in the case of a portfolio of projects. An NPV greater than zero implies value creation through the investment, while a negative NPV clearly indicates that project should not be embarked upon since no value is created. Money will be better spent elsewhere or in a savings account. A positive DPI indicates how much value is being created per investment dollar and hence is a measure of investment efficiency and will generally rank projects the same way as NPV does. The Internal rate of return (IRR, ROR or DCFROR) is still a popular investment metric in industry and is an indication of the size of the cash flows generated but, importantly, also the timing of those cash flows. Usually, a project IRR should exceed the cost of capital or the Company's reinvestment rate, if not; money should be invested in another opportunity which will guarantee higher returns. Payout or payback is rarely used now as a project performance metric except in situations in which there is considerable concern that the project environment will be negatively impacted so much so that initial investment may not be recovered. Small companies are especially sensitive to this consideration since they have limited capital and persist in using this particular metric.

All the aforementioned metrics however suffer from the fact that they do not indicate the level of risk in the investment under consideration. Implicit in a deterministic model is the assumption that the various project variables in the economic model will be as estimated. In reality, this is not the case. Project outcomes usually vary from anticipated due to the fact that many of the project variables are uncertain and not under the control of the decision maker. To accommodate changes in Project variables, sensitivity or scenario analysis is carried out showing to demonstrate the range of the project outcomes. The usefulness of sensitivity analysis is limited by the number of project variables and more explicit risk analysis must be done to quantify the project level of risk. This is all the more compelling in the oil and gas industry in which most of the major project variables of *Reserve* sizes, *Production* level, *Price* and *Fiscal* parameters are subject to significant uncertainty. The more explicit risk quantifying methods of expected value analysis (EPV), Monte Carlo simulation not only show the *magnitude of value created* by the investment, but also the *probability distribution* of the value creation (NPV) and/or some other investment metric under consideration that Management chooses to base their investment decision on. When accounting for risk by taking a fraction of the project, usually referred to as the working interest, the NPV metric is proportioned in the ratio of working interest taken – most Nigerian Joint venture assets (JV) are structured that way. For an NNPC/IOC asset structured 60/40 in which NNPC owns 60% and contributes that proportion in exploration, development and operations, NPV due to NNPC from an asset is simply 60% of total NPV, because the working interest is a linear function of the cash flows. Other metrics, such as the rate of return (ROR) do not exhibit the same linear behavior as the NPV and must be estimated appropriately from the economic model cash flows.

3.12 Mean-Variance Analysis

Consider a simple two outcome prospect



This is typical of wild cat drilling in which the outcomes are discovery with present value, V or dry hole, with a loss of C, the cost of the drilling operation and any other upfront exploration costs. The value, V in this gamble is the same as the net present value (NPV) from deterministic (DCF) analysis. The probabilities of the two events are P_s and P_f and sum up to one (1).

The Expected Present value $EPV = \mu = P_s V - P_f C$ …………………………………… (3.19)

When $P_s V > P_f C$, the mean or expected value is positive.

The mean is also the first moment of the distribution of project outcomes.

The second moment of the distribution of outcomes $E_2 = P_s V^2 + P_f C^2$ ………………… (3.20)

Variance $\sigma^2 = E_2 - \mu^2 = P_s V^2 + P_f C^2 - \left(P_s V - P_f C\right)^2$ ……………………………..…. (3.21)

$\sigma^2 = (V + C)^2 \left(P_s P_f\right)$ ……………………………………………………………... (3.22)

Volatility $v = \frac{\sigma}{\mu} = \frac{(V+C)\left(P_s P_f\right)^{1/2}}{\left(P_s V - P_f C\right)}$ ……………………………………………………...… (3.23)

Volatility, v is an indication of the stability of the mean, a small value of v (v <<1) denotes small uncertainty of the Expected Present Value while, v>>1, implies significant uncertainty in the expected value.

The Performance Index, PI is mean or expected value, μ per unit of standard deviation σ

$PI = {}^{\mu}/_{\sigma}$ ……………………………………………………………………………. (3.23a)

Observe the PI is the inverse of the volatility. The PI has a property similar to discounted profit to investment (DPI) which measures value created per unit of investment (the investment efficiency criterion). The performance index gives an indication of the expected value created per unit of standard deviation (of returns). The usefulness of this metric lies in project ranking – for instance, if two projects have the same expected value (mean) and different standard deviations, the project with the lower mean is preferred (Decision Rule 1). Conversely, if two projects have the same standard deviation, but different means (or expected value), the project with the higher mean is preferable (Decision Rule 2). The higher the PI, the more desirable the project should be from an investment stand point.

When a fraction, W of the prospect is taken, rather than the whole prospect, the preceding analysis is modified only slightly.

$EPV(W) = P_s(WV) - P_f(WC) = W\left[P_sV - P_fC\right]$ ……………………………………..… (3.24)

$EPV(W) = f(W, P_s, V, C)$ …………………………………………………………..….. (3.25)

Expected present value, is a linear function of W, the fractional working interest when investor takes a part of the prospect, and is maximized at 100% working interest (W=1).

3.13 Mean Semi-Variance Analysis

Mean semi-variance analysis is premised on the fact that Risk can be defined as the chance of a project outcome being lower than a certain threshold or downside, an unpleasant surprise, the possibility of loss of the money that has been invested. So in this instance, we are only concerned with deviations below a particular level of NPV, DPI or rate of return (ROR). Conversely, if the project comes in better than expected, that is a pleasant surprise or opportunity for more benefits or reward from the investment. The deviations that we will estimate in this instance are the deviations above the expected value, the mean. In both cases, the means are the same; it is the variability or the variances that defer. In one it is the total variance, while in the other we have semi-deviations below the mean and semi-deviations above the mean.

Mean, $\mu = P_sV - P_fC$ ……………………………………………………………………... (3.26)

In Semi-variance considerations, we seek to minimize deviations below the mean and maximize deviations above (the mean). The semi-deviation below the mean is given by:

$${\sigma_s}^2 = \sum P_i[Min(x_i - \mu), 0]^2 = P_s[Min(V - \mu), 0]^2 + P_f[Min(-C - \mu), 0]^2$$

${\sigma_s}^2 = P_f[C^2 + 2\mu C + \mu^2]$ ………………………………………………….…………… (3.27)

For the upside or “pleasant surprise”, maximizing the deviations above the mean:

$$\sigma_{sup}{}^2 = \sum P_i[Max(x_i - \mu), 0]^2 = P_s[Max(V - \mu), 0]^2 + P_f[Max(-C - \mu), 0]^2 \quad \text{......... (3.28)}$$

$$\sigma_{sup}{}^2 = P_s[V^2 - 2\mu V + \mu^2] \quad \text{......... (3.29)}$$

The addition of the two semi deviations gives;

$$\sigma_s{}^2 + \sigma_{sup}{}^2 = P_f[C^2 + 2\mu C + \mu^2] + P_s[V^2 - 2\mu V + \mu^2] \quad \text{......... (3.30)}$$

$$= P_s V^2 + P_f C^2 - \mu^2 = \sigma^2$$

$$\sigma_s{}^2 + \sigma_{sup}{}^2 = \sigma^2 \quad \text{......... (3.31)}$$

Expectedly, the sum of the deviations below and above the mean equals the total deviations around the mean.

The downside semi-deviation can be expressed in terms of the Project parameters thus:

$$\sigma_s{}^2 = P_f[C^2 + 2\mu C + \mu^2] = P_f\left[C^2 + 2\left(P_s V - P_f C\right)C + \left(P_s V - P_f C\right)^2\right] \quad \text{......... (3.32)}$$

$$\sigma_s{}^2 = P_f P_s{}^2[V + C]^2 \quad \text{......... (3.33)}$$

$$\sigma_s = P_s P_f{}^{1/2}[V + C] \quad \text{......... (3.34)}$$

Similarly, the semi-deviation above the mean, σ_{sup}, is:

$$\sigma_{sup}{}^2 = P_s[V^2 - 2\mu V + \mu^2] = P_s\left[V^2 - 2\left(P_s V - P_f C\right)V + \left(P_s V - P_f C\right)^2\right] \quad \text{......... (3.35)}$$

$$\sigma_{sup}{}^2 = P_s P_f{}^2[V + C]^2 \quad \text{......... (3.36)}$$

$$\sigma_{sup} = P_f P_s{}^{1/2}[V + C] \quad \text{......... (3.37)}$$

Ratio of the Upside or Opportunity Standard Deviation to the Downside Standard deviation, τ:

$$\tau = {}^{\sigma_{sup}}/_{\sigma_s} = \frac{P_f P_s{}^{1/2}[V+C]}{P_s P_f{}^{1/2}[V+C]} = \left[\frac{P_f}{P_s}\right]^{1/2} \quad \text{......... (3.38)}$$

Equation 3.38 shows that the ratio of upside standard deviation to that of the downside is solely dependent on the success and failure probabilities (P_s, P_f) and not on the success or failure values (V, C), which is counter intuitive. Intuitively, an increase in the value of the upside, V while keeping the costs, C the same should make the ratio of opportunity to the downside to increase, but Equation (3.38) predicts otherwise. The reason for this is that the choice of the mean, μ as the "threshold" between the upside and downside is a "moving" one, keeping the ratio between the upside and downside constant as correctly predicted by (3.38). However, the choice of a fixed or "static threshold" for example zero (0) eliminates this "problem".

Making zero, the "benchmark" or target expected worth of the risky prospect, the following are the relationships for upside and downside semi-standard deviations, σ_s, σ_{sup} respectively:

$$\sigma_s{}^2 = \sum P_i[Min(x_i - 0), 0]^2 = P_s[Min(V - 0), 0]^2 + P_f[Min(-C - 0), 0]^2 \quad \text{……….. (3.39)}$$

$$\sigma_s{}^2 = P_s[0]^2 + P_f[(-C - 0)]^2 = P_f C^2 \quad \text{………………………………………….… (3.40)}$$

$$\sigma_s = P_f{}^{1/2} C \quad \text{……………………………………………………………………….. (3.41)}$$

Similarly

$$\sigma_{sup}{}^2 = \sum P_i[Max(x_i - 0), 0]^2 = P_s[Max(V - 0), 0]^2 + P_f[Max(-C - 0), 0]^2 \quad \text{……. (3.42)}$$

$$\sigma_{sup}{}^2 = P_s[(V - 0)]^2 + P_f[0]^2 = P_s V^2 \quad \text{…………………………………………...… (3.43)}$$

$$\sigma_{sup} = P_s{}^{1/2} V \quad \text{………………………………………………………………………… (3.44)}$$

The ratio of the upside to the downside standard deviations τ is now given by the following relationship:

$$\tau = {}^{\sigma_{sup}}/_{\sigma_s} = \left(\frac{P_s}{P_f}\right)^{1/2} \left[\frac{V}{C}\right] \quad \text{…………………………………………………..…… (3.45)}$$

This shows that τ is a function not only of the probabilities, but also of the failure and success values, more reflective of our intuition than Equation 3.38. We have thus eliminated the "moving mean" problem in decomposing the standard deviations between the upside and the downside.

The choice of zero as the "benchmark" for decomposing the total deviation into upside and downside is for illustrative purposes only. We could have chosen another "benchmark" such as "value< $10 million", or "20% of project success value, V", whatever is more appropriate as an investment benchmark in the circumstances and for a particular Company. The practical implication of this decomposition using a "fixed" or "static" benchmark will be more fully discussed in the *Analysis and Discussion* section of this thesis using a numerical example taken from Lerche and Mackay (1993). Discussion is limited to a two outcome example also for illustrative purposes. While the analysis of a prospect with more than two outcomes will be more involving, the fundamental analytical procedures of estimating the total variance and decomposing it into downside and upside variances (and hence standard deviations) is the same and no new fundamental insights beyond what is already presented in this theoretical framework will be gained.

3.14 Mean-Variance, Mean-Semi Variance-Analysis for Portfolio of Projects

Some properties of the mean and variance of a random variable are especially useful in Portfolio analysis.

Expected Value - Linearity of Expectation

$E(X + Y) = E(X) + E(Y)$ ……………………………………………………..…… (3.45a)

$E(cX) = nE(X)$ ………………………………………………………………….... (3.45b)

where c is a constant

The expected value of the sum of any finite number of random variables is the sum of the expected values of the individual random variables. It is not required that the individual random variables be mutually independent.

$E(XY) = E(X)E(Y)$ ……………………………………………………………….. (3.45c)
if X and Y are mutually independent

Variance – Additive Property of Variance of Independent Random Variables

The impact of the number of projects in a portfolio on the total portfolio variance is also considered in this research. When a portfolio contains a number of projects with the same variance, the total portfolio variance is given by

$V(cX) = c^2V(X)$ ………………………………………………………………...….. (3.45d)

$V(X + c) = V(X)$ ……………………………………………………………….… (3.45e)

where c is a constant

$V(X + Y) = V(X) + V(Y)$ ……………………………………………………... (3.45f)

The variance of the sum of any number of mutually independent random variables is the sum of the individual variances.

The mean variance efficiency criterion pioneered by Markowitz (1954) essentially seeks the maximization of the objective function, F for a portfolio of investments by trading off value (EV or μ) for risk (variance, σ^2) and can be represented mathematically thus:

$$F = \mu - \lambda\sigma^2 \quad \text{(3.45g)}$$

Where μ and σ represent the mean and Standard Deviation of one portfolio from a set of available portfolios and λ is a constant. By varying the value of λ from 0 to ∞, we can generate the set of mean-variance efficient portfolios. This is the foundation of portfolio diversification proposed by Markowitz.

3.15 Expected Utility–Certainty Equivalent (CE) or Risk Adjusted Value (RAV) Analysis

Utility theory, Certainty Equivalent or Risk Adjusted Value Analysis is premised on the fact that people are not impartial to money or risk neutral which expected value or mean variance analysis assumes. The foundation for the theoretical framework for Utility theory was laid by Von Newman and Morgenstern in the "Theory of Games and Economic Behavior" (1949). Expected Utility Hypothesis stipulates that a decision maker has risk preferences represented by Utility functions, U(x) and makes decisions in order to maximize expected utility, EU(x) where E is the expectation operator based on the subjective probability distribution of x. In this research thesis, we will only consider x to be monetary values. To estimate Expected Utility therefore, we need to estimate the probabilities of the outcomes as well as the risk preference for each outcome. The relevant probabilities can be estimated using sample information and/or subjective assessments.

Utility theory is based on some fundamental assumptions (**Chavas 2004**) viz:

Assumption 1: **Ordering and Transitivity**: For any random variables, x_1 and x_2, only one of the following holds true

$x_1 \geq x_2$, $x_1 \leq x_2$ or , $x_1 \approx x_2$

if $x_1 > x_2$ and $x_2 > x_3$ then $x_1 > x_3$ (transitivity)

Assumption 2: **Independence**

For any random variables, x_1, x_2, and x_3, and any α ($0 \leq \alpha \leq 1$), then $x_1 \leq x_2$ if and only if

$[\alpha\ x_1 + (1 - \alpha)\ x_3] \leq [\alpha\ x_2 + (1 - \alpha)\ x_3]$

(The preferences between x_1 and x_2 are independent of x_3)

Assumption 3: **Continuity**

For any random variables, x_1, x_2, and x_3, and ($x_1 \leq x_3 \leq x_2$), there exist numbers α and β, ($0 \leq \alpha \leq 1$), ($0 \leq \beta \leq 1$), such that

$x_1 <[\alpha\ x_2 + (1 - \alpha)\ x_1]$ and $x_3 >[\beta\ x_2 + (1 - \beta)\ x_1]$

(A sufficiently small change in probabilities will not reverse a strict preference)

Assumption 4:

For any risky prospects x_1, x_2 satisfying $Pr[x_1 \leq r: x_1 \leq r] = Pr[x_2 \geq r: x_2 \geq r] = 1$ for some sure reward r, then $x_2 \geq x_1$

Assumption 5:

- For any number r, there exist two sequences of numbers $s_1 \geq s_2 \geq \ldots$ and $t_1 \leq t_2 \leq \ldots$satisfying $s_m \leq r$ and $r \leq t_n$ for some m and n.
- For any risky prospects x_1 and x_2, if there exists an integer m_0 such that [x_1 conditional on $x_1 \geq s_m$: $x_1 \geq s_m$] $\geq x_2$ for every $m > m_0$, then $x_1 \geq x_2$. And if there exists an integer n_0 such that [x_1 conditional $x_1 < t_n$: $x_1 \leq t_n$] $\leq x_2$ for every $n > n_0$, then $x_1 \leq x_2$.

The proofs of these five assumptions are given in the work of Von Neumann-Morgenstern and in most risk analysis references and will not be repeated here. Suffice to state that, when all these five assumptions hold, then for any risky prospects x_1 and x_2, there exists a preference function U(x) representing individual risk preferences such that

$x_1 \geq x_2$ if and only if $EU(x_1) > EU(x_2)$

where U(x) is defined up to a positive linear transformation

The assumptions provide the axiomatic framework on which preference theory rests and characterizes individual behavior under risk. The behavior of the individual and their decisions thereof ***will be consistent with maximizing expected utility (of rewards).*** The framework therefore can be used to predict and recommend the appropriate decisions to be taken in a risky situation if the associated probabilities and the utilities of the outcomes can be established.

There are many forms of utility or preference functions. However, preference functions can be classified into three broad groups:

- Risk Seeking,
- Risk Neutral or (Expected Value maximizing) or
- Risk Averse.

Figure 3.1 shows the profiles of the three broad classes of utility functions, the risk averse is concave downwards indicating a positive risk premium, while the risk seeking function is concave upwards. Risk neutrality (or the expected value decision making) shows indifference to increasing success or failure and is represented by the straight line through the origin. Most investors are risk averse and their preference functions have been found to be fairly accurately defined by the exponential utility function:

$U(x) = e^{-rx}$ ……………………………………………………………………………… (3.46)

Where x = terminal wealth and r = risk aversion level =1/millionths

Another form of the exponential Utility function is:

$U(x) = 1 - e^{-rx}$ ………………………………………………………………………… (3.47)

Figure 3.2 plots the two forms of the exponential function shown by eq. 3.46 and 3.47.

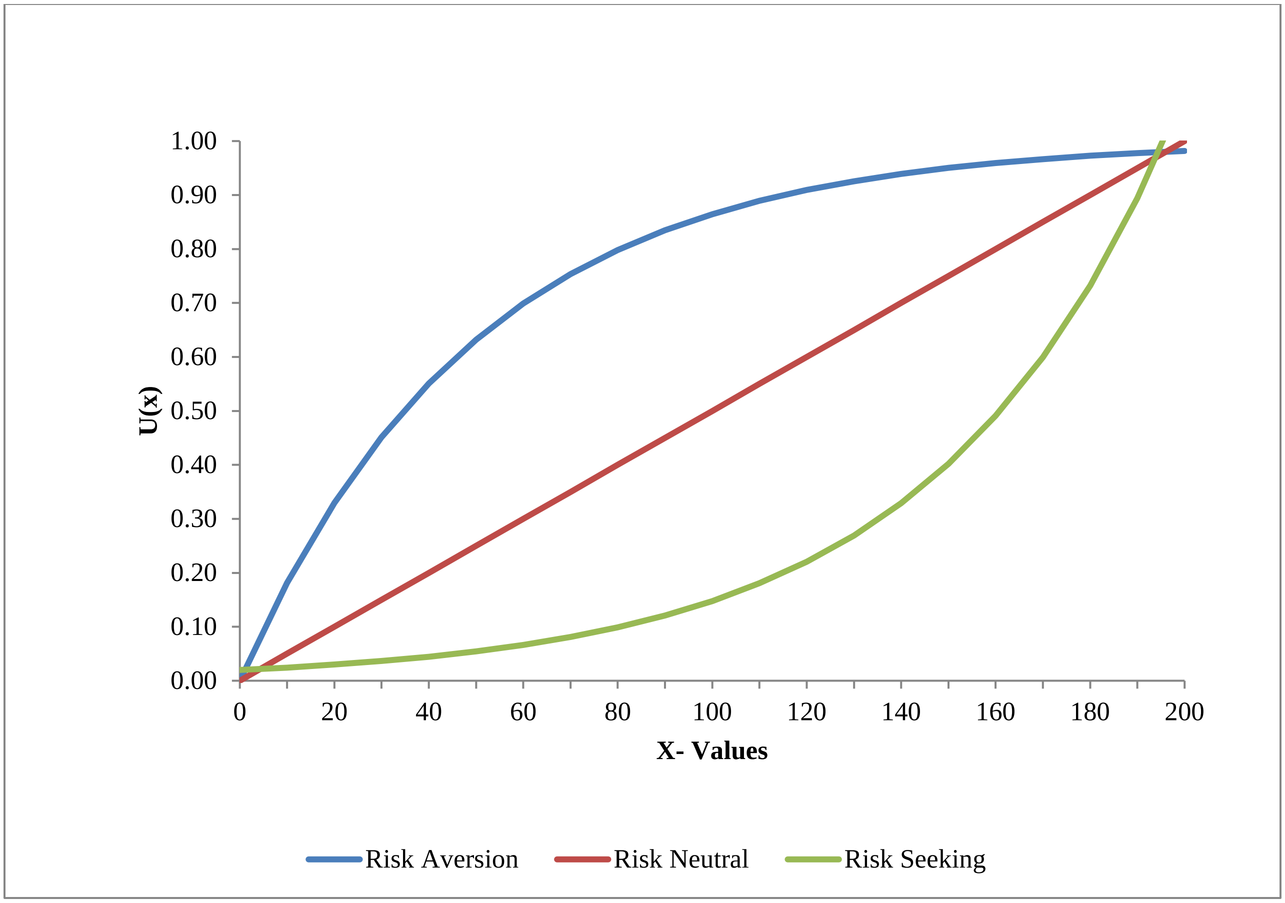


Figure 3.1 Investor Risk Attitudes

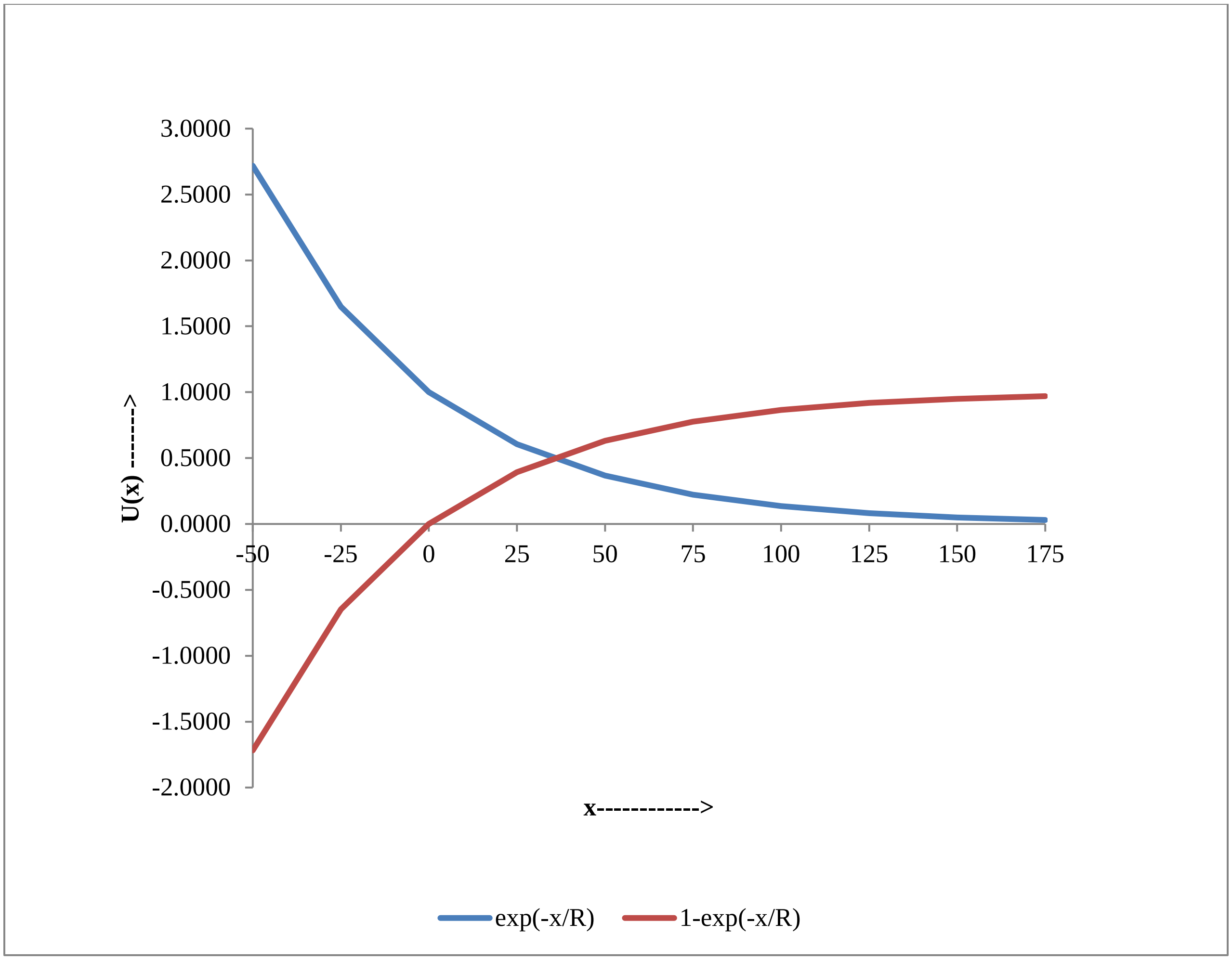


Figure 3.2 Exponential Preference Function forms

Hyperbolic analytic functions are also commonly used to model preference functions:

$U(x) = \tanh(x)$ ……………………………………………………..…………… (3.48)

$U(x) = 1 - \tanh(x)$ ……………………………………………………….….…… (3.49)

The hyperbolic forms display a gentler slope in high loss situations, compared to the exponential forms which tend to exaggerate the dislike for losses. After all, people are willing and do take informed risks when they have all the information about a risky situation available to them for instance, in Oil exploration business which is actually calculated risk taking.

3.16 Properties of Preference Functions

If the Preference function is continuous and twice differentiable, then:

- U(x) is concave if and only if $\frac{\partial^2 U}{\partial x^2} \leq 0$ for all x
- U(x) is convex if and only if $\frac{\partial^2 U}{\partial x^2} \geq 0$ for all x

A very important property of concave (convex) functions is stated by

Jensen's Inequality (Chavas 2004):

If U(x) is a $\begin{Bmatrix} Concave \\ Linear \\ Convex \end{Bmatrix}$ $function\ of\ the\ random\ variable, x\ then$

$U[E(x)] \begin{Bmatrix} \geq \\ = \\ \leq \end{Bmatrix} E[U(x)]$ where E is the expectation operator

The risk aversion coefficient, r is defined thus:

$$r = -\frac{U''(x)}{U'(x)}$$

The **Risk premium**, $R = EV - CE$

where EV is the Expected Value and CE is the Certainty Equivalent.

The Risk Premium represents a discount of the Expected Value, EV to account for the risk involved in a gamble or risky situation under consideration by an investor.

- R > 0, indicates Risk Aversion
- R = 0, indicates Risk Neutrality (Expected Value Maximization)
- R < 0, denotes Risk Seeking attitude

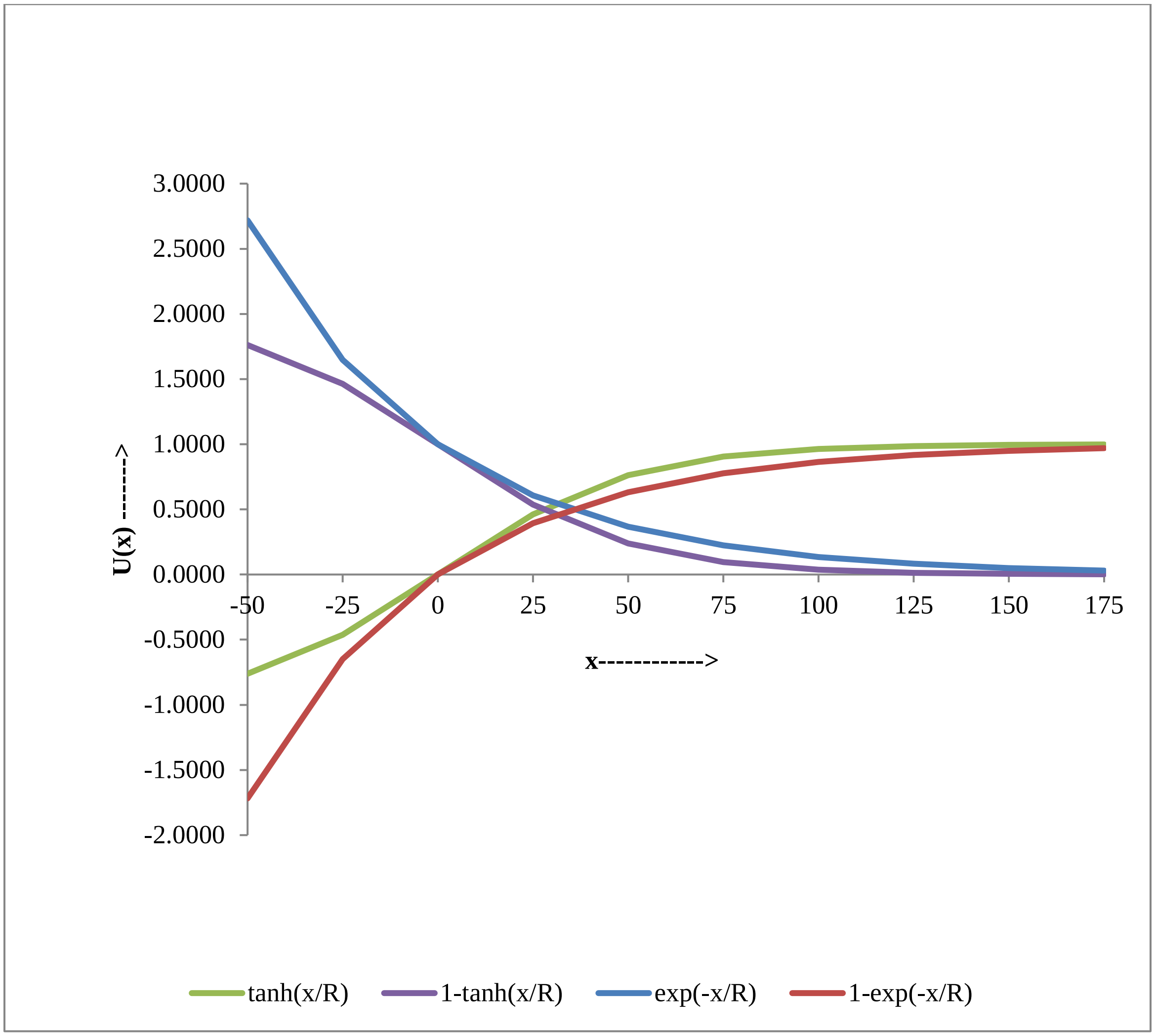


Figure 3.3 Exponential and Hyperbolic Preference functions

3.17 Risk Adjusted Value Analysis-Use of the Exponential Utility Function

Consider a preference function of the form

$$U(x) = e^{-rx} \quad \ldots\ldots (3.50)$$

$$U'(x) = -re^{-rx}$$

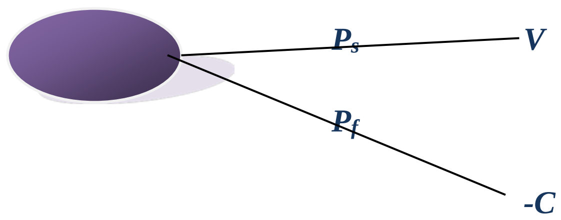
$$U''(x) = r^2 e^{-rx}$$

$$\frac{U''(x)}{U'(x)} = \frac{r^2 e^{-rx}}{-re^{-rx}} = -r \quad \ldots\ldots (3.51)$$

Risk Tolerance RT (or R) is defined as the inverse of r

$$RT = {}^{1}/_{r} \quad \ldots\ldots (3.52)$$

Recall the two outcome prospect



The Risk Adjusted Value can be represented by the general relationship:

$$RAV = -\left({}^{1}/_{r}\right)\ln\left(\sum P_i e^{-rV_i}\right) \quad \ldots\ldots (3.53)$$

Using the exponential form Preference function

$$U(x) = e^{-rx} = e^{-\frac{x}{RT}} \quad \ldots\ldots (3.54)$$

The Expected Utility for the 2-Outcome prospect shown is

$$EU(x) = P_s e^{-\frac{WV}{RT}} + P_f e^{\frac{WC}{RT}} \quad \ldots\ldots (3.55)$$

$$RAV = -RT\ln\left[P_s e^{-WI*V/RT} + P_f e^{WI*C/RT}\right] \quad \ldots\ldots (3.56)$$

Therefore Optimum Working Interest is (setting $\frac{\partial(RAV)}{\partial WI} = 0$)

$$W_{opt} = \frac{RT}{V+C}\ln\frac{P_s V}{P_f C} \quad \ldots\ldots (3.57)$$

Risk Adjusted Value at Optimum Working Interest is therefore:

$$RAV_{Wopt} = -RT\left[P_s\left(\frac{P_f C}{P_s V}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_s V}{P_f C}\right)^{\frac{C}{V+C}}\right] \quad \ldots\ldots (3.58)$$

For specific values of Success Probability Ps, Success Value, V and Cost, C, the foregoing shows, the Risk Adjusted Value at Optimum Working Interest varies linearly with the Risk Tolerance, RT since the expression in the bracket will remain constant. Thus:

$$RAV_{Wopt} = K * RT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.59)$$

where K is a constant representing the expression in the bracket in equation C8.

The "Grossed Up" Risk Adjusted Value

$$RAV_{Gross} = \frac{RAV_{Wopt}}{W_{opt}}$$

$$RAV_{Gross} = \frac{-RT\left[P_s\left(\frac{P_f C}{P_s V}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_s V}{P_f C}\right)^{\frac{C}{V+C}}\right]}{\frac{RT}{V+C}\ln\frac{P_s V}{P_f C}} = \frac{-(V+C)\left[P_s\left(\frac{P_f C}{P_s V}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_s V}{P_f C}\right)^{\frac{C}{V+C}}\right]}{\ln\frac{P_s V}{P_f C}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \ (3.60)$$

Grossed up Risk Adjusted Value depends solely on the magnitude of reward V, loss, C and success probability, P_s and significantly, independent of the level of Risk Tolerance, RT.

From Equation (3.57) the Risk Tolerance,

$$RT = \frac{W_{opt}(V+C)}{\ln\frac{P_s V}{P_f C}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.61)$$

The historical level of working interests taken by a Company can be considered as the Company's Optimum Working Interest, W_{opt} . Though not explicitly quantified, the Organization's Risk Tolerance (RT) can be estimated and will be the apparent risk tolerance of the Company (ART).

3.18 Paradox of Aversion to Incremental Reward (PAIR)

The aversion to deviation however, sometimes leads to a Paradox of aversion to incremental reward (PAIR). Since all risk is considered undesirable, deviations resulting from unexpected good returns or high gain situations also reflect negatively in most risk measures, including the Performance Index (PI). PI has an inverse relationship with standard deviation (eq. 23a), the higher the standard deviation the worse the PI of a risky asset, regardless of the nature of the deviation.

This research shows explicitly the impact of PAIR in both the expected value-variance and expected utility contexts and in particular show that, because of PAIR, the performance index metric must be used with caution in risky investment decision making. Deviation from expected return need not necessarily lead to perception that a project is poor from an investment standpoint.

The deviation may be on the upside – a situation of unexpectedly high returns or more appropriately described as "High Gain" situations.

There is a need to decompose total variance into upside and downside. A realistic investment performance measure will take into account the nature of the variation from the expected and reflect such accordingly otherwise; it will lead to wrong investment decisions

In an expected utility maximization context, the impact of PAIR is shown in optimum working interest determination. In this case, there is decreasing working interest recommendation with increasing success value – which is also a paradox and counterintuitive to what most rational investors will do.

Consider the 3 two outcome prospects shown in (Projects a, b and c). The failure cost is zero in each, so these are riskless since the investment required is zero. The Expected Values, variance (and standard deviations) and Performance Indices (PIs) are shown in Table 3.1

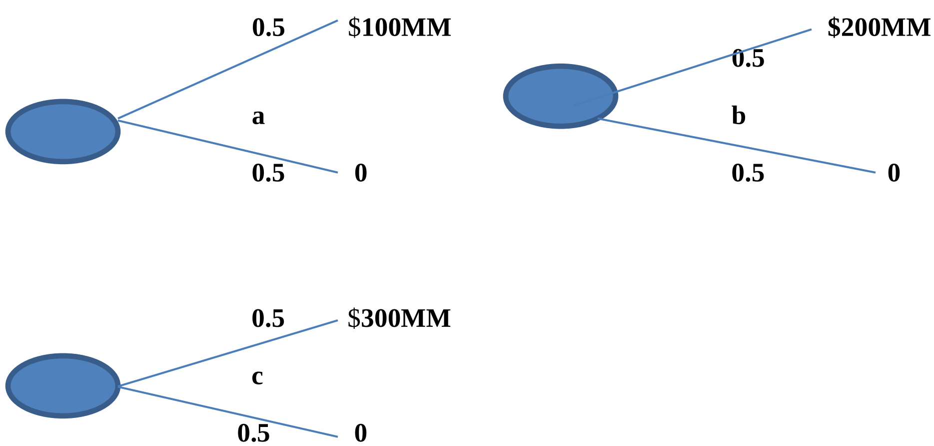


Table 3.1 Estimated

Riskless Asset Parameters	**a**	**b**	**c**
Expected Value, EV(μ)	\$50.0MM	\$100.0MM	\$150.0MM
Variance(σ^2)	2,500	10,000	22,500
Standard Deviation(σ)	50.0	100.0	150.0
Performance Index, PI(μ/σ)	**1.000**	**1.000**	**1.000**

Expressions to estimate the expected value and standard deviation for a 2-Outcome prospect are given in the Appendix. For instance, for project (a),

Expected Value, $\mu = (0.5)\ (100MM) + (0.5)\ (0) = \$50\ MM$

Variance, $\sigma^2 = \Sigma P_i (x_i - \mu)^2 = 2{,}500$

Standard Deviation, $\sigma = 50$ MM

Performance Index, PI = $\mu/\sigma = 1.000$

The Performance index is a numerical value of 1.000 for each of the three projects, though the value of success tripled from $100MM for project (a) to $300 MM for (c). PI clearly ignores the increasing value of success and cannot differentiate the superiority of project (b) over (a) or project (c) over (a). All three projects are ranked the same in terms of value created per unit of standard deviation. The increasing value created is exactly matched by the increasing variance. In real life investment situations, the three projects will not be the viewed the same by the rational investor. Project (c) will be viewed more favorably because the discerning investor recognizes that the increasing variance is on the upside. The methodology of estimating the PI does not distinguish between downside and upside variance. This is the Paradox of aversion to incremental reward and can sometimes lead to incorrect ranking, and investment decision making.

PAIR occurs in the use of the PI because the measure (PI) fails to distinguish between "Good variance" (opportunity) and "Bad "variance (risk). A method for modifying PI to correct for PAIR is shown in Chapter 4.

3.19 Expected Utility Maximization - PAIR

Recall Equation (3.57) $W_{opt} = \frac{RT}{V+C} \ln \frac{P_s V}{P_f C}$

This shows that optimum working interest is a direct function of the Risk Tolerance, RT and an inverse function of the value of success, V and failure cost, C. The inverse relationship between working interest and cost of failure is intuitively expected and understandable. However, the inverse relationship of working interest with success value is counter intuitive and predicts that the more the success value the less working interest be taken after some value of working interest (the optimum working interest) – which is the Paradox of aversion to reward, the primary focus of this research study.

Recall from (3.22), variance is given by $\sigma^2 = (V + C)^2 (P_s P_f)$

$(V + C) = \sigma/(P_s P_f)^{1/2}$ …………………………………………………………………. (3.62)

Therefore $W_{opt} = \frac{RT(P_s P_f)^{1/2}}{\sigma} \ln \frac{P_s V}{P_f C}$ …………………………………………………..… (3.63)

In this research study, the inverse relationship between optimum working Interest and variance is explicitly shown (3.60).

From (3.63), Optimum Working Interest ***varies inversely*** as the square root of the variance (Standard Deviation). Working Interest increases until the optimum working interest level is reached and then decreases (with variance). The problem with this relationship is that the increase in variance may be a result of exceptional high returns or significant increase in the value of success, V in the 2-Outcome prospect. The relationship recommends less working interest be taken with increasing success value, V – this is the paradox of aversion to incremental reward in an EU context. The flaw in the relationship is because the variance shown in Equation 3.60 is the total variance which includes upside deviation from the expected return (mean or expected value). Deviation from unexpected high returns does not fit into the traditional definition of risk the investor seeks to avoid – unexpected high gain should be an opportunity and occurs not infrequently in risky investments. Upside deviation is "Good Risk", opportunity or windfall and in these situations, the more stake the investor should take as working interest. The traditional expected utility maximization procedures do not distinguish between upside and downside deviation and hence PAIR. This study therefore recommends that risk be decomposed into upside and downside risk - the motivation for the Mean-semi-variance analysis in section 3.13.

3.20 Proposed Hybrid Models

Recall from Expected Value Analysis of the 2-Outcome Prospect,

$$EPV(W) = P_s(WV) - P_f(WC) = W[P_s V - P_f C]$$

$$W = \frac{EPV}{[P_s V - P_f C]}$$

Working interest, W is a linear function of expected present value and increases with increasing expected value, EV. The higher the success value, the higher the expected value and hence, the higher the working interest recommended.

The hybrid model this study proposes is of the form:

$$\boldsymbol{RAV} = \begin{cases} -\boldsymbol{RT}\ \ln\left[\boldsymbol{P_s e^{-WI*V/RT}} + \boldsymbol{P_f e^{WI*C/RT}}\right] & \boldsymbol{0 < W < W_{opt}} \\ \boldsymbol{W\left[P_s V - P_f C\right]} & \boldsymbol{W > W_{opt}} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.64)}$$

The model combines the positive attributes of expected utility by defining the appropriate working Interest up to the Optimum value and then recommends increasing working interest beyond optimum, as expressed in the expected value relationship, thereby correcting for the Paradox of aversion to incremental reward (PAIR).

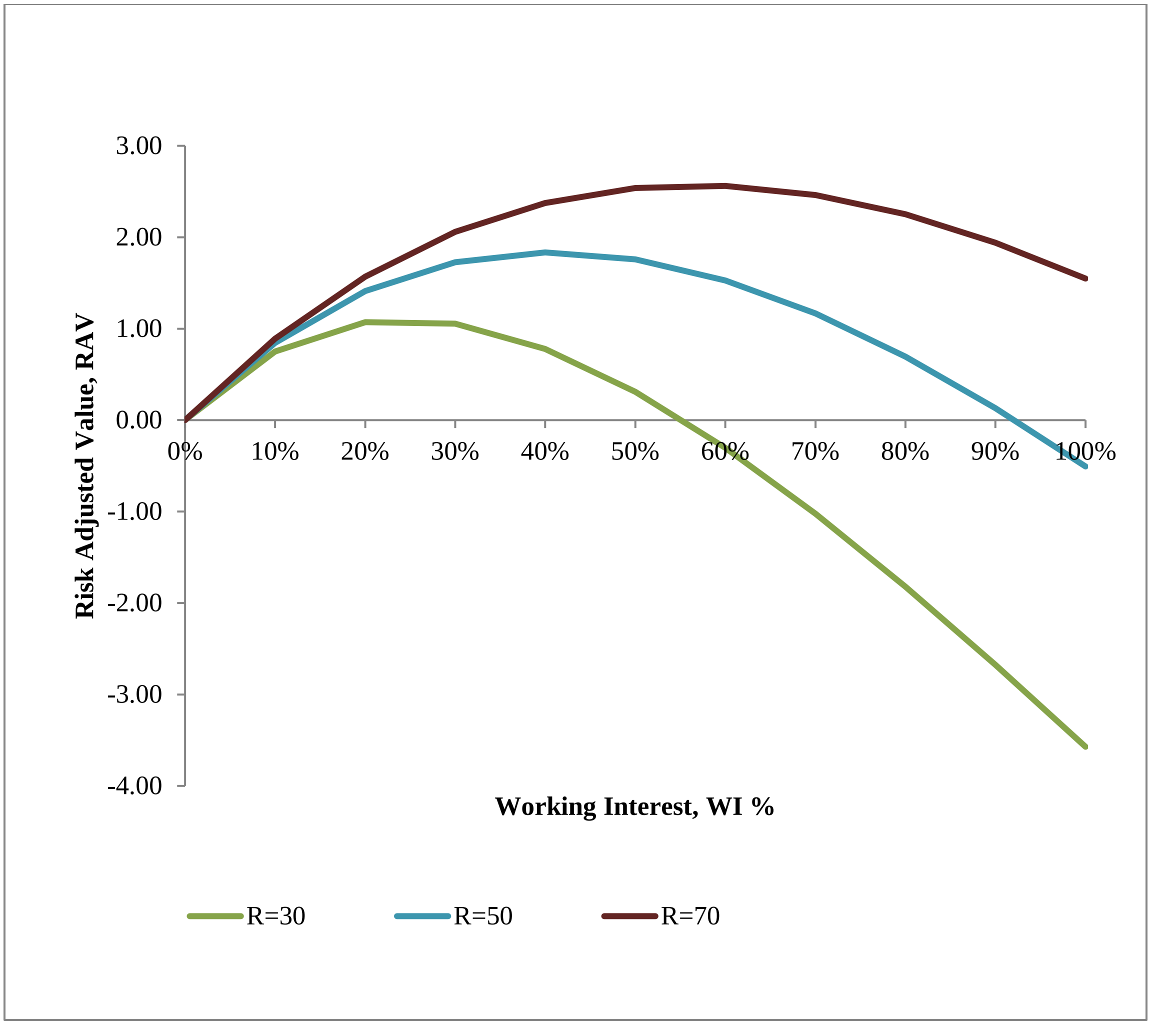


Figure 3.4 RAV versus Working Interest, WI – Exponential

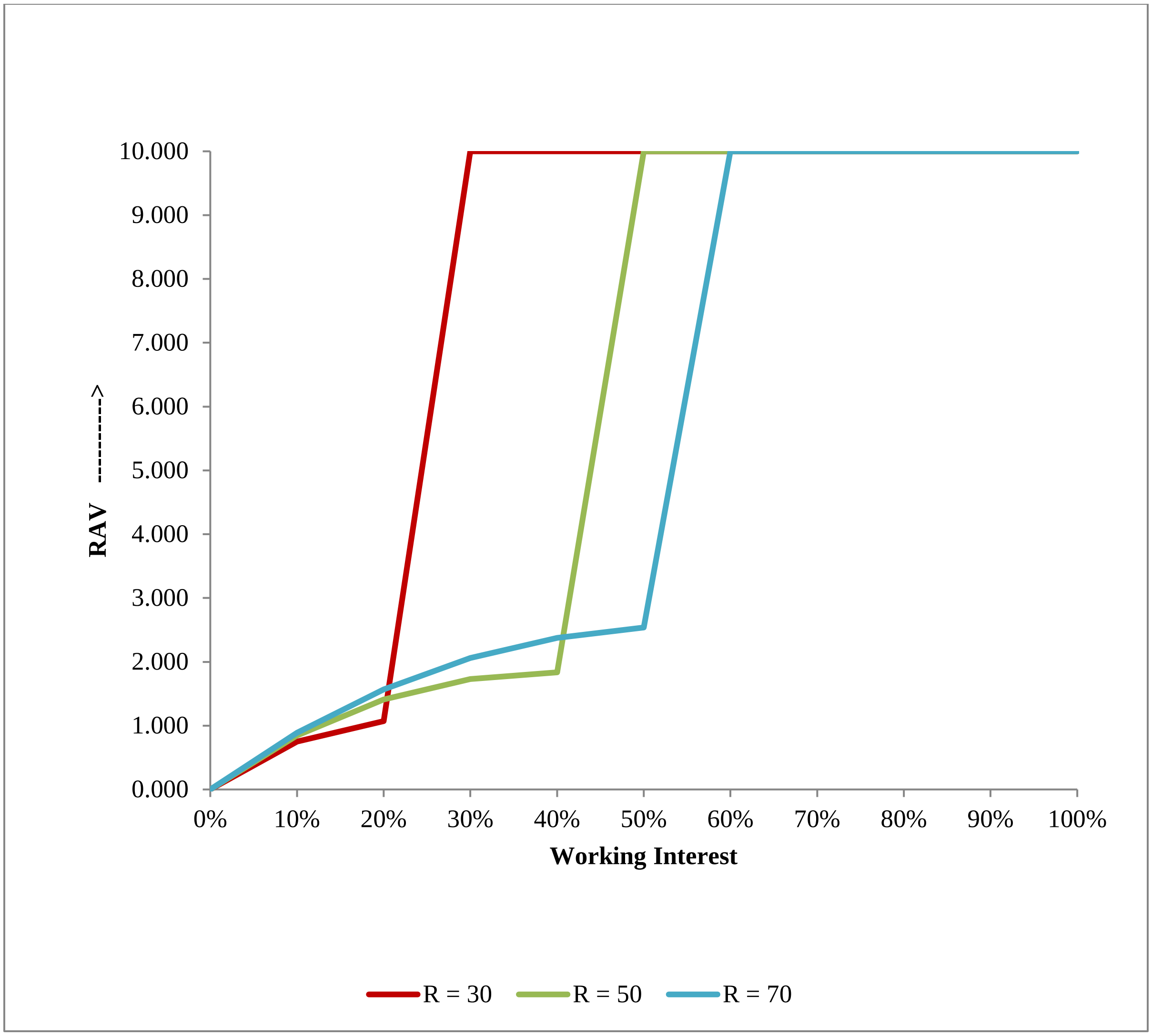


Figure 3.5 RAV versus Working Interest-Exponential/Linear Hybrid

A fuller discussion of the Hybrid Model – combination of the Exponential and Linear Utility Models is given in the Analysis and Discussion section. Figure 3.4 shows the RAV for the Exponential Risk Model, while Figure 3.5 shows the proposed Hybrid (Exponential/EV) Model for R values of 60, 80 and 90 Million dollars respectively which shows more explicitly the transition from Exponential to EV (Straight Line) when choosing participating working interest level in a risky prospect Generally, the higher the Risk Tolerance, R, the higher optimum working interest lies in the expected utility maximization region. The Paradox of aversion to incremental reward occurs more quickly for lower R levels and hence the transition to expected value (EV) regions also occurs more rapidly (Figure 3.6).

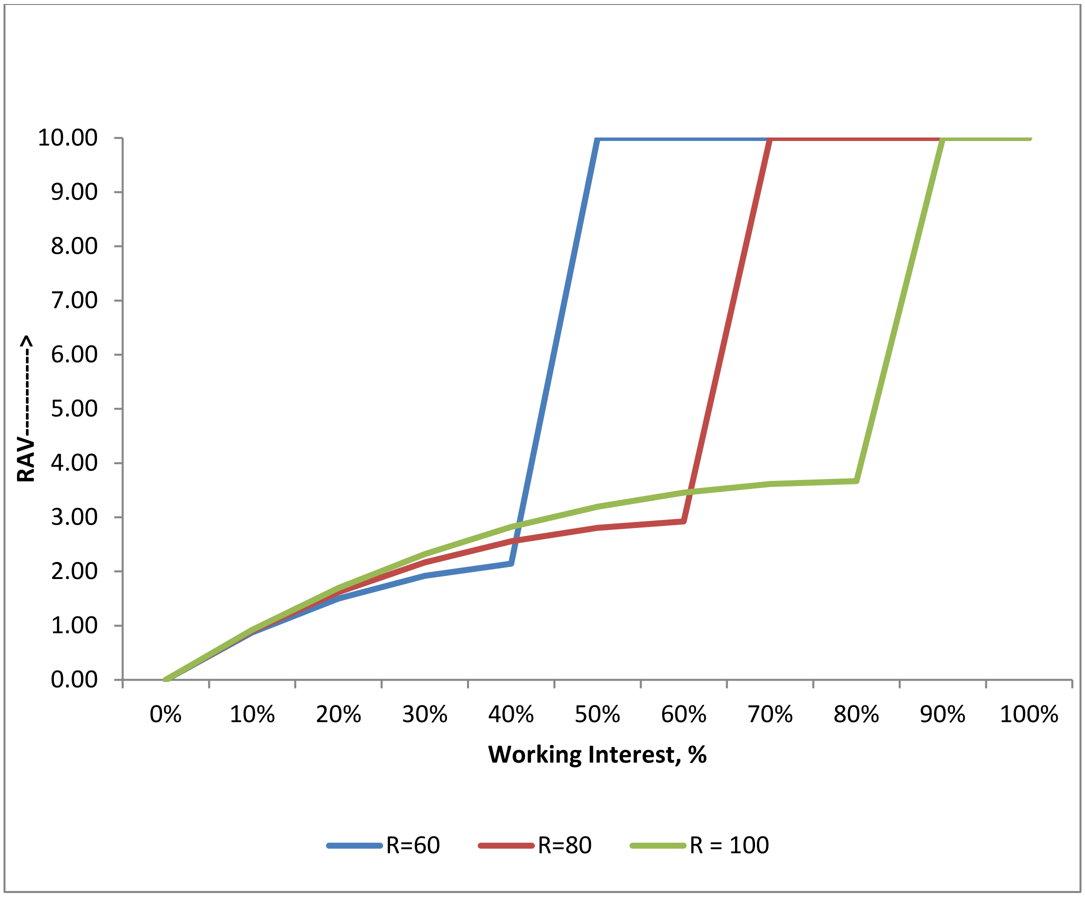


Figure 3.6 RAV **versus Working Interest - Exponential/Linear Hybrid**

3.21 Risk Adjusted Value Analysis - Use of the Hyperbolic Preference Function

Use of Hyperbolic Utility Function of the form:

$$U(x) = 1 - tanh(x) \quad (3.65)$$

Where x = terminal wealth and r = risk aversion level =1/millionths

We also use the two outcome prospect

The Expected Utility (EU) of can be expressed by the following:

$$EU(x) = P_s\left[1 - \tanh\left(\frac{\mathrm{WV}}{\mathrm{RT}}\right)\right] + P_f\left[1 - tanh\left(\frac{-WC}{RT}\right)\right] \quad (3.66)$$

$$EU(x) = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \quad (3.67)$$

The Certainty Equivalent (CE) of this expected utility is the Risk Adjusted value and assuming it is also of the exponential form (Lerche and McKay), it can be represented as:

$$e^{-\frac{RAV}{RT}} = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \quad (3.68)$$

$$RAV = -RT\ ln\left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \quad (3.69)$$

The Risk Adjusted Value RAV) is also a nonlinear function of the Working Interest, W. Differentiating RAV with respect to W and equating to zero, RAV has maximum value at Working Interest expressed implicitly by:

$$cosh\left(\frac{W_{opt}C}{RT}\right) = \left(\frac{P_f C}{P_s V}\right)^{1/2} cosh\left(\frac{W_{opt}V}{RT}\right) \quad (3.70)$$

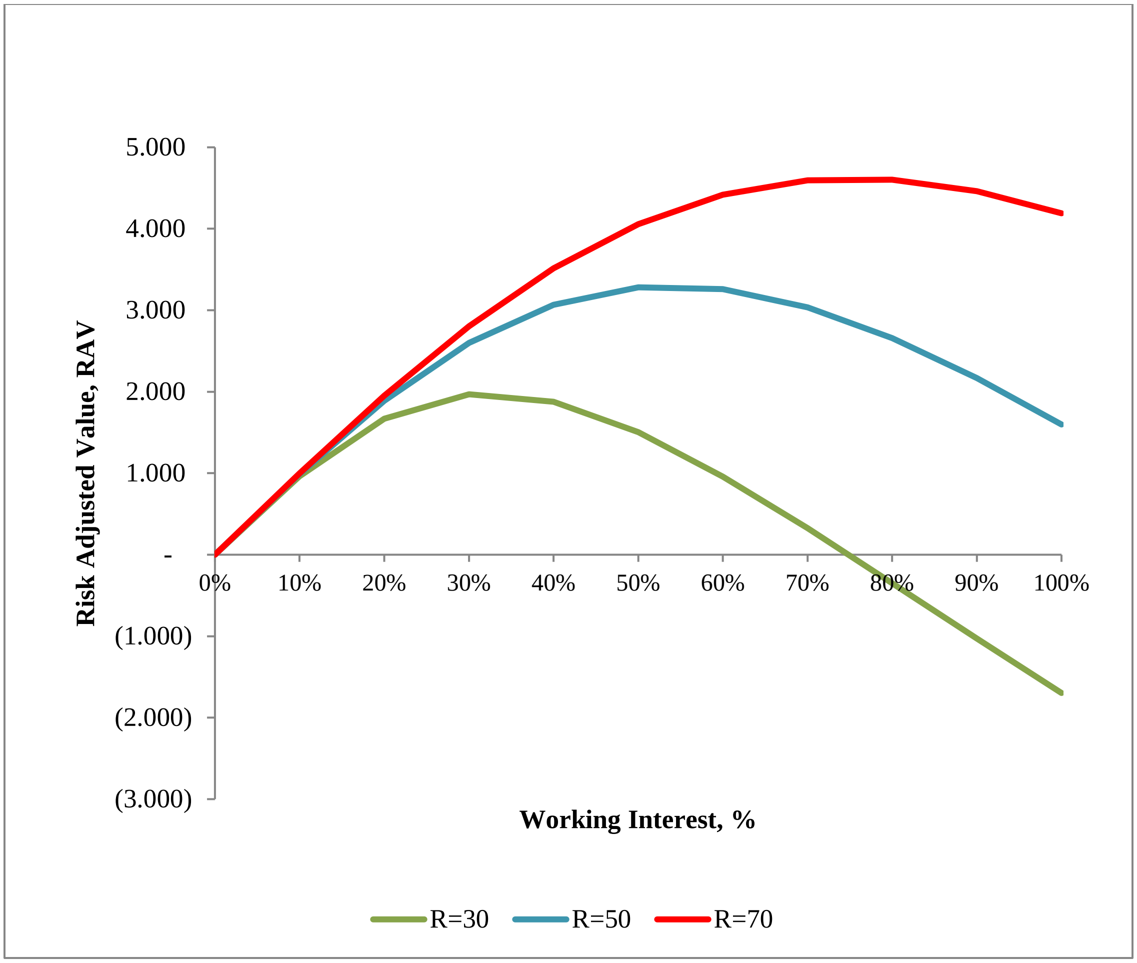


Figure 3.7 RAV versus Working Interest - Hyperbolic Model

The Hyperbolic Model also suffers from the Paradox of aversion to reward. Working Interest peaks at a certain level and then decreases because of the inverse relationship with Risk Adjusted Value, though in this instance, it is not very obvious since optimum working interest is expressed implicitly in Equation (3.67)

A similar treatment is applied to the RAV analysis analogous to the hybridization of the exponential model. Thus combining the hyperbolic and the linear models, a hybrid model of Risk Adjusted Value that corrects for the Paradox of aversion to reward can be expressed thus:

$$\boldsymbol{RAV} = \begin{cases} -\boldsymbol{RT}\ \boldsymbol{ln}\left[1 - \boldsymbol{P_s tanh}\left(\frac{\boldsymbol{WV}}{\boldsymbol{RT}}\right) + \boldsymbol{P_f tanh}\left(\frac{\boldsymbol{WC}}{\boldsymbol{RT}}\right)\right] & \boldsymbol{0 < W < W_{opt}} \\ \boldsymbol{W}\left[\boldsymbol{P_s V} - \boldsymbol{P_f C}\right] & \boldsymbol{W > W_{opt}} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.71)$$

We will use the approximation of the exact implicit expression for Optimum Working Interest given. Thus Optimum Working Interest can be represented by:

$$W_{opt} = \frac{1}{2}\left(\frac{RT}{V}\right)\ln\left(\frac{4P_sV}{P_fC}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.72)$$

Figure 3.7 shows the RAV profile for different risk tolerances using the hyperbolic model, while Figure 3.8 shows the proposed combination of the Hyperbolic and Linear (EV) risk adjusted value models. Just as in the Exponential/EV model, transition to the Expected value region of the RAV profile occurs more rapidly for lower risk tolerance levels.

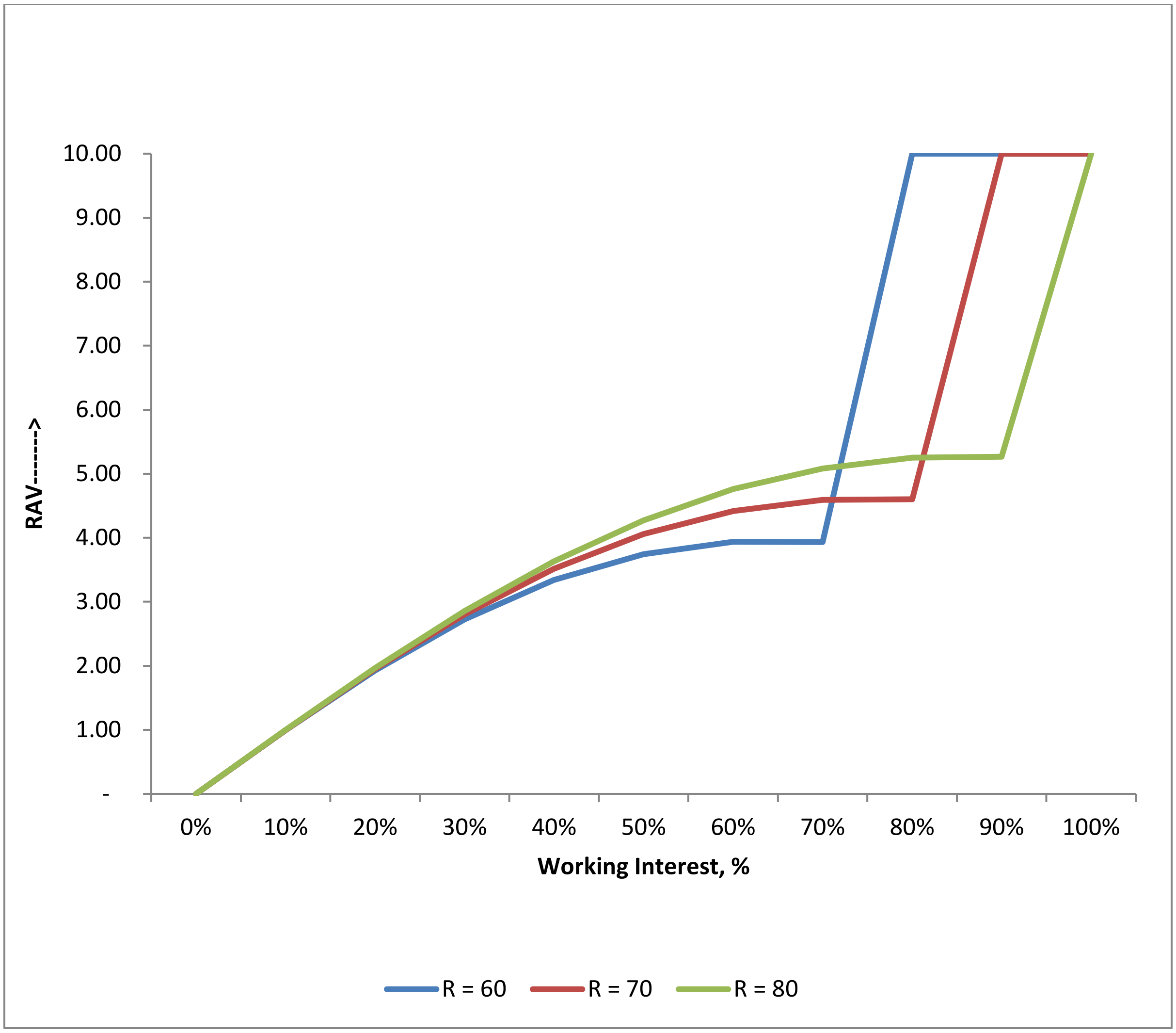


Figure 3.8 RAV versus Working Interest-Hyperbolic/EV Hybrid

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Results and Discussions

The results and discussions of the findings of this research work will proceed along the following lines:

I. Demonstrating the procedure to correct for Paradox of aversion to reward (PAIR) in an expected value (EV) and developing a modified performance index that correctly ranks risky prospects

II. Correcting for PAIR in expected utility context - analysis and application of the proposed hybrid models- exponential/linear and hyperbolic/linear models to determine working interest for single prospect and portfolio of prospects under unlimited and limited capital situations

III. Sensitivity Analysis and Monte Carlo Simulation to determine the relative impact of risk tolerance, RT, success value, V, cost C and success factor, Ps on Risk adjusted value (RAV)

IV. New applications of RAV analysis – exploring the impact of differing levels of risk tolerance on Bid (asset) values by participating partners in a bidding group or Farm out arrangement.

A Numerical Example by Lerche and Mackay will be used to illustrate the mean-variance and Mean-Semi-Variance Models cited in the Theoretical Models in the previous section

The same Numerical example will be used to illustrate the Exponential and the Hyperbolic Risk Preference Models in the determination of Optimum Working Interest for risky assets and the attendant problems associated with these models especially the Paradox of Aversion to Reward

4.2 Correcting for PAIR in EV Context and Modification of the Performance Index (PI)

Recall the three (3) two outcome prospects shown (Prospects a, b and c) from Chapter 3. The Prospects are shown in **Figure 4.1**, burdened with failure cost of $15 MM each which reflects actual risky investment situation-there is tangible wealth exposure to risk (of loss). The expected values, standard deviations and performance indices are recalculated and shown in Table 4.1

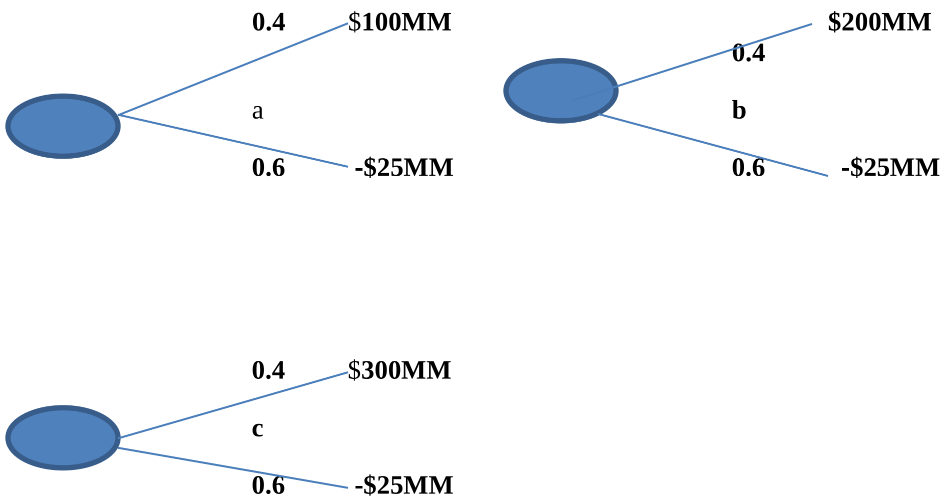


Figure 4.1Risky Prospects (a, b, c)

Table 4.1- Correcting for PAIR	**a**	**b**	**c**
Expected Value, EV(μ)	$42.5MM	$92.5MM	$142.5MM
Variance(σ^2)	3,306	11,556	24,806
Standard Deviation(σ)	57.5	107.5	157.5
Performance Index, PI(μ/σ)	**0.739**	**0.860**	**0.905**

Table 4.2 Decomposing Variances	**a**	**b**	**c**
Expected Value, EV	42.50	92.50	142.50
Upside Variance, σ_{sup}^2	1,653	5,778	12,403
Downside Variance, σ_{sd}^2	1,653	5,778	12,403
Total Variance, σ^2	3,306	11,556	24,806
$\tau = \sigma_{sup}/\sigma_{sd}$	***1.000***	***1.000***	***1.000***
PI'	1.05	1.22	1.28

The PI increases from Prospect (a) to (c) reflecting the increasing success values. In terms of ranking, PI correctly ranks (c) higher than (b) and (a) the least favorable. However, the increase in PI from Prospect (a) to (b) is a mere 16% though the success value has doubled, and from Prospect (a) to (c) is 22% though success value tripled from $100MM to $300MM. While relationship between the PI and success value is not linear, the flaw highlighted in PI estimation in the riskless example shown in **Section 3.18** cannot be ignored. The correct ranking notwithstanding, PI is not reflecting the magnitude of the increase in success values from Prospect (a) to (c). To the extent that it can impact investment decision making, a formal process to differentiate the uncertainty structure of a risky investment into upside and downside and incorporate such in the performance index will be beneficial.

For a two outcome risky prospect with P_s, P_f representing chances of success and failure, V and C are success value and failure costs respectively (Appendix A)

Downside variance $\sigma_{sd}{}^2 = P_f P_s{}^2 [V + C]^2$ ……………………………...……… (4.1)

Upside variance $\sigma_{sup}{}^2 = P_s P_f{}^2 [V + C]^2$ ……………………………………. (4.2)

The ratio of the upside and downside standard deviations can be easily estimated from the following relationship

$$\tau = {}^{\sigma_{sup}}/_{\sigma_{sd}} = \frac{P_f P_s{}^{1/2}[V+C]}{P_s P_f{}^{1/2}[V+C]} = \left[\frac{P_f}{P_s}\right]^{1/2} \quad \text{(4.3)}$$

The ratio, τ is shown to be solely dependent on the failure and success chance factors and independent of the value of success, V and failure cost, C. The ratio remains the same at 1.000 though success value V tripled from $100MM to $300MM (Table 4.2). This is as a result of the "shifting" of the mean or expected value (EV) from $42.50MM for Prospect a to $142.50MM for Prospect c. Estimating the PI based only on downside variance, in this instance does not eliminate PAIR. Hence, the process of decomposing the variance must be carefully done to avoid the "shifting mean" problem.

The "shifting" of the mean can be avoided by choosing a particular threshold value, for example zero (0) or any other value corresponding to some level that the Decision Maker or investing institution explicitly sets as objective, similar to a "Hurdle rate". Choosing zero as the threshold, the downside and upside standard deviations of the 2-outcome risky prospects are given in the following expressions:

$$\sigma_{sd} = {P_f}^{1/2} C \quad (4.4)$$

$$\sigma_{sup} = {P_s}^{1/2} V \quad (4.5)$$

$$\tau = {}^{\sigma_{sup}}/_{\sigma_{sd}} = \left(\frac{P_S}{P_f}\right)^{1/2} \left[\frac{V}{C}\right] \quad (4.6)$$

τ not only depends on the success and failure chances, but also on the success value, V and failure cost, C. Table 4.3 shows a constant value for the downside semi-deviation, a more accurate reflection of the fact that only the success value has been changed from Prospect (a) to (c); the failure cost has been kept constant at $15MM. The upside variance shows dramatic increase from (a) to (c) also more reflective of the increase in success value. Expectedly, τ increases from 4.01 to 13.44. More importantly, the modified PI increases as the success value increases which more accurately reflects the definition of that metric and the investment characteristics of the three risky prospects. Generally, keeping all the other characteristics of the risky prospect constant, the higher the success value, the higher the Performance Index, as it should be.

Table 4.3

Modifying PI Based Solely on Downside Variance	**a**	**b**	**c**
Expected Value, EV	42.50	92.50	142.50
Upside Variance, σ_{sup}^2	5,000	20,000	45,000
Downside Variance, σ_{sd}^2	113	113	113
Total Variance, σ^2	5,113	20,113	45,113
$\tau = \sigma_{sup}/\sigma_{sd}$	***6.67***	***13.33***	***20.00***
PI'	4.01	8.72	13.44

4.2.1 Correction for PAIR in Expected Utility (EU) Context – Analysis of Prospect A

This section shows PAIR and its correction in an expected utility maximization (EU) context. The proposed hybrid models are applied in determining working interest using a numerical example from Lerche and Mackay (Risky prospect A) and a portfolio of prospects (A-E)

<u>Risky Prospect A</u>

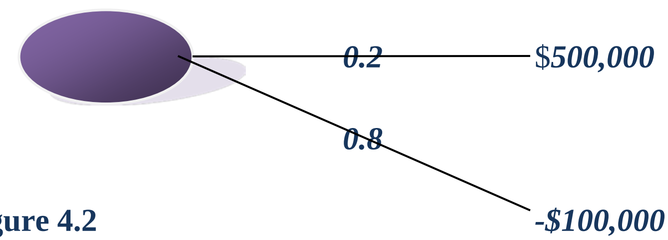


Figure 4.2

The Risk Tolerance, **RT** for this Company = **<u>$1,000,000</u>**

Working Interest, WI = 100% for mean-variance Calculations.

The first step in the analysis of the uncertainties in this risky prospect will be the decomposition of the variance into upside and downside semi-deviations to assess which of the two deviations is contributing most to the total uncertainty in this prospect.

Table 4.4

Expected Value, Variance, Semi-Variance Calculations for Risky Prospect A

Expected Value @ 100% WI =	\$20,000
Total Variance σ_T^2 =	57.6×10^6
Standard Deviation, σ_T =	240,000
Downside Variance, σ_s^2 =	11.52×10^6
Downside Semi-Deviation, σ_s =	107,331
Upside Variance, σ_u^2 =	46.08×10^6
Upside Semi-Deviation, σ_u=	214,663
Ratio of Upside to Down Side Deviation, τ =	2.000

Specifying Zero (0) as "Threshold"

Downside Semi-Deviation, σ_s =	89,443
Upside Semi-Deviation, σ_u=	223,607
Ratio of Upside to Down Side Deviation, τ_0=	2.5

For the risky prospect, decomposition of total variance shows that upside variance is four times downside variance (since ratio of upside semi-deviation to that of downside = 2). Thus:

$\sigma_T{}^2 = \sigma_u{}^2 + \sigma_s{}^2$

Since $\sigma_u{}^2 = 4\,\sigma_s{}^2$

$\sigma_s{}^2 = \sigma_T{}^2/5.$

Only 20% of the total variation is due to downside risk, while 80% is due to upside uncertainty. The decomposition of the uncertainty structure for this prospect is repeated in the lower half of the table using zero as the “threshold”. The result is a ratio of Opportunity to Risk of 2.5 (even higher than the 2 that was previously estimated) indicating prospect has even more upside potential. Knowledge of the opportunity to risk ratio gives an indication of the likelihood of the occurrence of the PAIR in an EU maximization context. This prospect (τ=2.5) shows most of the variance is on the upside. The Risk adjusted value (RAV) is plotted against the working interest in Figure 4.3 using the exponential and hyperbolic preference models and the stipulated Company Risk Tolerance RT = $1 Million.

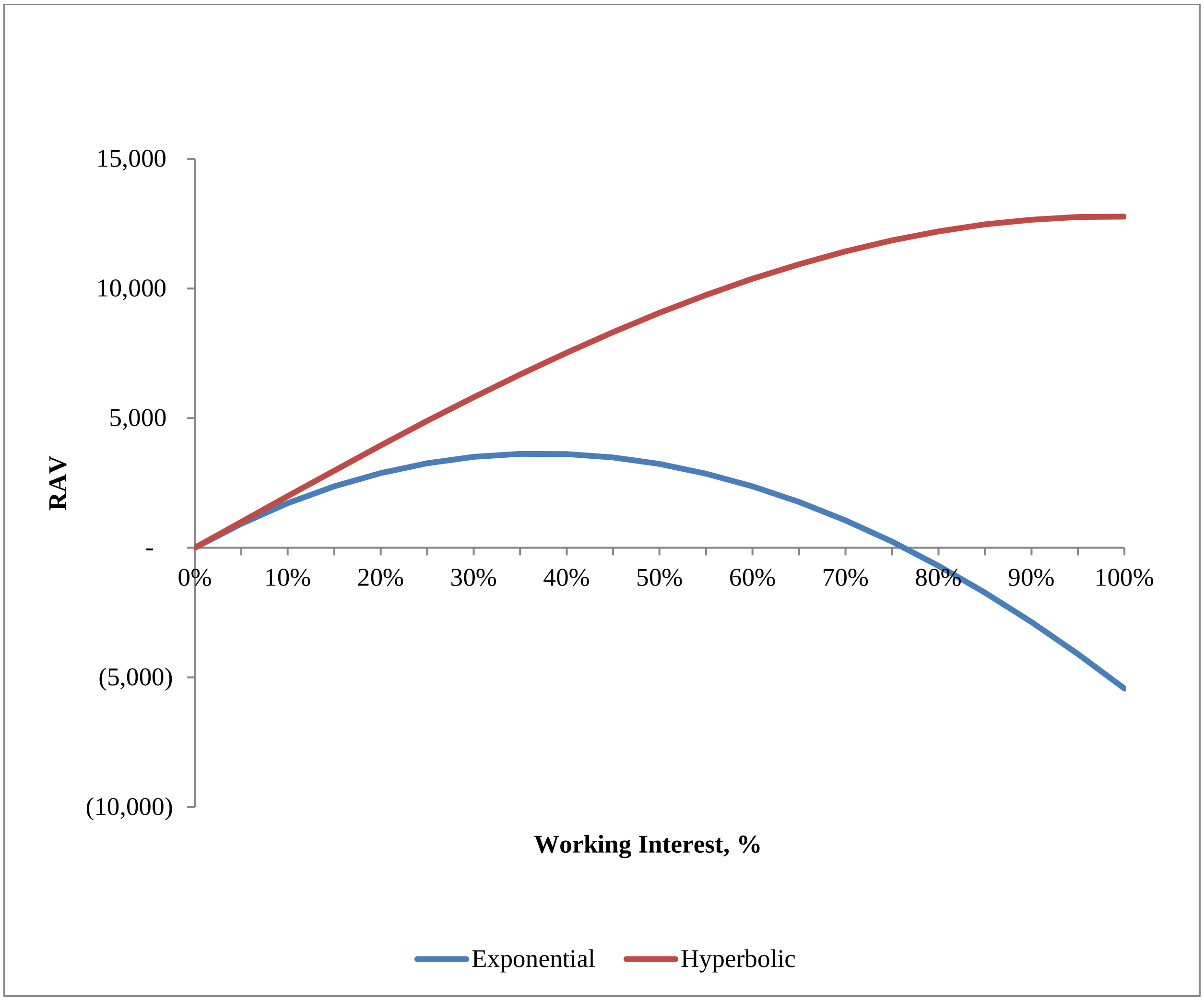


Figure 4.3- RAV versus WI - Exponential and Hyperbolic Models-Prospect A

For the exponential model, the Risk adjusted value increases from a working interest above zero until a value of about 40% (exact value is 35%) and then declines. For the hyperbolic model, the RAV continues to rise for almost all values of working interest. The exponential model predicts taking a smaller working interest than the hyperbolic. In the light of the decomposition of variance into upside and downside for this particular prospect, (and recalling that the Opportunity to Risk Ratio, τ=2), the hyperbolic model reflects more the potential for the upside than the downside. In fact, the hyperbolic model recommends taking a 100% working Interest for this model (the same as the expected value model predicts) with a value of RAV=$12.77 Million. In the extreme, working Interest levels above 80% show negative Risk Adjusted Values for the exponential model.

4.3 Proposed Hybrid Models – Exponential/Expected Value, Hyperbolic/Expected Value Models

Two hybrid Models correcting for the Paradox of Aversion to reward in the determination of working interest in Risky ventures were proposed in Chapter Three (**Section 3.20**). The two models combine the positive attributes of maximizing expected utility by defining the appropriate working interest up to the optimum value and then recommend increasing working interest beyond optimum, as expressed in the expected value relationship. The fundamental basis of expected utility that have been established from numerous prior studies and the expressed linear relationship between working interest and expected value are embedded in the hybrid models. The hybrid models therefore embody the theoretical foundations and tenets of expected utility maximization and the fundamental logic underpinning the expected value concept.

The first proposed Model is a hybrid of the **Exponential preference** function and **Expected value** and is given in the following relationship:

$$\boldsymbol{RAV} = \begin{cases} -\boldsymbol{RT}\ln\left[\boldsymbol{P_s e^{-WI*V/RT} + P_f e^{WI*C/RT}}\right] & \boldsymbol{0 < W < W_{opt}} \\ \boldsymbol{WI * \left[P_s V - P_f C\right]} & \boldsymbol{W > W_{opt}} \end{cases} \quad \textbf{……………………...…. (4.7)}$$

This model is the Exponential/Expected Value model or the Exponential/Linear model. The RAV versus working interest relationship is shown in Figure 4.4 for three Risk Tolerance levels (RT = $1Million, $1.3 Million and $1.5 Million). For the Tolerance levels considered, beyond the Optimum Working Interest, RAV rapidly increases in accordance with the linear relationship between Expected Value and Working Interest in the EV Model. The higher the Risk Tolerance, RT the higher the Optimum Working Interest lies in the Expected Value Maximization region. Figure 4.4 also shows RAV increases with increasing working interest until a working interest of about 60% (for RT=$1.5 Million), when RAV equals a maximum value (or the Expected Value) for the prospect.

The second proposed model is a hybrid of the Hyperbolic Tangent preference function and Expected value and is given in the following relationship:

$$\boldsymbol{RAV = \begin{cases} -RT\ ln\left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] & 0 < W < W_{opt} \\ WI * \left[P_s V - P_f C\right] & W > W_{opt} \end{cases}} \quad \text{………………..} \ \textbf{(4.8)}$$

Working Interest is determined using Expected Utility (Hyperbolic preference function) up to the Optimum Working Interest and there is a transition to the Expected Value Model beyond the Optimum Working Interest. The RAV versus working Interest Relationship is shown in Figure 4.5 for three Risk Tolerance Levels (RT = $1 Million, $1.3 Million and $1.5 Million). The RAV versus working interest relationship profile in this Hyperbolic/EV Model resembles two linear models. This is consistent with the earlier observation that the Hyperbolic Preference function is a more Optimistic, more risk tolerant function than the Exponential.

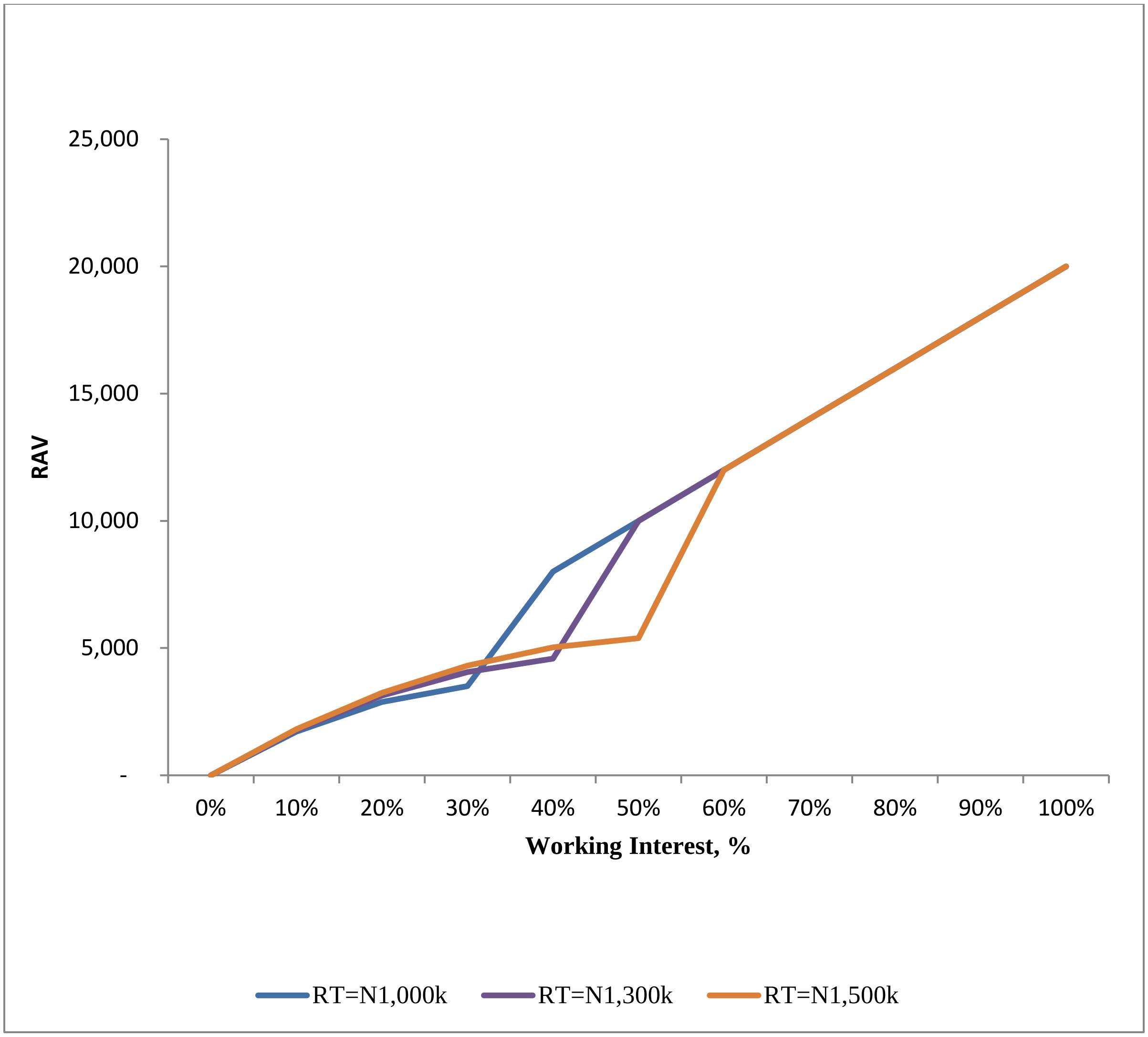


Figure 4.4 RAV versus WI - Exponential/Linear Model – Prospect A

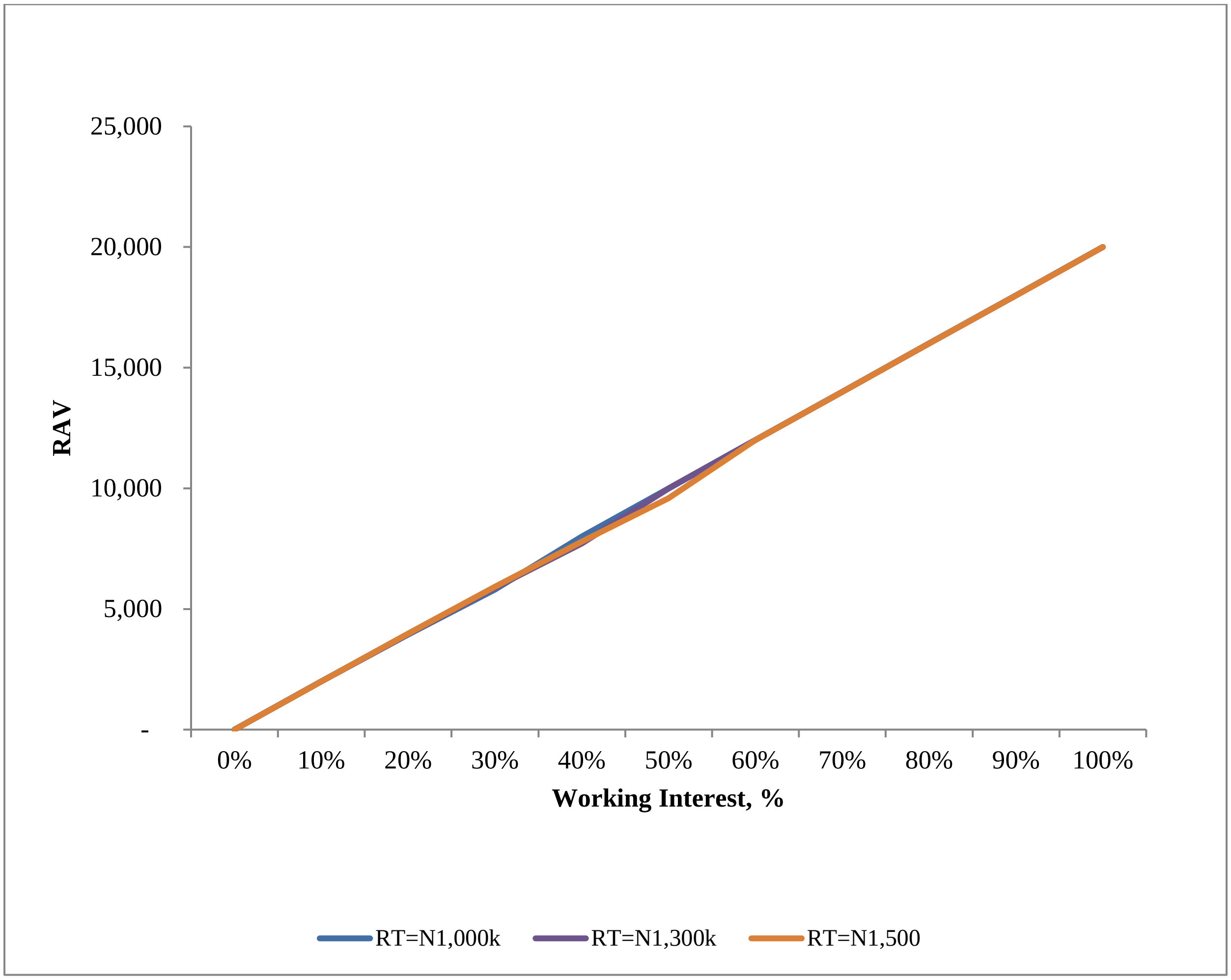


Figure 4.5 RAV versus WI - Hyperbolic/Linear Model- Prospect A

Figure 4.6 shows the two proposed hybrid models for the risky prospect under consideration. The Hyperbolic/EV model consistently predicts higher risk adjusted values for the range of working interest in the expected utility maximization range, confirming the optimistic nature of the hyperbolic preference function relative to the exponential. The "kink" shows the transition from the utility maximization to expected value and is very pronounced for the exponential, much less pronounced for the hyperbolic. The hyperbolic model essentially "tracks" the expected value model – for this particular risky prospect. Another prospect with much less opportunity to risk ratio, τ would show a marked transition from utility maximization to expected value. This observation will be investigated for Prospect B (which is a lot riskier) in the Portfolio of Prospects shown in Table 4.5 and Figure 4.8.

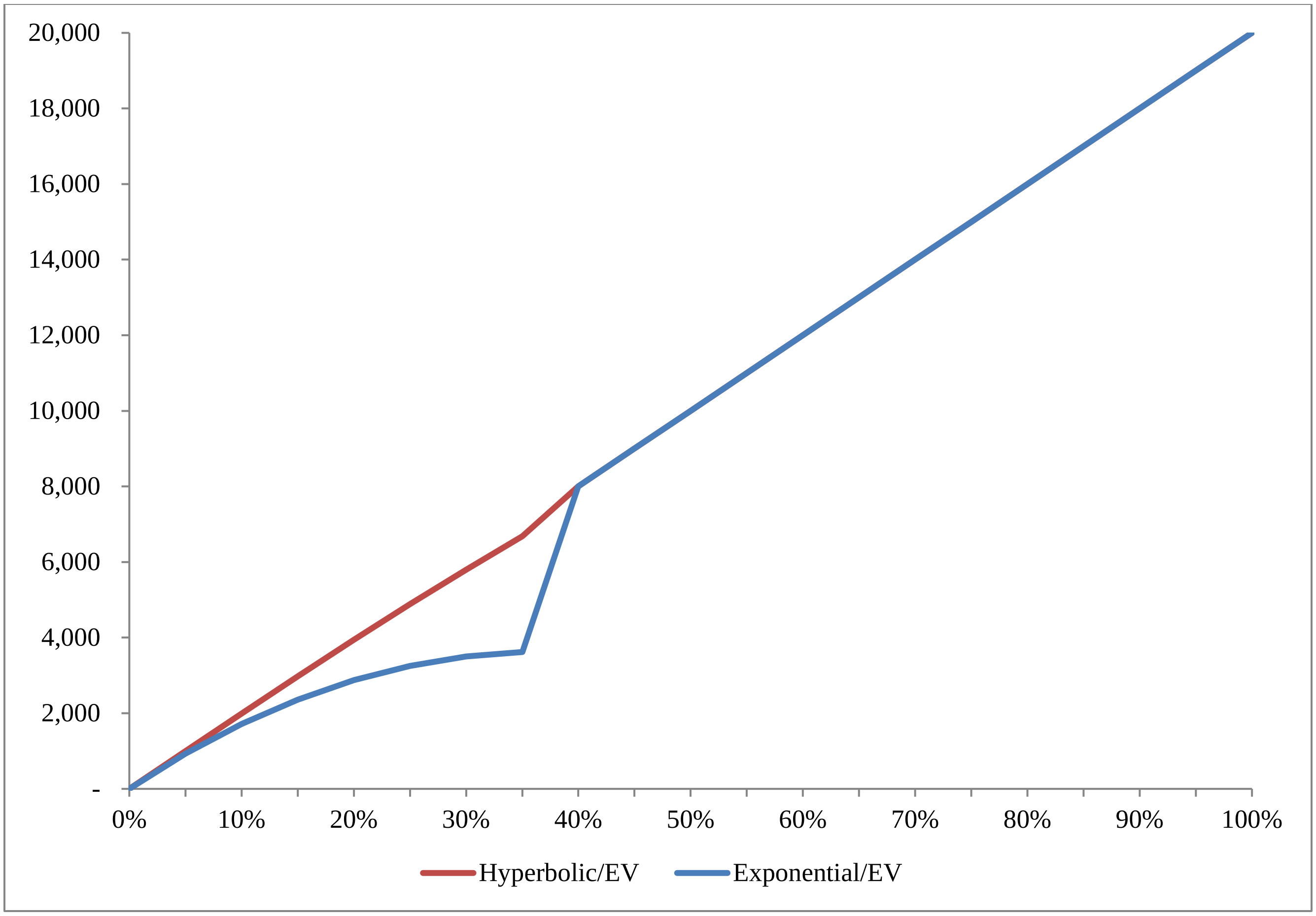
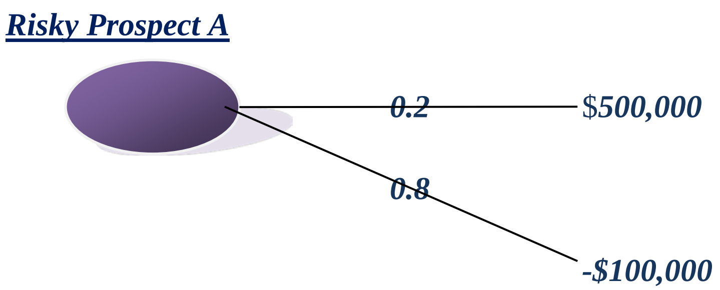


Figure 4.6 Exponential/EV and Hyperbolic/EV Model Profiles-Prospect A

4.4 Determination of Working Interest for Two Risky Prospects – A and B

Two risky prospects are analyzed for their riskiness and hence determination of the appropriate working interest to be taken by an investor using the Exponential, Hyperbolic and the Hybrid Models proposed. Prospect A has characteristics exactly like the risky prospect in Figure 4.2, while for Prospect B, the success probability Ps is slightly higher at 25%, the success value , V is four times larger than Prospect A at $2 Million dollars and the Cost is also larger at $0.5 Million dollars. We expect Prospect B to have a larger variance than A because of the range of the Success and Failure outcome values. However, as previously noted in this research, decomposing the total variance into the upside and downside gives an insight into how much of an opportunity or risk the two prospects present. This is shown in Table 5.3



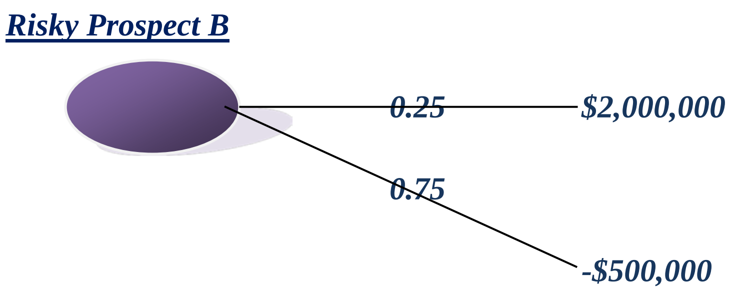


Figure 4.7 Lottery Diagrams of Two Risky Prospects – A and B

Table 4.5

Comparison of Expected Value (EV), Variance and Semi-Variance Risky Prospects A and B	Prospect A	Prospect B
Success Probability=	20%	25%
Failure Probability =	80%	75%
Net Present Value of Success, V=	500,000	2,000,000
Cost of Failure, C=	100,000	500,000
EV, Variance Calculations		
Expected Value @ 100% WI =	20,000	125,000
Standard Deviation, σ =	2.40.E+05	1.083.E+06
Downside Semi-Deviation=	107,331	541,266
Upside Semi-Deviation=	214,663	937,500
Ratio of Upside to Down Side Deviation, , τ=	2.000	1.732
Specifying Zero (0) as "Threshold"		
Downside Semi-Deviation, σ_s =	89,443	433,013
Upside Semi-Deviation, σ_u=	223,607	1,000,000
Ratio of Upside to Down Side Deviation, τ_0=	2.50	2.31

Prospect B's expected value is considerably larger than that of A ($125,000 compared to $20,000). However, total variance is also five times larger. Decomposing the total variance, shows that there is slightly more opportunity than risk in Prospect A (the Opportunity to Risk ratio for A is 2.5 while that for Prospect B is 2.31). Prospect B is therefore riskier than Prospect A. The risk adjusted value profiles for the two Prospects are shown in Figure 4.8. The Risk Adjusted Value for Prospect B peaks very quickly at 10% and then declines very rapidly. In actual fact, the decline in the RAV for Prospect B declines very sharply into negative values as working interest increases beyond 25%. The Optimum Working Interest for Prospect B therefore is about 10% with a corresponding RAV of $6,888. Prospect A's working interest as earlier determined is 35% with a Risk Adjusted Value of $3,619.

Working Interest determination for both Prospects was then done using the hyperbolic preference model at the same level of Risk Tolerance of $1 Million. The results are shown in Figure 4.9. Expectedly, the working Interests determined utilizing the hyperbolic model are higher than for the exponential model, and particularly for a prospect with significant downside risk like B, the RAV does not go into negative values as quickly as when the Exponential Preference function is used. The Hyperbolic Preference Model is more "tolerant" of high loss situations than the Exponential – the "stability management" characteristic of the Hyperbolic Preference Model. Not surprisingly, Optimum Working Interest recommended for Prospect B is 30% (much higher than the 11.5% predicted by the Exponential Model). For Prospect A, the RAV profile peaks at 100% (compared to the 35% predicted by the Exponential Model). The Hyperbolic Model RAV for Prospect A essentially approximates the expected value model which is linear in the EV-Working Interest relationship. Notice Prospect A's RAV continues to increase until 100% but slowly with each increment in working interest, there is still a positive difference between it and the Expected Value (the risk premium), an indication of Risk aversion.

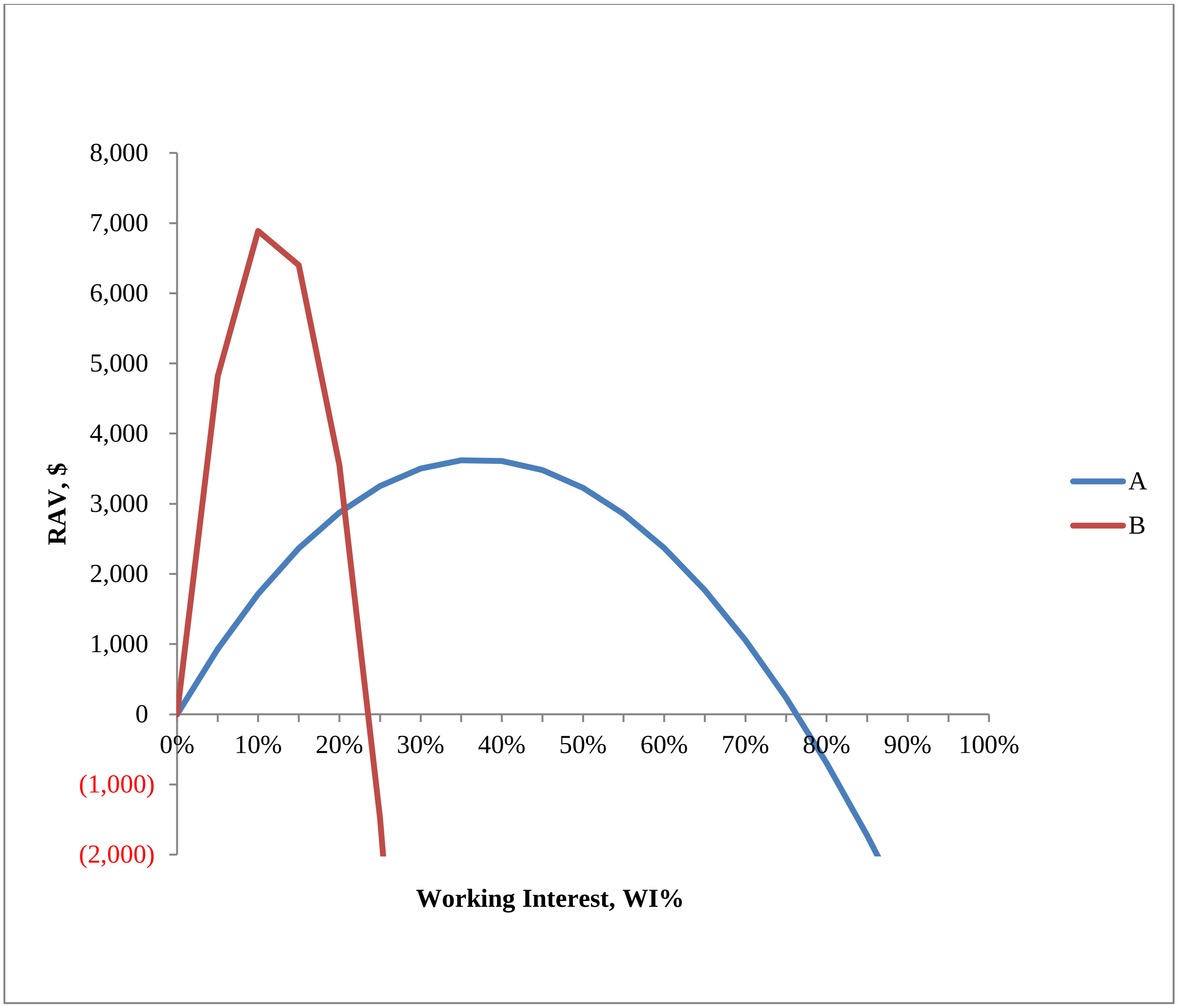


Figure 4.8- RAV versus WI for Prospects A and B- Exponential Model

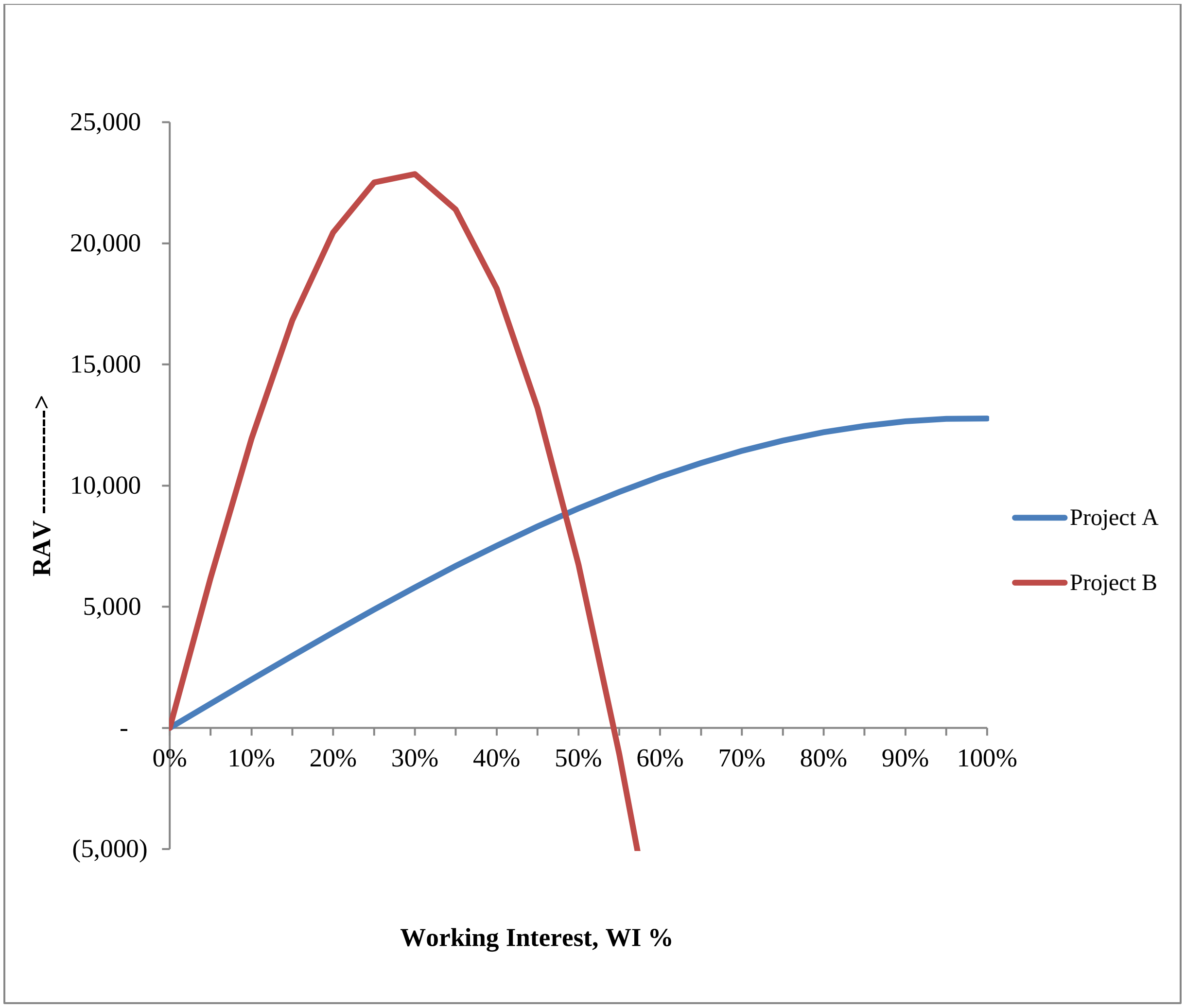


Figure 4.9- RAV versus WI for Prospects A and B-Hyperbolic Model

Figures 4.10 and 4.11 show RAV profiles for the proposed for A and B using the Exponential/EV Hybrid Model for RT = $1Milion and $5 Million. Prospect B completely dominates A both in the Exponential Preference function region, as well as the Expected Value region for the two risk tolerance levels. While Prospect B is seemingly riskier, the values of the success and failure outcomes and the success factor, Ps are much higher than for Prospect A. The higher the working interest, the more pronounced the domination (of Prospect B over A). Compare Prospects B expected value to that of A ($125,000 versus $20,000), a factor of 6. In Figures 4.8 and 4.9, the dominance of Prospect B over A Risk Adjusted Values for the Exponential and the Hyperbolic preference functions only occur between 0% and the Optimum Working interest – beyond the Optimum working interest, the profile of B drops rapidly due to impact of the larger failure cost, C. The Hybrid Models therefore would lead to a different decision by the investor (B is preferred to A).

Figures 4.8 and 4.9 also show the significant effect of Risk Tolerance (RT) on the RAV profile. The Optimum Working Interest for Prospect A is 15% (for RT = $1 Million) while the Optimum Working Interest for RT=$5 Million is 50%. The higher the risk tolerance, RT, the higher the optimum working interest determined – this is to be expected, since the risk tolerance of $5 Million can more easily accommodate the failure (cost) level of $500,000 than the tolerance level of $1 Million. This explains why the Optimum working interest for the lower risk tolerance level is at a low of 11.5%. The same effect occurs for Prospect B – at a higher risk tolerance of $5 Million, optimum working interest is 95% versus 37.2% at the lower risk tolerance of $1 Million.

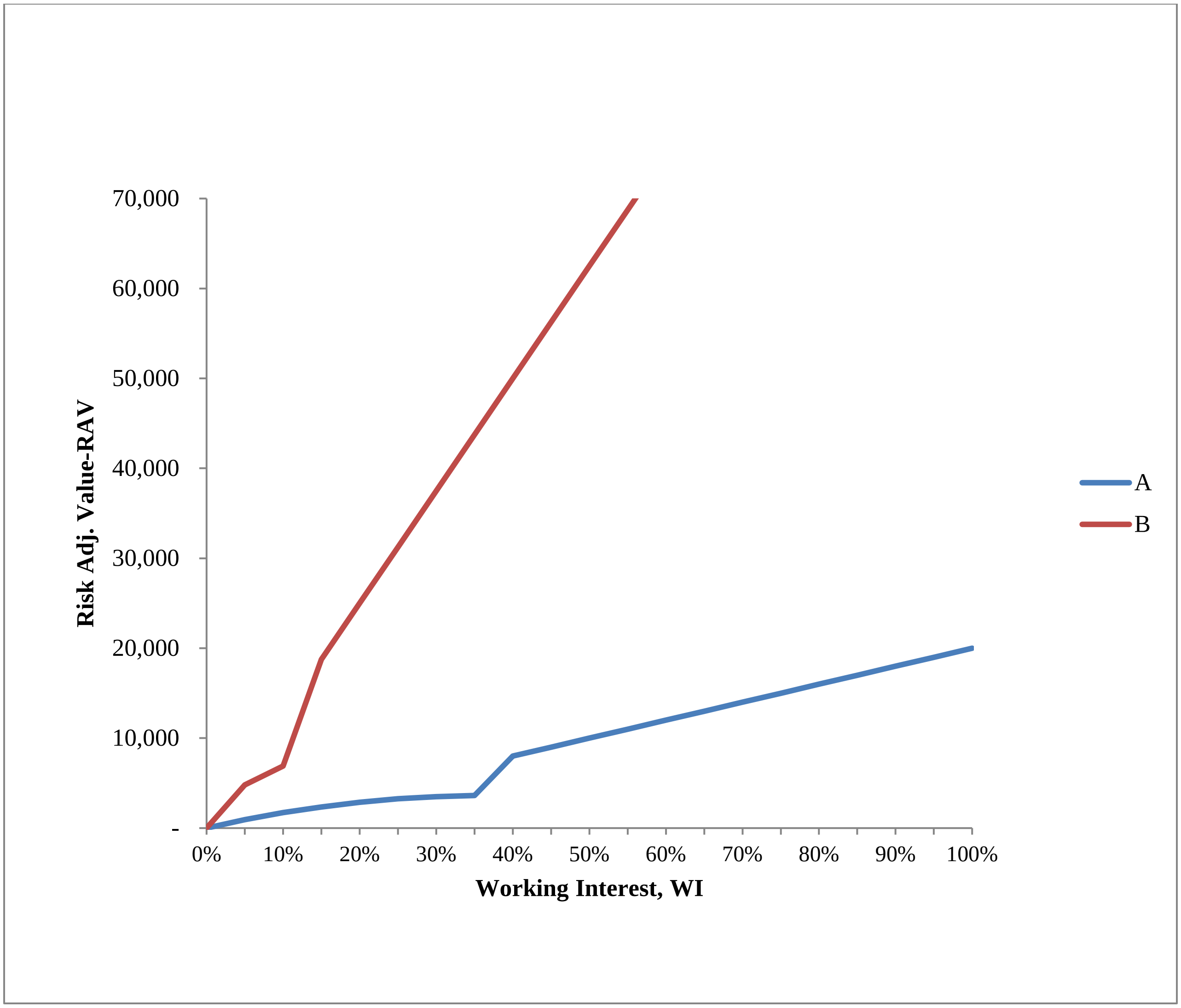


Figure 4.10 RAV versus WI - Exponential/EV Model-RT = $1MM

In summary, Figures 4.8, 4.9, 4.10 and 4.11, demonstrate the complex interaction between Success and Failure Outcome values, V and C, Success (chance factor), Ps and Risk Tolerance, RT. While the relationships derived have been quite helpful in giving us insights into how the risk adjusted value depends on each of the variable, it is still challenging to delineate the relative effect of each variable on the RAV. As Moore et al, recommends, we will have to resort to a simulation process in which the variables are described with probability distributions and the output RAV is also presented as a probability distribution. The probability distribution of RAV will not only indicate the range of possibilities that the outcome for a risky venture will eventually yield, but also the relative contributions of the key variables to the total uncertainty in the venture. The simulation model and results are discussed in Section 4.6

For now, we will extend our portfolio analysis and management to a set of five (5) Risky prospects. The analysis are the same as applied to A and B, only the scale is different – five (5) prospects as against two (2). The analysis can be extended to any number of risky prospects; the methodology remains the same, only the "scale" changes.

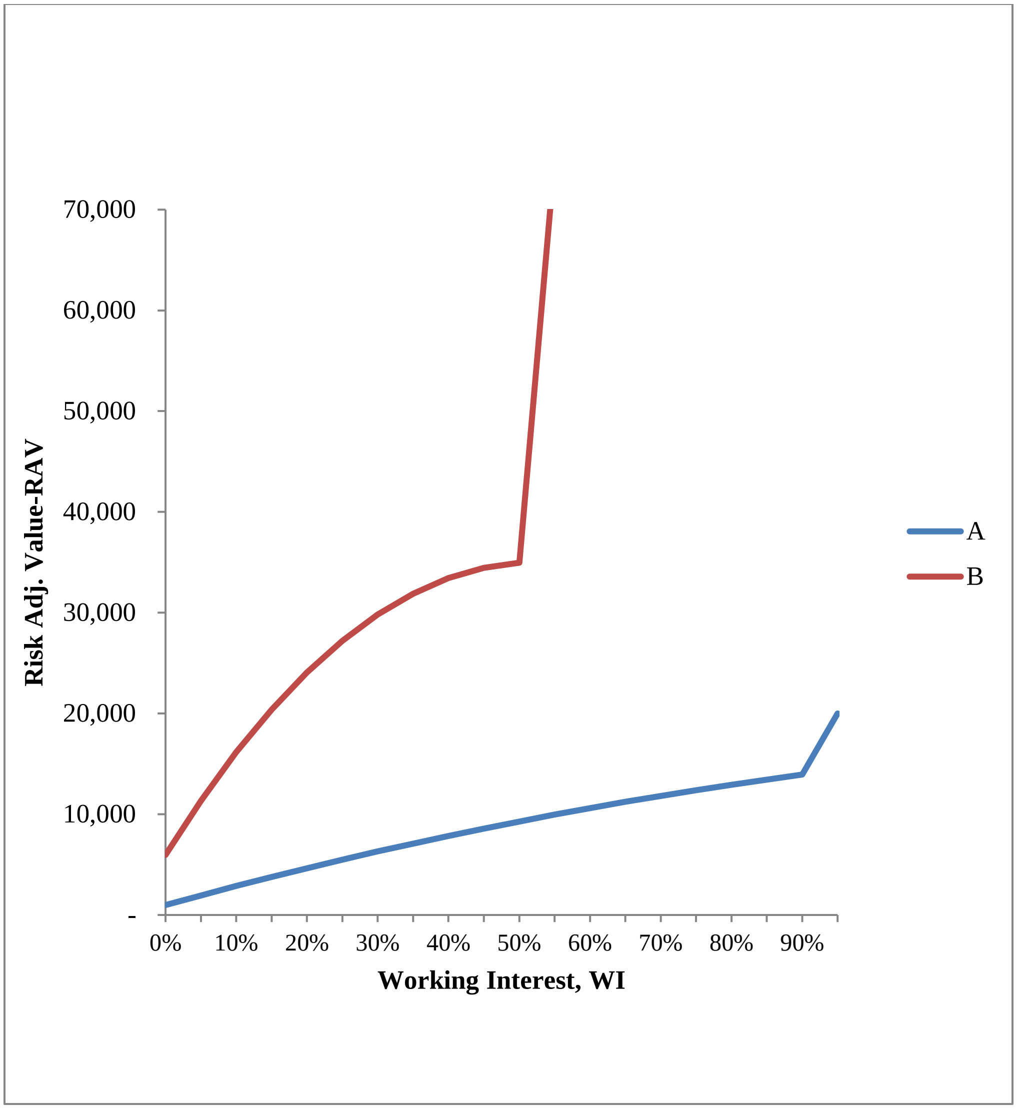


Figure 4.11 RAV versus WI-Exponential/EV- RT = $5 MM

4.4.1 Determination of Working Interests for a Portfolio of 5 Ventures

Table 4.3 lists the Prospect parameters for five Risky Ventures, A, B, C, D and E – from Mackay et al. The portfolio contains the two risky prospects A and B that have been analyzed previously in this chapter, as well as three others for a full portfolio. The Prospects vary in terms of Success value, Cost (or Investment size), and Chance of success, Ps (and hence chance of Failure). All of the Prospects consist of two discrete outcomes. Prospect B (previously analyzed) requires the largest investment of $0.5 Million and has the highest Success value of $2 Million while Prospect D is a relatively small Prospect costing just a $1,000 with a payoff of $5,000 but an 80% chance of success. Prospect E has a 50% Chance of Success with Success Value of $140,000 and a failure cost of $125,500. We thus have for the five Prospects very widely differing levels of Success Values, Failure Costs and Chances of success. The Prospects are deliberately limited to two outcomes for simplicity in analysis. The outcomes could be increased to more than two and even to a continuous distribution of outcomes from the best case to the worst-the analysis remains the same. In the case of a continuous distribution of outcomes, the distribution can be "discretized" into three distinct outcomes with three respective chances (or probability of occurrence).

Table 4.6

Comparison of Expected Value, Variance and Semi-Variance for Risky Portfolio of 5 Ventures

	Prospect A	Prospect B	Prospect C	Prospect D	Prospect E
Success Probability=	20%	25%	15%	80%	50%
Failure Probability =	80%	75%	85%	20%	50%
Net Present Value of Success, V=	500,000	2,000,000	700,000	5,000	140,000
Cost of Failure, C=	100,000	500,000	100,000	1,000	125,500
Risk Tolerance ($)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
EV, Variance Calculations					
Expected Value @ 100% WI =	20,000	125,000	20,000	3,800	7,250
Standard Deviation, σ =	2.40.E+05	1.083.E+06	2.857.E+05	2.400.E+03	1.328.E+05
Downside Semi-Deviation=	107,331	541,266	110,635	2,147	93,868
Upside Semi-Deviation=	214,663	937,500	263,363	1,073	93,868
Ratio of Upside to Down Side Deviation=	2.000	1.732	2.380	0.500	1.000
Specifying Zero (0) as "Threshold"					
Downside Semi-Deviation, σ_s =	89,443	433,013	92,195	447	88,742
Upside Semi-Deviation, σ_u=	223,607	1,000,000	271,109	4,472	98,995
Ratio of Upside to Down Side Deviation, η_0=	2.50	2.31	1.08	2.13	0.51
Working Interest , % =	100%	100%	100%	100%	100%
Risk Adjusted Value (Exponential) Model	(5,428)	(239,312)	(13,788)	3,797	(1,536)
Risk Adjusted Value (Hyperbolic)) Model	12,770	(100,370)	5,955	3,807	7,149

The differing characteristics of the Prospects reflect the variety of asset opportunity types that would confront an integrated oil and gas enterprise: from exploration to full field development, where technical risk levels and costs are relatively significant to well intervention programs with minimal technical risk, little investment outlays and corresponding modest returns or “low hanging fruits”. In the oil and gas Industry, technical risks, especially in the exploration phase refers to the uncertainties surrounding the possibility of an accumulation of petroleum deposits, the fluid and rock characteristics which in turn will affect the recovery mechanisms as well as the size of the accumulation or the commerciality of the prospect. Facility expansion investments, though may entail large capital outlays are nevertheless less risky than exploration prospects. The dominant risk in Facility investment projects are cost overruns and price risk. Development prospects have intermediate level risks that are mostly size and cost related- still some uncertainties remain unresolved as to the size of discovery which determines the size of the production facilities, well drilling costs and price.

Risk adjusted value estimates for all five (5) Prospects using the Exponential and the Hyperbolic Models are plotted versus working interest levels in Figures 4.12 and 4.13 at a risk tolerance level RT of $1Million. The RAV profiles show the classic shape of increase in RAV from zero up to a maximum and then decrease for all but one of the Prospects (Prospect D). Prospect D essentially approximates expected value- the success and failure values are relatively small compared to the risk tolerance level of $1 Million and the RAV profile essentially recommends taking 100% of the prospect if possible. Working interest chosen is expectedly constrained between 0% and 100% of a prospect – for real (and not financial equities) Prospects that are under consideration in this study, negative working interests are not considered.

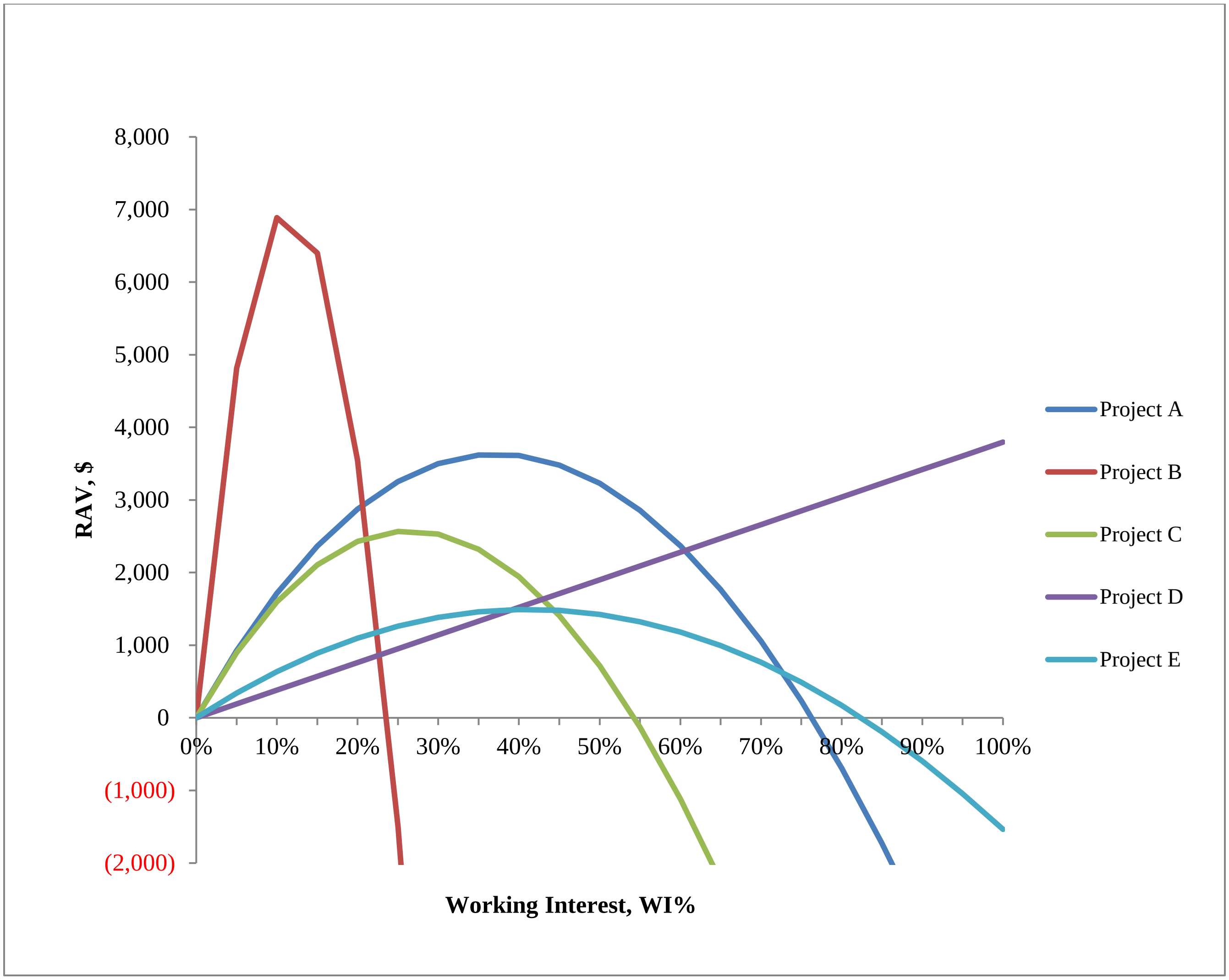


Figure 4.12 RAV versus WI for Portfolio - Exponential Model

Table 4.7 – Portfolio WI Determination – Exponential Model

Prospect	Optimum WI, W_{opt}	Expected Value @ W_{opt}	RAV at W_{opt}	Required Investment
A	37.2%	7,440	3,619	37,200
B	11.5%	14,375	7002	57,500
C	26.4%	5,280	2,567	26,400
D	100.0%	3,800	3,797	1,000
E	41.2%	2,987	1,491	51,706
		33,882	18,362	**173,806**
Portfolio Risk Premium =		**15,520**		

Table 4.8 – Portfolio WI Determination – Hyperbolic Model

Prospect	Optimum WI, W_{opt}	Expected Value @ W_{opt}	RAV @W_{opt}	Required Investment
A	100%	20,000	12,770	100,000
B	30%	37,500	22,858	150,000
C	70%	14,000	8,768	70,000
D	100%	3,800	3,807	1,000
E	100%	7,250	7,149	125,500
		82,550	**55,352**	**446,500**
Portfolio Risk Premium =		**27,198**		

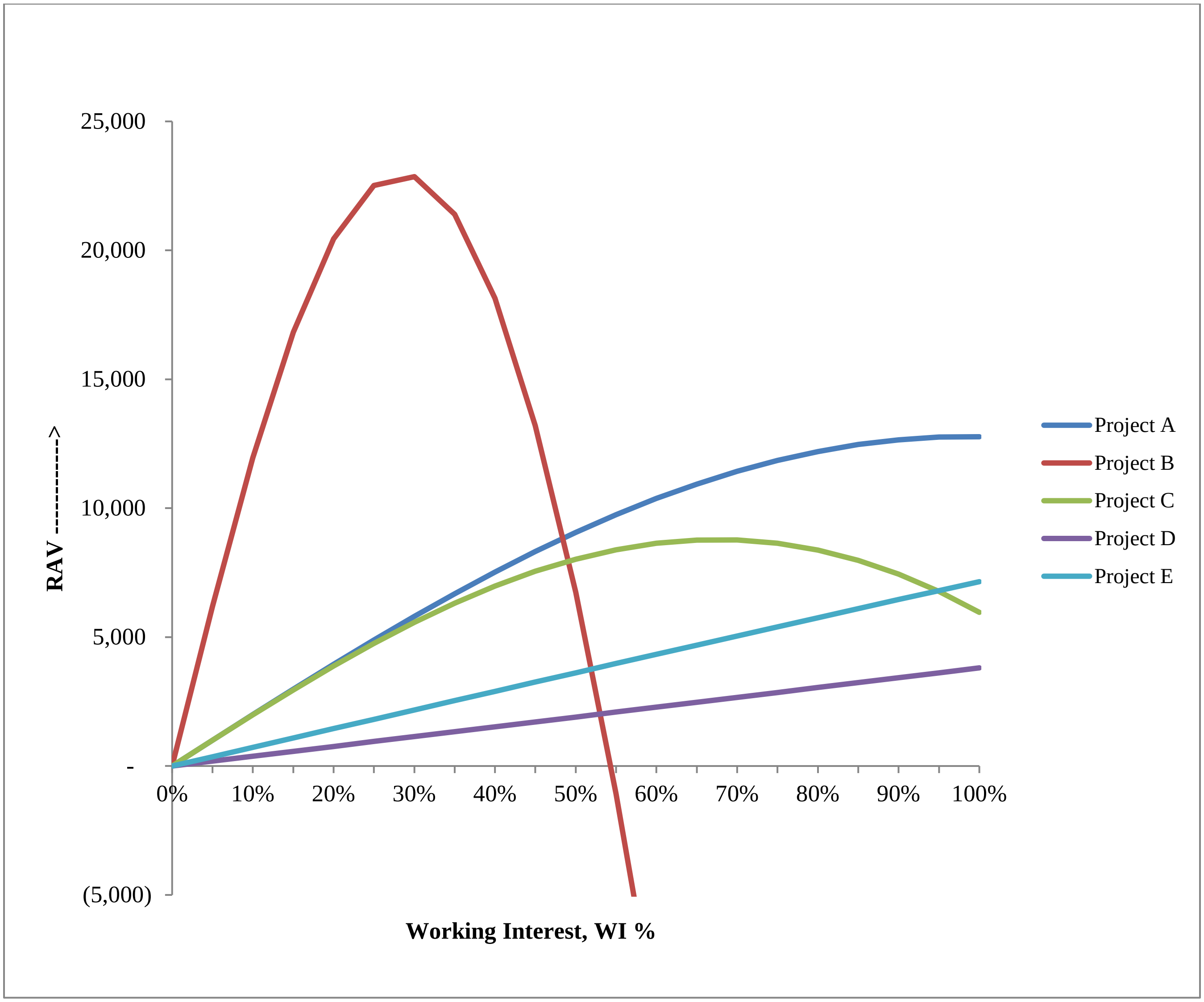


Figure 4.13- RAV versus WI for Portfolio -Hyperbolic Model

The Optimum working interest for each prospect is shown in Tables 4.7 and 4.8 for the Exponential Model and the Hyperbolic Preference Models (respectively) as well as the corresponding Risk Adjusted Values and the required investments. The Expected Values at the Optimum Working Interest levels are also indicated in Tables 4.7 and 4.8.

The optimum working interest can be read off from the RAV profiles in Figures 4.12 and 4.13. However, the precise values calculated from the relevant RAV versus working interest relationships are the ones shown in Tables 4.7 and 4.8.

Tables 4.7 and 4.8 showed the significant differences in optimum working interest determined and hence investment capital required using the Hyperbolic and the Exponential Models. Across board, the Hyperbolic Model recommends much higher working Interest levels compared to the Exponential. Even for the most "risky" investment, Prospect B, the Hyperbolic Model recommends taking a 30% of Prospect B compared with 11.5% Optimum recommended by the Exponential Model. For Prospects A, D and E, the Hyperbolic Model recommends 100% working interest whereas only for Prospect D did the Exponential recommend 100%. Prospect A and E working Interest levels for the Exponential were considerably lower at 37.2% and 41.2% respectively.

Figure 4.14 shows the optimistic nature of the Hyperbolic Preference Model over the Exponential for all 5 prospects. Because of the higher working interests determined using the Hyperbolic, the required investment levels are correspondingly much higher - $446,500 for the entire portfolio versus $173,800 for the Exponential. The expected values and the risk adjusted values at optimum working interest are also much higher. This observation underscores the crucial importance of selecting the appropriate preference model to use in risk adjusted value or certainty equivalent analysis whether for single prospect evaluation or for a portfolio of prospects. In the case of a portfolio, the effect is even more dramatic since the differences add up from single prospect to the entire portfolio level. However, the effect of risk tolerance level, RT in optimum working interest determination should not be discounted. A lower level of working RT will result in different working interest values for the portfolio under consideration with correspondingly different expected and risk adjusted values. A recommendation of 100% optimum working interest for Prospect D using either the Exponential or the Hyperbolic Model is due to the fact that the failure and outcome values ($1,000 and $5,000) were too small and readily accommodated by a risk tolerance level of $1 Million.

The portfolio analysis just shown is for an unlimited Capital environment. For a limited capital, which is the reality for most individuals and Corporations (the number of available opportunities exceeding the financial resources of the entity) an optimization program will have to be used to select the appropriate working interest for each prospect in the portfolio in order to maximize the Portfolio's RAV with the given Capital constraint (available Capital Budget).

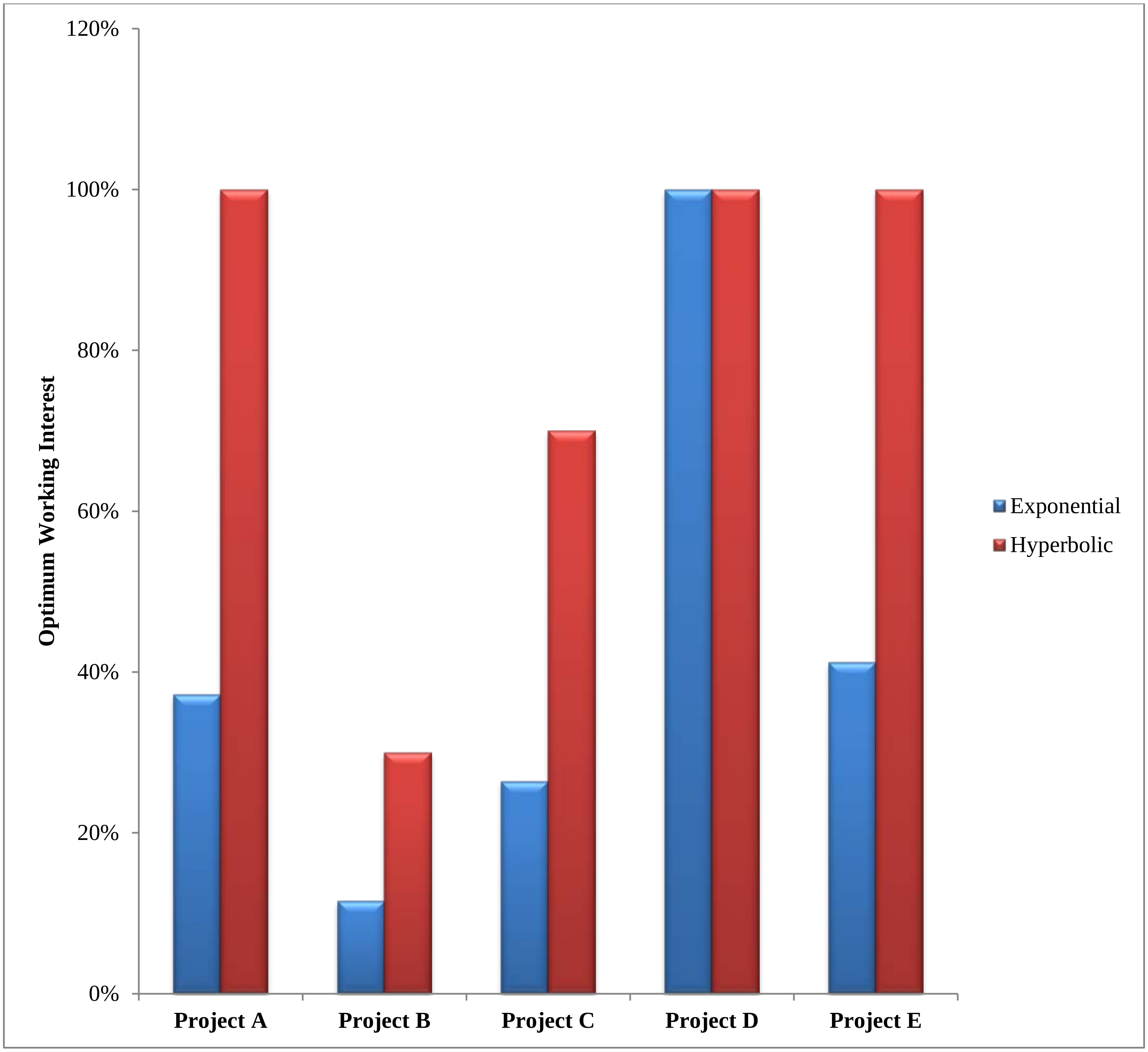


Figure 4.14- Optimum WI - Exponential versus Hyperbolic Preference Models

4.4.2 Portfolio Analysis of the Five Risky Prospects–Limited Capital Constraint Situation

Palisade's Decision Tools suite (Evolver) was used to perform the optimization for the Portfolio (of five prospects). Evolver is an Excel add-in that uses a linear algorithm to perform optimization. Investment capital was limited to $100,000 (Mackay) and optimization was performed using both the Exponential and the Hyperbolic Models to estimate the maximum RAV to be realized by varying the working interest levels for each of the five prospects in the Portfolio. The objective function therefore was the Total Portfolio Risk Adjusted Value while the variable was working interest. It should be noted that the working interest levels were contained between 0 and 1 (negative working interest levels and working interest greater than 100% were not allowed). The specified minimum and maximum working interest levels for real prospects are to be expected, working interest levels outside of this range are to be found in strictly financial investments like equities or derivative investments (financial options).

The results of the optimization performed are presented in Tables 4.9 and 4.10

Table 4.9 – Portfolio Optimization under Limited Capital Constraint (Exponential Model)

Portfolio Capital = $100,000

Preference Model : **Exponential**

Prospect	Working Interest	Expected Value at WI	RAV @WI	Required Investment
A	27.4%	5,477	3,388	27,383
B	9.1%	11,315	6,697	45,261
C	19.4%	3,884	2,400	19,418
D	100.0%	3,800	3,797	1,000
E	5.5%	401	374	6,938
		24,876	**16,656**	**100,000**

Portfolio Risk Premium = 8,220

Table 4.10 – Portfolio Optimization under Limited Capital Constraint (Hyperbolic Model)

Portfolio Capital =$100,000

Preference Model : **Hyperbolic**

Prospect	Working Interest	Expected Value at WI	RAV @WI	Required Investment
A	20.1%	4,021	3,964	20,105
B	13.0%	16,292	15,037	65,170
C	13.7%	2,745	2,705	13,725
D	100.0%	3,800	3,807	1,000
E	0.0%	0	0	0
		26,859	**25,513**	**100,000**
Portfolio Risk Premium =		**1,346**		

The portfolio optimization results show that the recommended working interest levels for each prospect (in the portfolio) were similar using both the Exponential and the Hyperbolic preference models. Both models recommend that very little of Prospect E be taken (actually the Hyperbolic Model predicts 0% working interest for Prospect E) while 100% of Prospect D is recommended. We had earlier observed that prospect D's level of investment required was so small compared to the risk tolerance of $1Million, such that the risk posed to the investor is minimal and inconsequential for all practical purposes. This observation is supported by the portfolio optimization results. Secondly, the RAV for the Optimized portfolio using the hyperbolic preference function is higher than the RAV value for the Exponential Model- this also to be expected, since the Hyperbolic Model has been identified as more "risk tolerant" than the Exponential preference function. The levels of recommended participation for Prospects A, B and C reflect their intrinsic risk profiles under the two preference models and show generally more of A be invested in than B and C.

The expected values for the optimized portfolio under the two preference functions are quite close ($24,876 for the Exponential and $26,859 for the Hyperbolic). However, the risk premiums differ quite significantly. The risk premium under the Exponential Preference function is $8,220 – a large value reflecting a very conservative model. The small risk premium for the hyperbolic shows the model is only marginally different from the expected value (risk neutrality).

The optimization by Evolver has a high degree of robustness. Several runs were made to test the uniqueness of the solutions obtained. The results were the same, although some differences in run times were experienced, indicating different times for convergence of solutions maximizing the objective function within the constraints specified in the Portfolio optimization problem. The high level of robustness of the solutions provides some degree of confidence to the portfolio analyst and the investment decision maker in the veracity of the recommended participation levels, knowing fully well that the solutions are not dependent on the optimization algorithm employed. The range in the computing run times experienced for Optimization were also well within acceptable limits that a reasonable modest computing resource level will accommodate quite well. For the number and level of investments considered in this study, the quality of solutions obtained is the major consideration; the cost of computing is quite minimal. For large Portfolios, however, the level and hence cost of required computing resources may well become a major consideration in Portfolio optimization.

4.5 RAV Sensitivity Analysis to important Risk Variables

This section presents the results of risk adjusted value (RAV) to the various risk variables such as Risk Tolerance (RT), success value, V, failure cost C and chance of success, Ps with a view to determining the relative impact of each of these variables to overall asset valuation.

4.5.1 RAV Sensitivity to Risk Tolerance, RT

This section explores the relationship between the optimum working interest and risk tolerance, RT. Risk tolerance indicates the level of risk the investor is willing to accept in investment decision making. Optimum working interest maximizes the value of the RAV to the investor given the prospects success values, cost and the chances of realizing the different outcomes. Generally, the higher the risk tolerance, RT the higher the working interest should be. Figure 4.2 shows the relationship between the optimum working interest and risk tolerance; RT is linear for both preference models. Recall for the exponential model, Optimum Working Interest is given by the relationship: $W_{opt} = \frac{RT}{V+C} \ln \frac{P_S V}{P_f C}$.

For constant values of V, C, P_s and P_f, Optimum Working interest is a linear function of Risk Tolerance as Figure 4.15 indicates. For the Hyperbolic Model, the relationship is also linear. For the hyperbolic preference model of the form$(x) = 1 - tanh(x)$, the optimum working Interest is given by the relationship:

$cosh\left(\frac{W_{opt}C}{RT}\right) = \left(\frac{P_f C}{P_s V}\right)^{1/2} cosh\left(\frac{W_{opt}V}{RT}\right)$ which can be approximated thus:

$W_{opt} = \frac{1}{2}\left({RT}/{V}\right) \ln\left(\frac{4P_s V}{P_f C}\right)$ such that $0 \leq W_{opt} \leq 1$.

Hence holding values of Ps, Pf, V and C constant in Equation **3.59**, the optimum working interest indicates a linear relationship with risk tolerance, RT as correctly shown in Figure 4.15

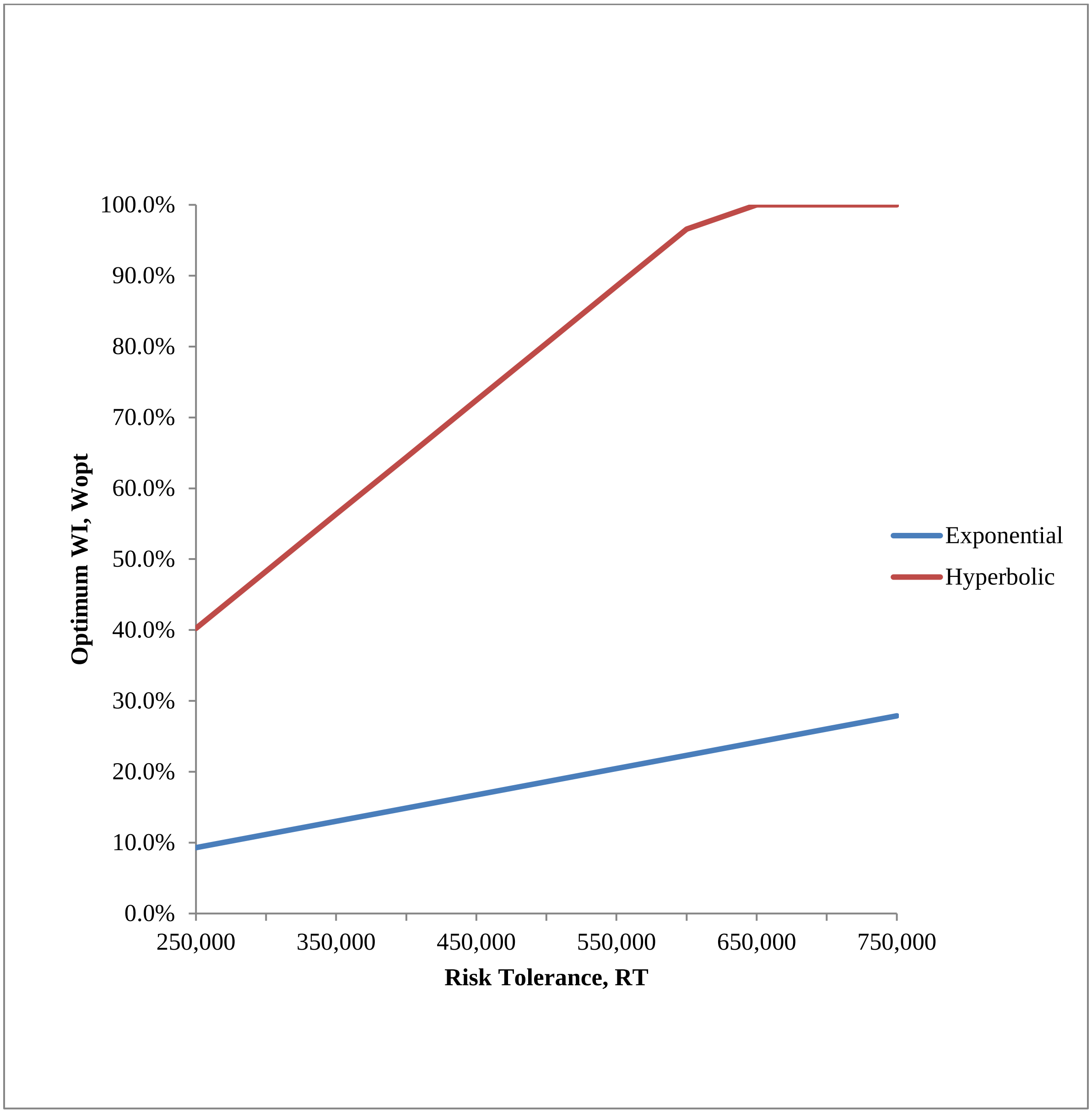


Figure 4.15 RAV Sensitivity-Optimum WI versus Risk Tolerance, RT

The optimum working interest determined using the hyperbolic model is consistently higher than that determined using the exponential for all levels of risk tolerance considered ($250, 000 through $750,000). In fact, at a RT value exceeding $650,000, the hyperbolic model recommends 100% working interest for the prospect (same as expected value). This finding is consistent with Lerche and Mackay in their analysis of prospects with negative expected values – “The Cozzolino formula tends to emphasize the negative fractional uncertainty around the mean value, while the hyperbolic tangent procedure tends to emphasize the positive fractional uncertainty”. The choice of the risk preference model plays a significant role in investment decision making and should reflect corporate philosophy on risk: conservative, neutral or aggressive.

4.5.2 RAV Sensitivity to Success Value, V and Probability of Success, P_s

The sensitivity of RAV of the prospect to success value, V and probability of success, P_s was also investigated in this research work. Figure 4.15 through Figure 4.19 shows the results of this analysis. Figure 4.16 illustrates the sensitivity of the prospect’s Risk Adjusted Values with Success Value; V. V is varied from the $500,000 to $1 Million dollars, keeping the other variables (P_s, P_f and C) constant. The Exponential Risk preference model is employed in Figure 4.16.

Figure 4.16 shows that with increasing success values, Optimum working interest determined for the risky prospect increases. This is to be expected- the more the upside opportunity, the more the investor should take in the prospect. Optimum working interest level increases till a value of 85%, when success value is $1 Million and the corresponding Risk Adjusted Value = $44, 520 (Table 4.11). The relationship of RAV with working interest is still non-linear for all values of V and significantly, the RAV “peaks” at optimum working interest and then declines (with increasing, V. This means that even at high values of V, the “Paradox of Aversion to Reward” still exists for the exponential preference model.

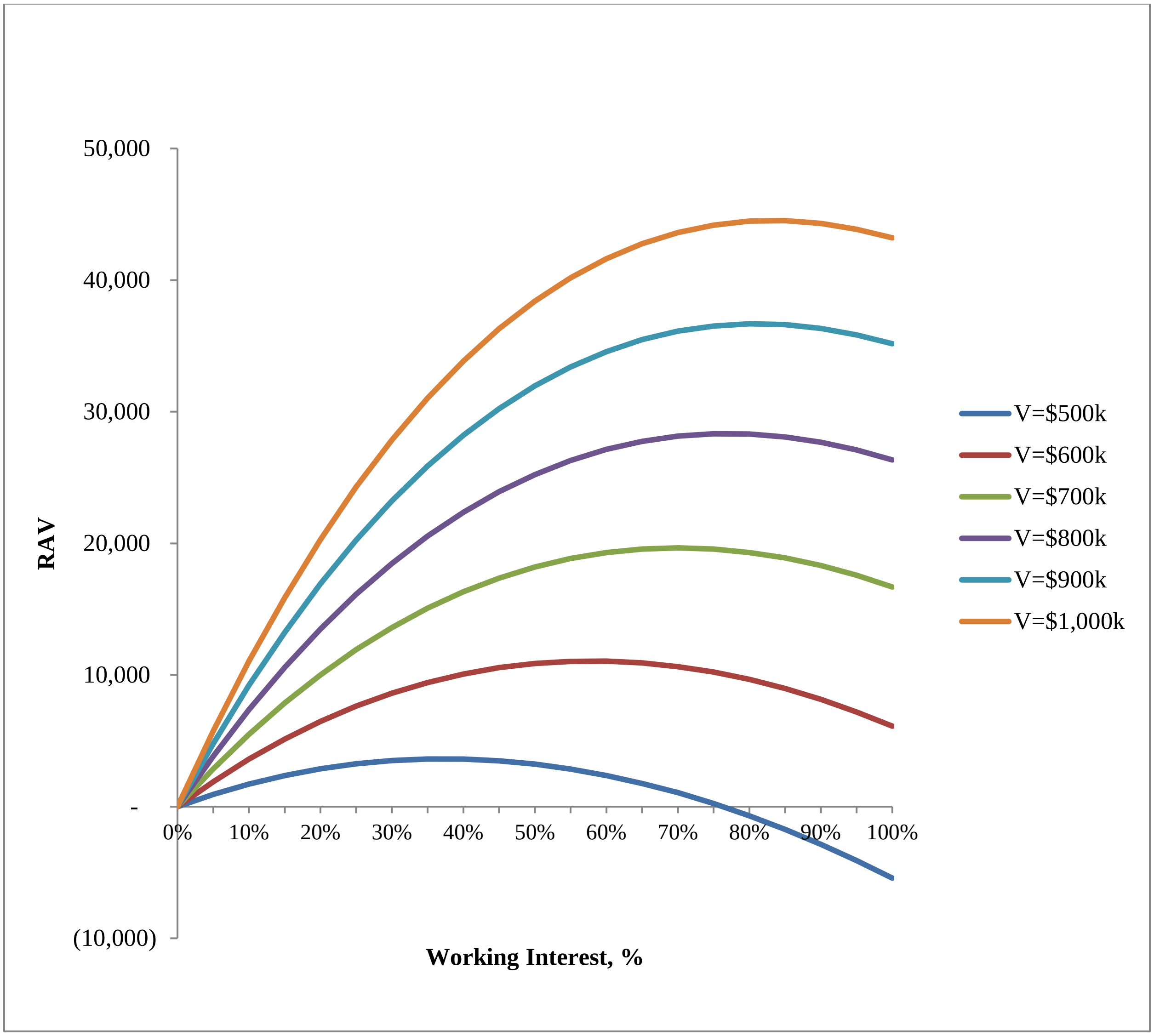


Figure 4.16 RAV Sensitivity to Success Value, V--Exponential Model

Table 4.11

Sensitivity of Optimum Working Interest and RAV to Success Value, V

Success Value, V	Optimum WI, W_{opt}	RAV at W_{opt}
500,000	35%	3,619
600,000	60%	11,056
700,000	70%	19,660
800,000	75%	28,328
900,000	80%	36,685
1,000,000	85%	44,520

Optimum working interest varies non-linearly with success value, V. However, the Risk adjusted value at optimum working interest varies linearly with success value, V (Figure 4.17). Recall from Equation 3.58, RAV at Optimum Working Interest is:

$$RAV_{Wopt} = -RT\left[P_s\left(\frac{P_f C}{P_s V}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_s V}{P_f C}\right)^{\frac{C}{V+C}}\right]$$

Hence for specific values of V, C, and Ps

$RAV_{Wopt} = K * RT$ – This is a linear relationship with K representing the expressions in the bracket.

Figure 4.17 and 4.18 show the impact of different levels of the chance of success, Ps and (hence the probability of failure) on the RAV relationship with working interest. The relationship is essentially linear for the Exponential model. All the curves start at zero, since for working interest of zero, regardless of the level of the chance of success, RAV is zero. The utility of Figure 4.17 essentially lies in the fact that it can aid in the knowledge of and specification of a minimum level of Ps acceptable to an investor or corporate organization considering a risky prospect. From 4.5, RAVs are essentially negative for Ps below 20%. This may serve as the minimum threshold or "cut off".

Figure 4.18 shows the impact of Ps differently from Figure 4.17. The chance of success is plotted on the x-axis for four levels of working interest 25%, 30%, 35% and 75%. The curves show the expected general relationship of RAV increasing with higher values of Ps, and the higher the working interest taken the higher the RAV that will be realized. However, this also occurs on the downside – when the prospect loses money, the higher the working interest taken, the higher the loss. At Ps = 10%, a WI=75% in the risky prospect leads to a larger loss.

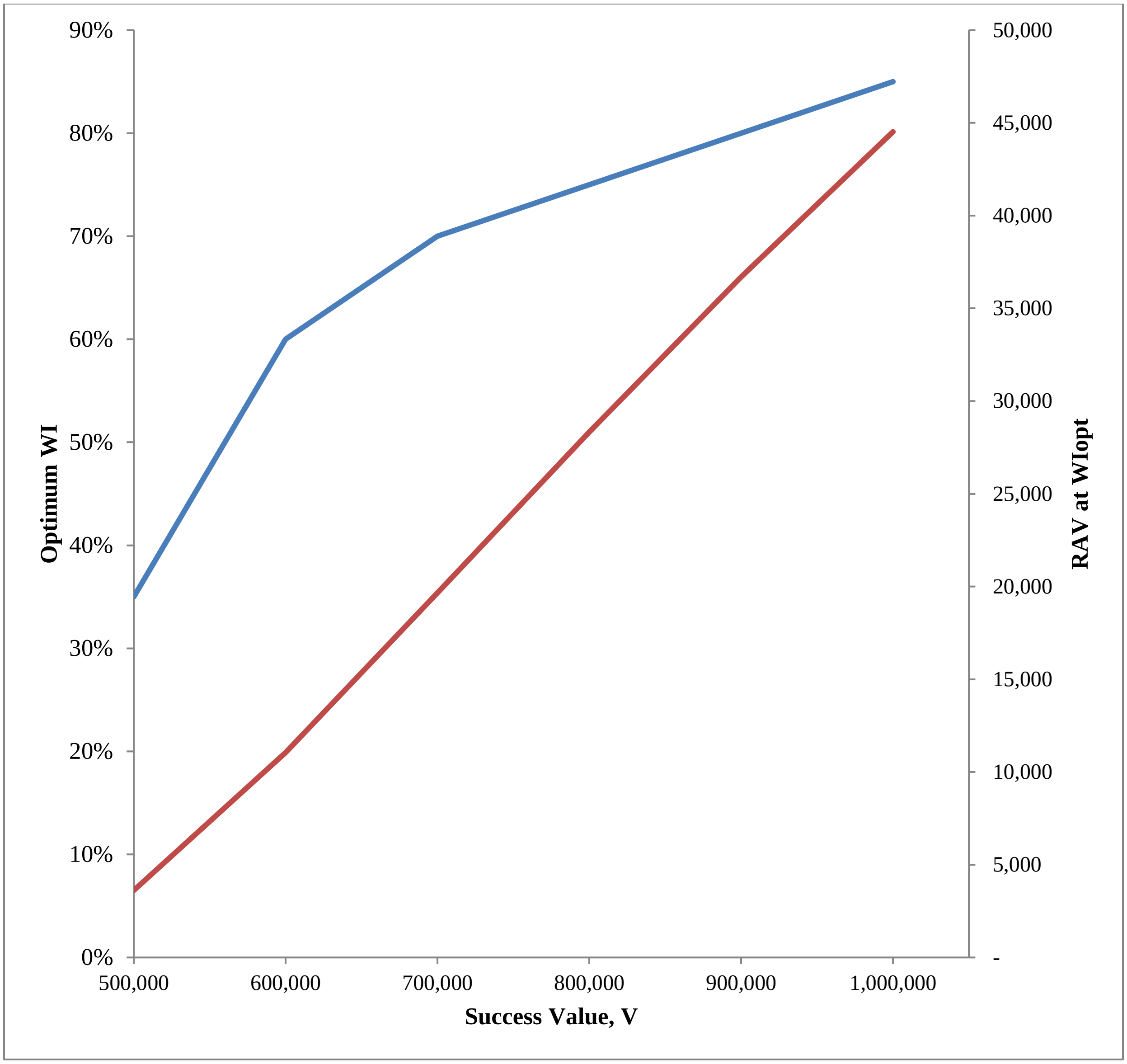


Figure 4.17 RAV, Wopt Sensitivity to Success Value, V

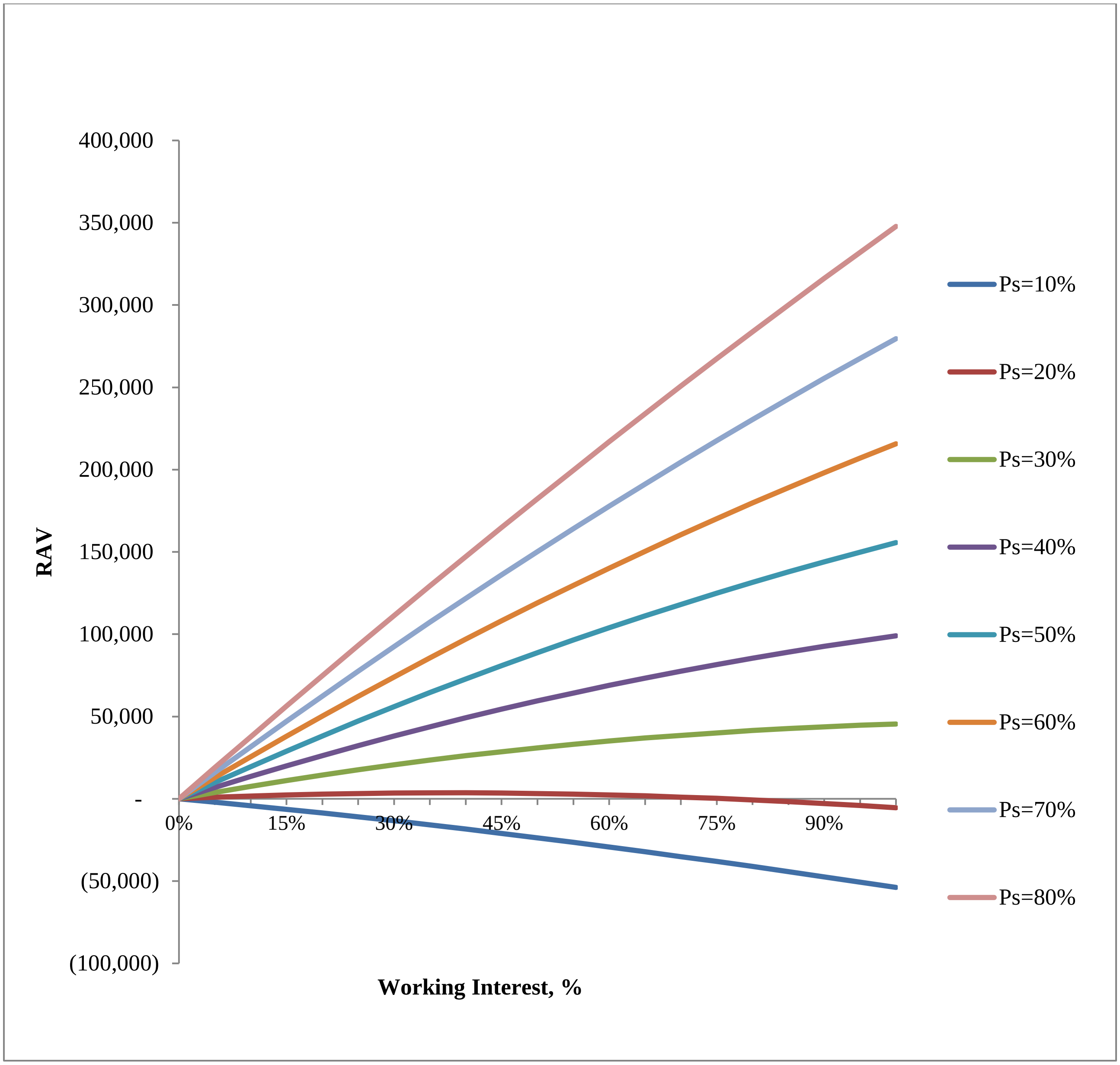


Figure 4.18- RAV Sensitivity to Success Probability, Ps

In utilizing fractional working interest as a risk mitigation strategy, the investor must entertain caution on the upside as well as the downside. However, the attraction to take more of the prospect because of the upside opportunity presented should be balanced or "moderated" by a knowledge of the optimum working interest calculated.

In Figure 4.19, the sensitivity of RAV to success value, V is explored for values between $500,000 and $1 million just as was done using the exponential preference model. Expectedly, RAV increases (non-linearly) with working interest. The characteristic here is that with increased working interest, the marginal increment of RAV decreases with working interest and nears a peak, but does not decrease as was the case with the Exponential preference model. Recall that the hyperbolic model reflects "more risk tolerance "than the Exponential. In fact, for each level of V considered, the more the working interest taken, the higher the RAV attainable. The maximum attainable RAV ($75, 354) occurs at 100% working interest for V=$1 Million. The Hyperbolic model, although non-linear in RAV versus working interest, recommends taking close to 100% just as the expected value (EV) model does. However, it will not be correct to conclude that it represents risk neutrality, since that implies RAV is equal in value to the Expected Value (EV) – no risk premium. The Hyperbolic tangent model still represents risk aversion, but of a more tolerant nature than the Exponential preference model.

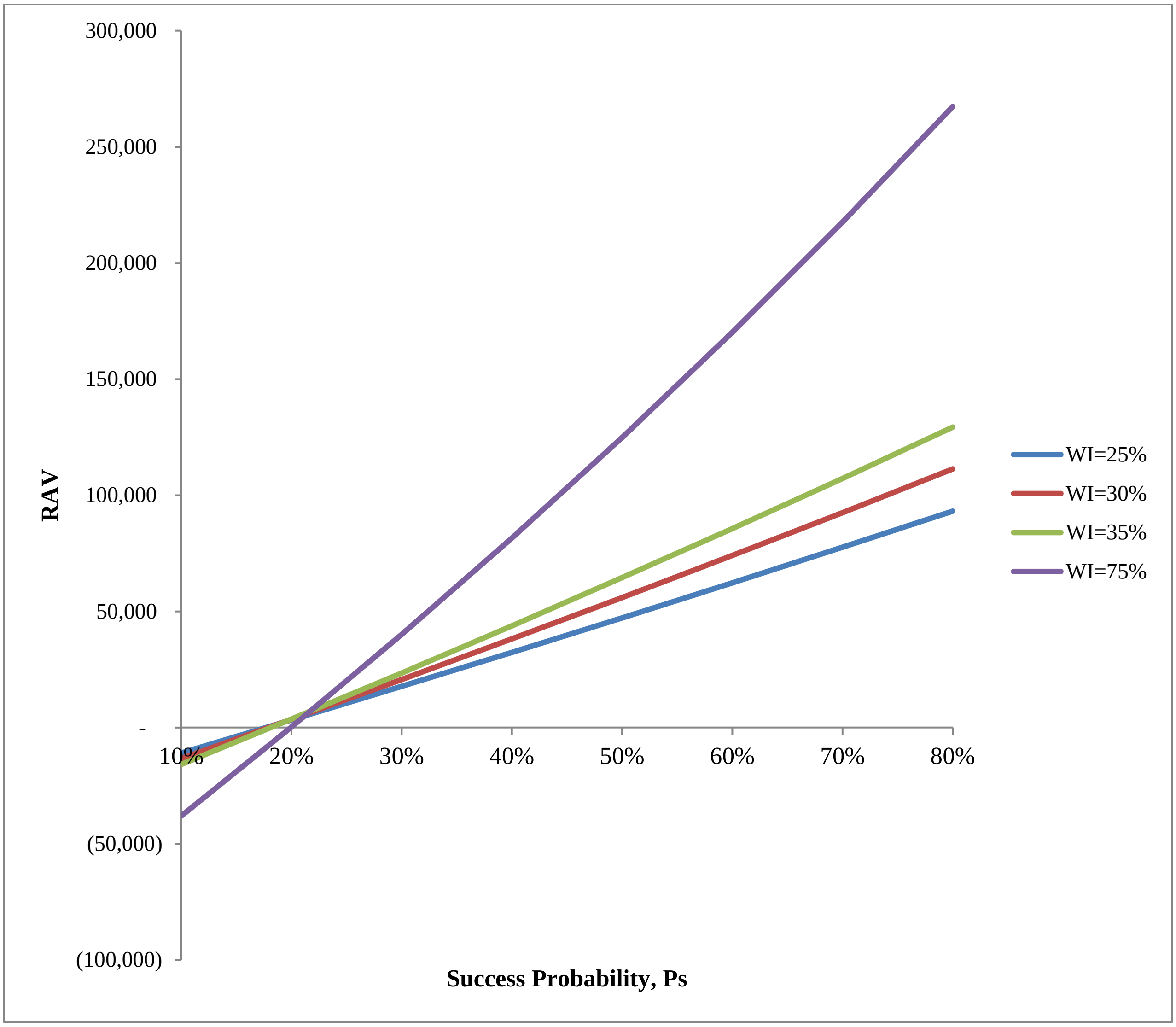


Figure 4.19 RAV Sensitivity to Chance of Success, Ps- Four WI levels

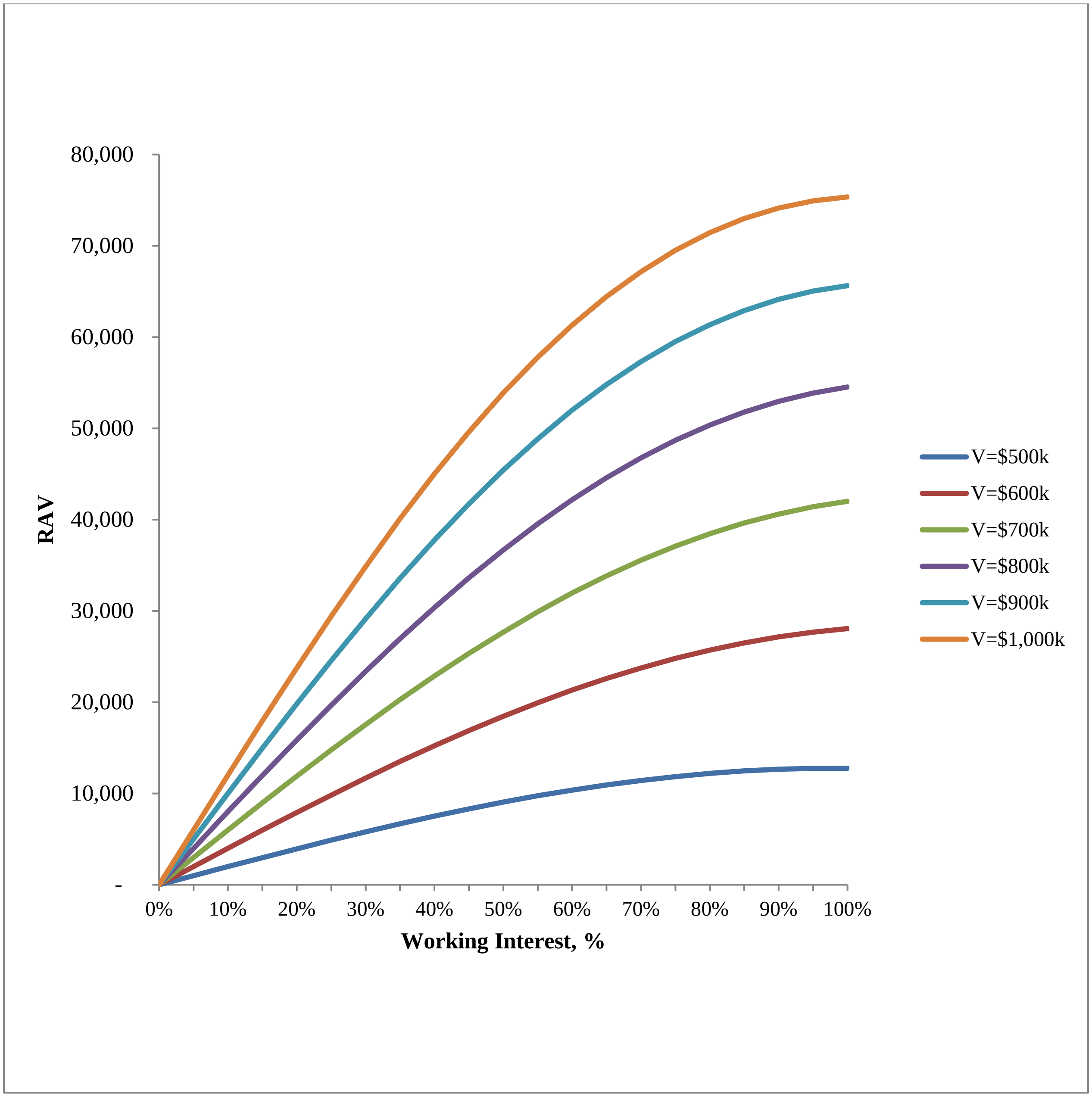


Figure 4.20- RAV Sensitivity to Success Value, V - Hyperbolic Model

4.6 Evaluating the Relative Impact of Success Values V, Failure Cost C, Chance Factor, Ps and Risk Tolerance in RAV Analysis

A simulation model was therefore constructed for Prospects A and B analyzed earlier in this research. Simple triangular distributions were used to define uncertainties of Success outcome value, V, Failure outcome value (Cost), C and Success factor, Ps. Risk tolerance is not strictly an uncertain variable, since the investor or Decision maker or in the case of an institution (or Company) can explicitly specify the level of risk to accommodate. It is here included as an uncertain variable mainly to investigate its effect on the Simulation model and as a way of highlighting its effect on the uncertainty distribution of RAV.

Another justification for including risk tolerance as an uncertain variable is the practical difficulty of determining what that value is. Some have suggested using a percentage of the Annual Exploration Company Budget as a proxy for risk tolerance (Rose 1994). Walls (1992) extensive research on the effect of RT on RAV suggest level between $1/6^{th}$ and 25% of the Capital exploration program in a study of bid evaluations for a number of Independents as well as Integrated exploration companies. Howard notes that risk tolerance is "approximately an investment in which the Corporation is indifferent to a 50-50 chance of winning that sum and losing half of it" (Howard 1988). Other studies point to the fact that the RT value cannot be the same for all parts or divisions of the Company. Frontier Exploration is riskier and should reflect that level of risk. Moore et al notes that "the concept of varying RT within a firm is intuitively reasonable, logical and a quantitative extension of the idea that each company has core areas or focus areas in which it wants to concentrate its investments without completely abandoning all other areas". Moore et al also observe in their fair market valuations for assets all over the World, that unit values of proved reserves varies from Country to Country and sometimes within a Country, reflecting Country Risk. By extension, they suggest that RT may vary to reflect Country Risk.

Mackay et al argue that prior working interest decisions taken by Company is a measure of the corporate risk tolerance level and call this the Apparent Risk Tolerance (ART). The ART was calculated by assuming that the prior working interests were at the optimum for the risky prospect under consideration. From their analysis of global exploration transactions Moore et al developed the following rules of thumb for choosing Risk Tolerance RT:

Table 4.12 Risk Tolerance Levels for Oil Exploration Firms

Exploration Firm/Program Description	***Risk Tolerance (RT)***
Major/Large Independent/Low Country Risk/Core Area	**$500 MM**
Super Major/Moderate Country Risk/Non-Core Area	**$200 MM**
Large/Independent/International/Core Business Unit	**$100 MM**
Large/Independent/International/Frontier and/or High Country Risk	**$50 MM**
Medium/Independent/Frontier and/or High Country Risk	**$20 MM**

The software utilized to build the simulation model is @Risk Simulation software – a Microsoft Excel add-in which offers features that capture uncertainties with several probability distribution functions.

4.6.1 Parameters for the Simulation Model

Simple triangular distributions were employed to model the uncertainty of the important variables of Success chance factor, Ps, Success Value, V, Cost of failure, C and Risk Tolerance. The minimum and maximum values of the variables were set at 50% on either side of the mean (for each variable) and simulation runs of 5000 iterations were run using @Risk simulation software. Figures 4.20 and 4.21 show the resulting probability distributions for RAV for Prospects A and B (the same numerical examples in the two Prospect portfolio) using the Hyperbolic and the Exponential preference functions. The relative impacts of the variables in the uncertainty structure of the Prospects are also shown to the right of the RAV distributions. A number of significant observations can be inferred from the simulation runs:

- For both Prospects (A and B) and regardless of the preference function applied, Cost of Failure, C contributes most to the uncertainty in Risk Adjusted Value
- Risk Tolerance least impacts the RAV distribution for Prospect A for both preference functions.
- Risk Tolerance comes next to Cost, C in terms of impact, for Prospect B using the Exponential Model and after the success chance factor, Ps for the hyperbolic preference function. Success Value, V and Chance Factor, Ps in that order are of relative importance to the uncertainty distribution in Prospect A for both preference functions.
- Risk Tolerance comes next to Cost, C in terms of impact, for Prospect B using the Exponential Model and after the success chance factor, Ps for the hyperbolic preference function. Success Value, V and Chance Factor, Ps in that order are of relative importance to the uncertainty distribution in Prospect A for both preference functions.

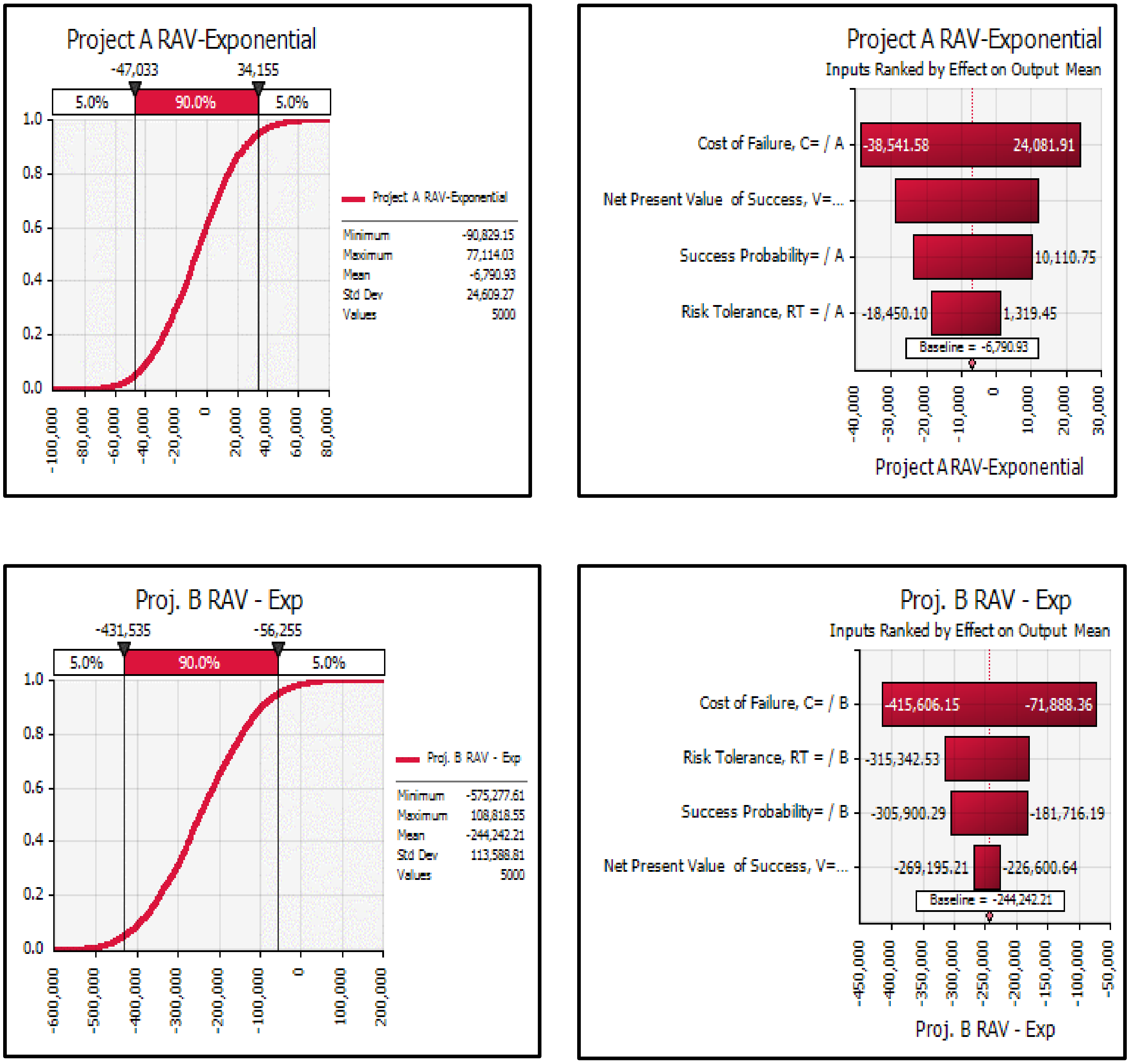


Figure 4.21 **RAV Probability Distributions and Relative Weightings of Variables-Exponential**

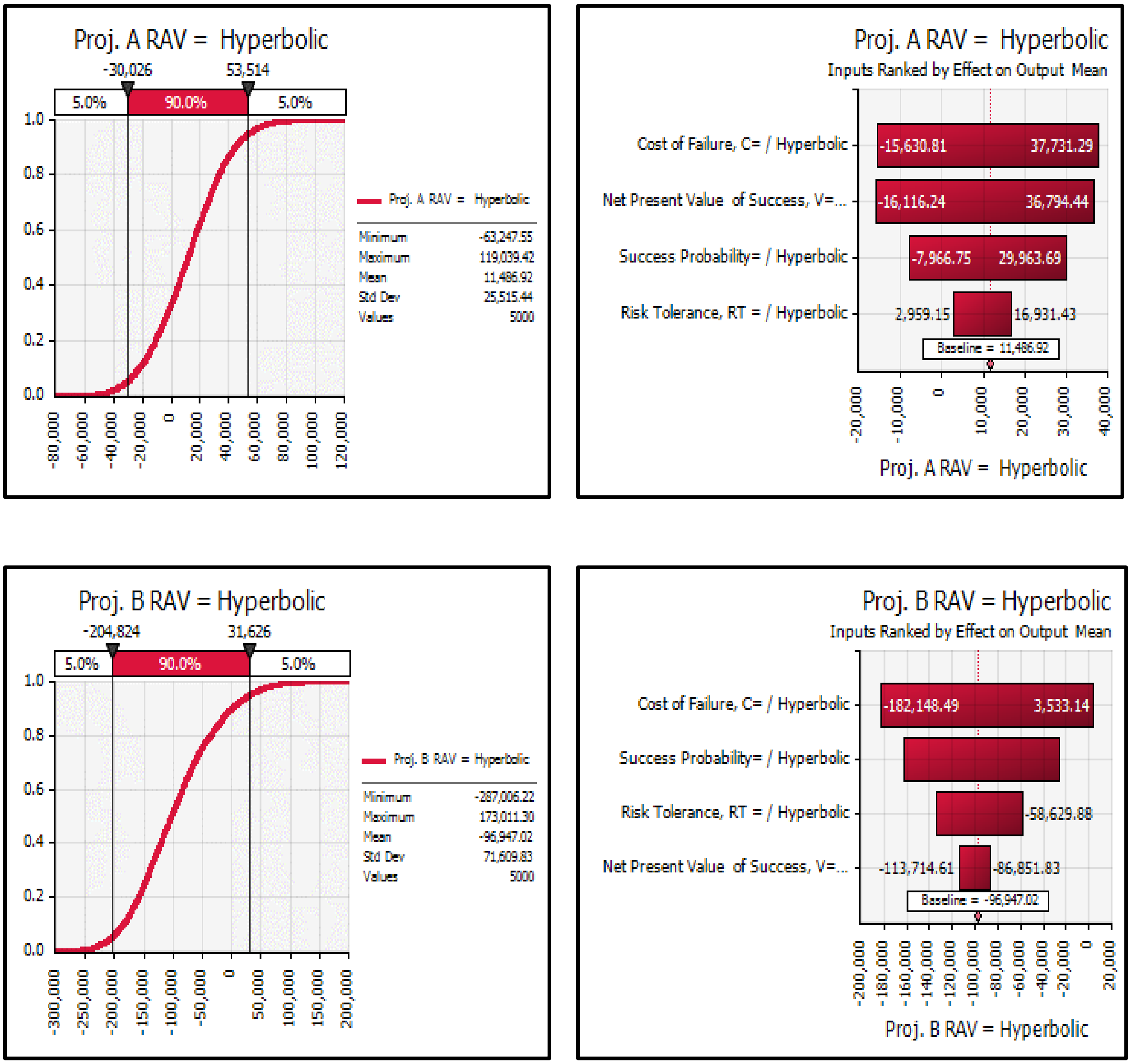


Figure 4.22 – RAV Probability Distributions and Relative Weightings of Variables-Hyperbolic

In summary, the simulation results show that in computing Risk adjusted value or Certainty equivalent, an accurate estimation of the failure cost is very important. This is consistent with the findings of Moore et al and assumes great significance given the tendency or bias to underestimate project cost in Oil and Gas asset valuations especially in Budget constrained environments where each project has to compete for available Capital. This finding is also consistent with Moore et al, who concluded that RAV estimates using preference functions are robust unless the cost of failure are significantly underestimated. The relative impacts of Success Value, V and Success chance, Ps is not clear cut and appears to depend on the individual project under consideration. Moore et al found that Optimum working interest, increases with increasing chance factor Ps, provided the value of Ps is not unduly optimistic. The impact of the Risk tolerance level, RT on RAV is relatively low for in Prospect A, the plausible explanation is the fact that the risk tolerance level is far greater than the values of both the Success Value V and Cost of failure, C.

The details of the Simulation Model showing the input and output variables probability distributions, as well as the results are contained in Appendix F.

4.7 Practical Application of RAV Analysis in Determining Bid Value and Allocation of WI

Moore et al first pointed out that Risk adjusted value analysis and working interest determination may significantly impact how Companies arrive at Bid value. This will suggest that Companies perception of their optimum working interest in a Risky prospect could serve as an analytical tool to allocate working interests and align individual partners in a Bidding group or Farm out arrangement. Since risk tolerance, RT is also a variable in optimum working interest relationship, differing levels of risk tolerance will impact value that a Company will bid for an asset, and hence the size of signature bonuses, entry fees and other revenue to the Government or the Company farming out a risky prospect.

This study, therefore explores how

i. Risk aversion and hence risk tolerance, RT levels influences RAV and hence Asset value for "Seller" and "Buyer"
ii. Government or Farmor (Company soliciting Farminee) can maximize its take in a Bid Round or Farm out arrangement by targeting a specific group of Companies with a particular level of Risk Tolerance.

A Risky prospect is thus analyzed for Risk Adjusted Value analysis under three scenarios:

1. Target Company is a Super Major having the Capital that can accommodate the entire Cost of the prospect in its Portfolio
2. Target Companies are two (2) large independent Companies jointly bidding for the Prospect
3. Target Companies are four (4) medium Independents

The prospect under consideration is one in which 200 Million barrels is expected as recoverable reserves, in an environment of $10 per barrel development cost and an oil price of $60/Barrel, constant for the entire life of the field. The other field parameters are shown in Table 4.13. Typical Niger Delta Field chance of success, Ps approaching 35% is assumed. The Risk tolerance for the Super Major, Large/Independent and Medium size Companies are assumed to $6, $4 and $2.0 Billion respectively. The analysis will proceed along the following lines:

i. Determination of Prospect Expected Value and Optimum Working Interest
ii. Determination of Risk Adjusted Value at Optimum Working Interest and hence "Grossed Up" Risk Adjusted Value
iii. Determination of offer or purchase price by farmor to farminees based on the Risk Adjusted Value analysis

Table 4.13

RAV Implications for Bid Value - Using Exponential Preference function

	XYZ

Risky Prospect			
Expected Recovery (MMBBLs) =		200	
Development Cost/Barrel =		10	
Risk Tolerance (Million) =		500	
Net Value of Success, $/Barrel =		60	
Total Cost =		2000	
Total Success Value =		12000	
Probability of Success, p=	35.0%	35.0%	35.0%
Value , V =	12,000	12,000	12,000
Cost , C=	2,000	2,000	2,000
Risk Tolerance, R =	2,000	4,000	6,000
Working Interest, W=	16.75%	33.51%	50.26%
RAV=	218	436	655
Expected Value, EV=	2,900	2,900	2,900
Optimum Working Interest =	16.75%	33.51%	50.26%
Grossed Up RAV =	1,302	1,302	1,302

Table 4.13 shows the risky prospect having an expected value of $2.9 Billion, which will be worthy of consideration for an investment exposure of $2 Billion for many Investment decision makers. The Risk Adjusted value and optimum working interests for the three levels of Risk Tolerances are shown in Table 4.13 and Fig 4.23. Optimum working interests are 16.75%, 33.51% and 50.26% for the $2, $4 and $6 Billion risk tolerance levels.

For the medium size Independent/Companies, a proposal offering them 20% of the prospects will translate into an RAV equal to $200 Million (as shown in Fig 4.23) or $1.0 Billion for the entire prospect. If these potential farminees were offered 30% of the prospect, the corresponding RAV will be $134 Million or $450 Million for the entire prospect. The insight here is that, ***an offer close to the Optimum Working interest leads to a higher valuation of the prospect***. For the Large Independent, with Risk Tolerance of $4 Billion, an offer of 40% (just slightly higher than the Optimum working Interest of 33.51%) translates to an RAV of $424 Million or $1.06 Billion for the entire prospect. If this group of Companies were offered 60%, however, the corresponding RAV = $268 Million or $450 Million for the entire prospect (also much less than when a working Interest close to the Companies' optimum working interest was offered)

A similar conclusion is reached for the Super Majors category of Companies. For this group, the Optimum working interest is 50% with an RAV of approximately equal to $655 Million and hence a "Grossed Up" RAV of approximately $1.3 Billion for the entire prospect. An offer of 100% prospect for the Super Major shows a corresponding RAV value of just $279 Million. The assumption of course, is that all the decision makers are using the same analytical RAV analysis procedures and are quite in agreement as to Success values, Failure cost and Success chance factors. The requirement that the Companies use the same risk adjusted value analysis implies that they subscribe to the maximization of expected utility.

Disagreement or otherwise on the values of Success Chance factor, Value of Success and Failure cost is more of a technical or procedural consideration and not fundamental. Therefore, a transparent display of how these values are arrived at will go a long way in resolving disagreement between "Buyer" and "Seller" or amongst the participating parties in a proposed Bidding Group or Farm Out arrangement.

The preceding analysis shows that, due to differing levels of risk aversion, allocation of working interest to potential farminees and/or bidders for a risky prospect has some effect on bid value and hence the level of revenue that will eventually accrue to the farmor or entity offering the prospect for sale. Companies will prefer to "take" or invest close to their optimum working interest intuitively to minimize exposure to bankruptcy, entire loss of their exploration budget or just purely motivated by portfolio diversification considerations.

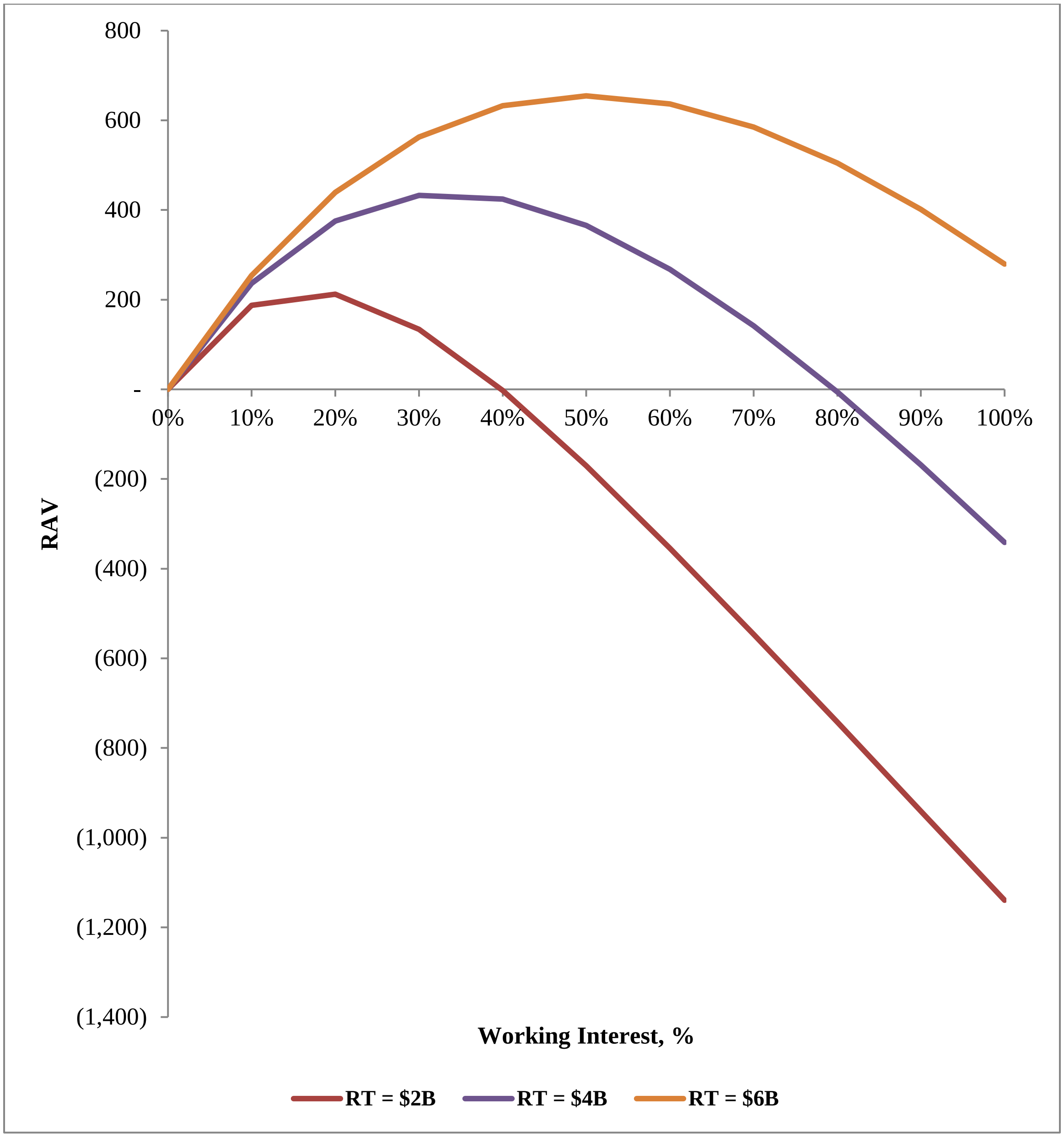


Figure 4.23- RAV versus Working Interest-Bidding Prospect

Because the Optimum Working interest is a linear function of Risk Tolerance, RT, the "Grossed Up" risk adjusted value is thus independent of RT. This is evident from Table 4.12 – irrespective of the level of Risk Tolerance the "Grossed Up" RAV is $1.302 Billion. The "Grossed Up" RAV therefore can serve as an analytical basis for the allocation of working interest, since all parties to the valuation can independently derive it, irrespective of the level of their aversion to risk and optimum working interest. The logic behind RAV analysis, and the exposure and level of application of this method of opportunity valuation in the oil and gas industry suggest that this will be a significant improvement over the "seeming" arbitrariness of working interest allocation that is currently the situation in the Industry. The literature is quite scanty on the analytical procedures employed by Bidding Companies on how individual working interests are allocated when putting a Bidding Group together – some non-quantitative considerations such as asset execution capacity and prior working relationships seem to play a major factor in putting a Bidding Group together. Using RAV analysis, all participants in the Group have the tools and expertise to independently estimate the "Grossed Up" RAV.

A similar analysis applies to Government Take realizable from Bid Rounds. In this case, the Government assumes the role of the Farmor and the potential bidders – the Farminees. Large Block offerings that will require significant investment capital outlays and technological know-how will attract only the large Companies or the Super Majors first, then the Large Independents next and lastly the medium size Companies. The possibility that the investment required will exceed investment at the optimum working interest level of the Super Major and hence will lead to a lower valuation compared to case in which the block is offered close to the Optimum working interest where it is valued the most, as shown in the preceding analysis. If blocks are offered in smaller parcels, they will likely be offered close to the optimum working interest(s) of more numerous potential bidders and (hence the highest valuation) and consequently maximum Government take in entry fees, Signature bonuses and other revenue.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The following are the conclusions of this research into improved working interest determination in risky Oil and Gas assets:

i. An analytical expression showing the inverse relationship between optimum working interest and total variance was derived in this study for a two outcome Risky prospect. Therefore this study takes the approach that a logical first step in Risk Analysis is the decomposition of total variance into upside and downside semi variances to determine if the risk in an investment consists more of Upside (opportunity) or Downside risk - measured by the Opportunity to Risk ratio. Thus:

ii. To correct for the aversion to the Paradox of Aversion to Reward, a two-step approach is proposed, this study proposes that :

   a. The working interest up to and including the Optimum working interest is first determined through the Maximization of Expected Utility and

   b. Beyond the Optimum Working Interest, the model switches to the Expected Value Model in which working interest is a linear function of Risk Tolerance.

iii. In order to test for the robustness of the RAV approach, the main input variables of Success Value, V, Failure Cost, C and Chance of Success, P_s were subjected to sensitivity analysis. The parameters were varied using discrete and simple probability distributions in the Simulation model. RAV analysis was found to be quite robust to the sensitivities of the various input parameters consistent with the findings of Moore et al.

iv. Portfolio Optimization Analysis was performed for a Group of five (5) Risky Assets (from the Literature – Lerche and MacKay) for Unlimited and Limited Capital situations using a linear Optimization Software (Solver in Palisades Decision Tools Suite). The result of the Optimization shows higher levels of working interest and hence required investment were recommended using the hyperbolic preference function. The Exponential preference function recommended lower working interest consistent with its more conservative nature in individual investor risk representation. The high level of robustness of the optimization in the recommended working interests for both the Limited and Unlimited Capital situations shows the solutions were independent of the solution algorithm employed.

v. To investigate the relative impacts of the important variables of Success Value, Failure Cost, Chance of Success and Risk Tolerance, a simulation Model was built and run for 10,000 iterations. Failure Costs turns out to be the most significant variable impacting the probability distribution of the Risky Asset's Risk Adjusted Value. In conducting Risk Adjusted Value Analysis therefore, an accurate estimation of Failure costs is of critical importance and assumes even more significance given the tendency or bias towards underestimation of Project Costs in the Oil and Gas Industry. Interestingly, the level of Risk Tolerance, RT did not significantly impact RAV probability distribution.

vi. The impact of risk aversion level (or Risk Tolerance RT) on Asset valuation of a Farm-Out arrangement was also investigated. Potential Farminees will intuitively desire to acquire an interest close to their working interest, at which point the Risk Adjusted Value is at the maximum to that particular investor. This implies that the intrinsic value of the asset or the "Grossed Up" Risk Adjusted Value is also at a maximum at the Optimum Working interest (the perception of the potential Farminee). An offer of larger than the Optimum Working interest will therefore lead to lower valuation or "Grossed Up" RAV. Risk Adjusted Value analysis therefore offers an analytical and transparent basis to allocate working interests for a group of investors, since all can independently carry out an estimate of the "Grossed Up" RAV, as long all the participants can agree on the values of the input variables of Success Value, Failure Cost and Success Chance factor.

vii. A similar conclusion can be drawn on the size of block offers by Government in a Bid round. The bigger the acreage, the more likely that the potential bidders will be the Super Majors or the Large Independents whose Risk Tolerance levels are markedly different from the Medium or Small Independents. The significantly differing Risk Aversion levels can lead to different valuations by potential bidders. Offering the blocks in smaller lots will attract many more investors (large and small) and potentially maximize value of returns to the Government.

viii. The two forms of the Preference functions used in this study, the Exponential and the Hyperbolic forms are commonly used to model investor risk avoiding behavior, where Risk Adjusted Value analysis have been adopted for Exploration Asset valuation. This does not preclude the use of other preference function forms especially where data is available on the preferences of the Individuals or Group of Individuals constituting the Investment Selection and Decision Board/Committee of a Corporation. The Hyperbolic Utility function is a little less conservative as a Risk avoiding preference function than the Exponential function, which was extensively studied and recommended by Cozzolino et al.

5.2 Study Limitations

This study is predicated on the fundamental theory of maximization of Expected Utility-the mathematical foundation of which was laid by Von Newman and Morgenstern **(1944).** Preference functions represent individual behavior towards risk and expected utility is the probability weighted average of the utilities of the outcomes in a risky situation. The rational investor's goal is maximization of expected utility of the risky situation. Non-acceptance of the fundamental basis of Utility theory for risky asset valuation will be a rejection of the basis for RAV analysis from which Optimum Working Interests were determined using the proposed Hybrid Models in this study. The Portfolio Optimization and results of the sensitivity analysis were all based on maximization of expected utility. A rejection of Utility theory therefore will be a rejection of these results and the conclusion therefrom.

Other limitations of the study include:

i. Use of Non-Monetary Criteria in Project Performance Evaluation. Monetary values have been used in the success and failure outcome values, which are then converted into utilities using the appropriate Preference functions for the Investor or Decision maker. This study therefore does not extend into opportunity valuations in which non-monetary criteria have to be incorporated into the analysis. These situations are generally referred to as Multi criteria decision situations in which non-monetary criteria are important and essentially have to be factored into Investment Decision making.

ii. The conclusion that RAV analysis can be used as an analytical basis of working interest allocation in Farm out or bidding situation is predicated on maximization of expected utility. If any of the parties to the Farm Out or Bid Group disagrees with the fundamental basis of RAV analysis, then this conclusion does not hold and another alternative process must be used to allocate working interest amongst the Group. An example will be, when one party uses Expected value and the others are Expected Utility maximizing. The two valuation procedures are close enough for valuation analysts to work their way through, the results will however be different from the one highlighted in this study.

5.3 Recommendations

I. Constant Risk Tolerance Values, RT have been used in this Research. There is a growing body of literature suggesting RT levels should vary according to Strategic Business Units (SBUs) of a Company or Exploration Region (e.g. Mature Basin, International or Frontier Region). This is not the focus of this study. In any case, the appropriate RT level to use can be specified by the Company or Decision Maker if the requisite information is available. This study recommends published rules of thumb for assigning Risk Tolerance, RT level for different sized firms based on the size of their Exploration Budgets. The Company or Investing Institution can commission research into determining the appropriate level of Risk Tolerance to use in risk adjusted value analysis for specific regions, especially where the investment capital required are significant relative to total firm value.

II. Incorporating the precise uncertainty in the Success Value is another major consideration in the risk analysis of Exploration ventures. Discrete values and simple triangular probability distributions have been used in this research. In some situations, the precise modeling of the uncertainty may justify the use of certain specific probability distributions-for example use of the lognormal distribution when the success leg is shown as Reserves. Incorporating the uncertainty in the success value introduces another level of complexity in RAV analysis and analytical methods may not be adequate in solving the RAV models that may be developed. Numerical solution methods may be resorted to when this level of complexity arise in RAV analysis.

REFERENCES

Akinpelu, L. O., Omole O.A. “Quantifying Errors in Present Value Factor Relationships for Restricted/Declining Production” SPE Paper 128357 presented at the 33rd NAICE, Aug. 3-5, 2009

Arps, J.J. and Arps, J.L. (Prudent Risk-Taking”, JPT, SPE July 1974, pp.711-716

Bailey, W. “ A Stepwise Example of Real Options Analysis of A Production Enhancement Project” SPE 78329 presented at the 13th European Petroleum Conference held in Aberdeen, Scotland, UK, 2002

Baird, B.F., Managerial Decisions under Uncertainty: An Introduction to the Analysis of Decisions Making, John Wiley and Sons, N.Y. (1989)

Brashear, J. P. et al “Interdependencies among E and P Projects and Portfolio Risk Management,” SPE 56574presented at the 1999 ATCE-Houston, Texas

Capen, E. C. et al 1971“ Competitive Bidding in High Risk Situations” JPT 23, 641-653

Capen, E. C. et al 1976 “Growth Rate – A Rate of return Measure of Investment Efficiency” JPT 28, 531-543

Chavas Jean-Paul: Risk Analysis in Theory and Practice, Elsevier Academic Press, san Diego, Ca, USA 2004

Cozzolino, J.M., “ A New Method for Measurement and Control of Exploration Risk,” SPE Paper 6632 March 1978

Cozzolino, J.M.,”A Simplified Utility Framework for the Analysis of Financial Risk” SPE Paper 6359, (1977)

Dixit A.K. and Pyndick, R.S. (1994) “Investment under Uncertainty” Princeton University Press, Princeton, N.J.

Dixit A.K. and Pyndick, R.S. (1995) The Options approach to Capital Investment. Harvard Business Review, May-June, pp. 105-115

Dougherty, E.L. and Lohrenz, J. (1976) Statistical Analysis of bids for federal offshore Leases, J. Pet. Technology 28, 1377-1390

Dougherty, E.L. and Nozaki, M. Analysis of Bidding Strategies. Pare SPE-4566, SPE 48th Annual Fall Meeting, Las Vegas, Nevada

Estrada, J "Mean-Semi-variance Optimization: A Heuristic Approach" IESE, Barcelona, Spain

Glanville, J.W. (1957) Rate of Return Calculations as a measure of investment opportunities, JPT 9, 12-19

Grayson, C.J. (1960). "Decisions under Uncertainty: Drilling decision by Oil and Gas Operators." Harvard University Press, Boston.

Greenwalt, W.A. "Determining Venture Participation" JPT, SPE, Nov 1981 pp. 2189-2195

Harbaugh, J.W., Doveton, J.H. and Davis, J.C. (1977). "Probability Methods in Oil Exploration." Wiley, New York.

Hightower, M. L. et al "Portfolio Modeling: A Technique for Oil and Gas Investors", SPE 22016 presented at the SPE Hydrocarbon Economics and Evaluation Symposium, Dallas, Texas, 1991

Lerche, I and Mackay, J, Economic Risk in Hydrocarbon Exploration, Academic Press, 525 B Street, San Diego, Ca 92101, USA, (1993)

Lerche, I. (1990) "Basin Analysis: Quantitative Methods," Vols 1and2, Academic Press, San Diego, CA

Lerche, I. (1991). "Oil Exploration: Basin Analysis and Economics." Academic Press, San Diego, CA

Lerche, I. (1994). "Anatomy of a Lease sale: Sub-salt bids and strategies at Gulf of Mexico lease sale 147, , Oil and Gas Journal, Oct. 3, pages 68-71.

Lerche, I. (1997). "Geologic Risk and Uncertainty in Oil Exploration." Academic Press, San Diego, CA

Lohrenz, J. and Bailey, A.J. (1995). Evidence and results of present value maximization for oil and gas development projects. SPE paper 30050 presented at the SPE Hydrocarbon Econ. Evaluation Symposium, Dalas, TX, 1995

Lohrenz, J. and Dickens, R.N. (1993). Option theory for evaluation of oil and gas assets: The upsides and downsides. SPE Paper 25837, 179-188

Mackay, J. A. "Utilizing Risk Tolerance To Optimize Working Interest", SPE Paper 30043, 1995

Markowitz, H., Portfolio Selection: Efficient Diversification of Investment, 2nd Edition, Cambridge, MA, USA

McCray, A.W. (1969) Evaluation of Exploratory Drilling Ventures by statistical decision methods; JPT 21, 1199-1209

McCray, A.W., Petroleum Evaluations and Economic Decisions, Prentice-Hall Inc, New Jersey, (1975).

Mian, M. A. , Project Economics and Decision Analysis , Volumes 1and2, Penwell Publishing Co., Tulsa, Ok, (2002)

Nersesian, R. (2013) Energy Risk Modeling, Palisade Corporation, Ithaca, New York, USA

Newendorp, P. Decision Analysis for Petroleum Exploration, Petroleum Publishing Company, Tulsa, Ok (1975)

Newendorp, P.D. and Root, P.J. (1968) Risk Analysis in drilling investment decisions, JPT 20, pp. 579-585

Quick, A. N. and Buick, N.A. "Viability and Portfolio Analysis as Tools for Exploration Strategy Development," JPT, SPE April, 1984, pp.619-627

Rose, P.R. (1987). Dealing with risk and uncertainty in exploration: How can we improve? AAPG Bulletin. 71, 1-16

Rose, P.R. (1994). Implementing risk analysis in exploration organizations: What works? What doesn't? American Assoc. of Pet. Geologists, 1994 Notes.

Rosenberg, B. F. "Risk versus Return Considerations for International Oil Exploration and Development Licenses" SPE Paper 14467, (1985) Smith, J. L. “Petroleum Property Valuation” , June 2003- Center for Energy and Environmental Policy Research, MIT

Smith, M.B. (1974) “Probability Estimates for petroleum drilling decisions” JPT 26, pp. 687-695

Moore, C. R. K. et al (2005) “Use of Risk Adjusted Values in Exploration Portfolio Management”, SPE Hydrocarbon and Economics Evaluation Symposium, Dallas, Tx

Von Newman, J. and Morgenstern, O. “Theory of Games and Economic Behavior”, Princeton University Press, Princeton, USA (1944)

APPENDIX A

Discounted Cash Flow (DCF) Analysis –

Example Application of DCF in Determining Value of Reserves

Let N_p represent Recoverable Reserves for a discovered or producing field in barrels, P the Oil Price and C, the production costs per barrel.

The Net Present Value (NPV) of the reserves can be analytically represented thus:

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+i)^t} \quad \text{.......... (A1)}$$

where CF_t are the cash flows in time t, i the interest (discount) rate.

On a continuous basis,

$$NPV = \int_0^T CF_t e^{-it} dt \quad \text{.......... (A2)}$$

The cash flows are generated by production q and the net value of the barrel (*Price, P less Costs,* C_p).

Net Present Value is thus:

$$NPV = \int_0^T q_i e^{-(D_i+i)t}(P - C_p)dt \quad \text{.......... (A3)}$$

If we assume production declines exponentially, q_i is the initial production, the net value of the barrel (P-C_p) does not change with time and T is reasonably long (project life exceeds 20 years for most Exploration and Development assets), NPV can be approximated thus:

$$NPV = \frac{q_i(P - C_p)}{(D_i + i)} \quad \text{.......... (A4)}$$

Recoverable reserves for exponential production can be expressed as

$$N_p = \frac{q_i - q_a}{D_i} \cong \frac{q_i}{D_i} \quad \text{.......... (A5)}$$

$$q_a \ll q_i$$

Production rate at abandonment, q_a is usually much less than initial production, q_i and can be neglected.

The in situ value of a barrel of Reserves ($/bbl) is therefore,

$$V_i = {}^{NPV}/_{N_p} = \frac{D_i}{(D_i + i)}(P - C_p) \quad \text{.......... (A6)}$$

Thus $V_i = f\left(D_i, P, C_p, i\right)$

If *Di* and *i* ≈ 10% (typical values) and assume costs are approximately 1/3rd of the Price, then

$V_i = \frac{1}{3}P$ …………………………………………………………………………….. (A7)

The in situ value of a barrel in place is approximately 1/3rd the Oil Price or price at the well head, P (a popular industry rule of thumb)

The preceding shows an analytical form of DCF analysis in which the cash flow stream and the discounting are treated as continuous. Most industry applications however, treat cash flows as discrete, usually on an annual basis and the discounting is also discrete (annual), except otherwise stipulated. Sometimes the discounting is done on a monthly basis particularly for very marginal production assets or "stripper wells". The economics of these assets coincide with the accounting reporting schedules which are usually quarterly or even monthly.

APPENDIX B

Expected Present Value (EPV) Analysis

The Expected Value approach includes the probability of occurrence of all the events in a prospect. Consider the following prospect:

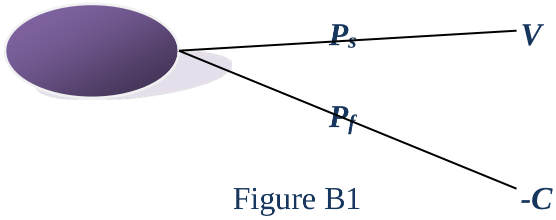


Figure B1

This is a two outcome prospect such as in wild cat drilling in which the outcomes are Discovery with present value, V or Dry hole, with a loss of C, the cost of the drilling operation and any other upfront Exploration costs. The probabilities of the two events are P_s and P_f and sum up to 1.0.

The Expected Present value, $EPV = P_s V - P_f C$ ……………………..………………... (B1)

Equation (B1) is positive as long as $P_s V > P_f C$.

The Expected Present value is also the mean of the distribution of the values of the outcomes, in this case discrete distribution of two outcomes, success and failure events. The mean is also referred to as the first moment of the project values. That is why the Expected value analysis is otherwise referred to as mean variance analysis after Markowitz.

The second moment of the distribution of outcomes, $E_2 = P_s V^2 + P_f C^2$ …………………...(B2)

Variance of the prospect, $\sigma^2 = E_2 - \mu^2 = P_s V^2 + P_f C^2 - \left(P_s V - P_f C\right)^2$ ……………..….(B3)

$\sigma^2 = (V + C)^2 \left(P_S P_f\right)$ ………………………………………………………………......(B4)

Volatility, $v = \frac{\sigma}{\mu} = \frac{(V+C)\left(P_S P_f\right)^{1/2}}{\left(P_S V - P_f C\right)}$ ………………………………………………………..….(B5)

Volatility, *v* is an indication of the stability of the mean, a small value of *v* (*v* <<1) denotes small uncertainty of the Expected Present Value while, v>>1, implies significant uncertainty in the Expected Value.

Considering a situation in which a fraction, W of the prospect is taken, rather than the whole prospect, the preceding analysis is modified only slightly.

$EPV(W) = P_s(WV) - P_f(WC) = W\left[P_s V - P_f C\right]$ ……………………………………….… (B6)

$EPV(W) = f(W, P_s, V, C)$ …………………………………………………………….. (B7)

Expected Present Value, is a linear function of W, the fractional working interest when investor takes a part of the prospect, and is maximized at 100% working interest (W=1).

Mean Semi-Variance Analysis

Mean semi-variance analysis is premised on the fact that Risk can be defined as the chance of a project outcome being lower than a certain threshold or downside, an unpleasant surprise, the possibility of loss of the money that has been invested. So in this instance, we are only concerned with deviations below a particular level of NPV, DPI or rate of return (ROR). Conversely, if the project comes in better than expected, that is a pleasant surprise or opportunity for more benefits or reward from the investment. The deviations that we will estimate in this instance are the deviations above the expected, for example, the mean. In both cases, the means are the same; it is the variability or the variances that defer. In one it is the total variance, while in the other we have semi-deviations below the mean and semi-deviations above the mean.

Returning to the 2-Outcome example

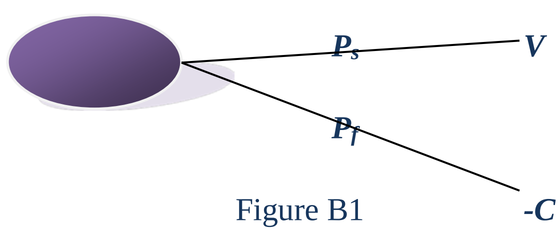


Figure B1

Mean, $\mu = P_s V - P_f C$ …………….(B1)

In Semi-variance considerations, we seek to minimize deviations below the mean and maximize deviations above (the mean):

$\sigma_s{}^2 = \sum P_i[Min(x_i - \mu), 0]^2 = P_s[Min(V - \mu), 0]^2 + P_f[Min(-C - \mu), 0]^2$……………..… (B8)

$\sigma_s{}^2 = P_f[C^2 + 2\mu C + \mu^2]$ …………………………………………………………... (B9)

For the upside or "pleasant surprise", maximizing the deviations above the mean:

$\sigma_{sup}{}^2 = \sum P_i[Max(x_i - \mu), 0]^2 = P_s[Max(V - \mu), 0]^2 + P_f[Max(-C - \mu), 0]^2$ ……..…(B10)

$\sigma_{sup}{}^2 = P_s[V^2 - 2\mu V + \mu^2]$ …………………………. ………………………………...(B11)

Adding the two semi deviations;

$\sigma_s{}^2 + \sigma_{sup}{}^2 = P_f[C^2 + 2\mu C + \mu^2] + P_s[V^2 - 2\mu V + \mu^2]$ …………………………...… (B12)

$= P_s V^2 + P_f C^2 - \mu^2 = \sigma^2$

$\sigma_s{}^2 + \sigma_{sup}{}^2 = \sigma^2$ ………………………………………………………………… .(B13)

As to be expected, the addition of the deviations below and above the mean equal the total deviations around the mean.

$\sigma_s{}^2 = P_f[C^2 + 2\mu C + \mu^2] = P_f\left[C^2 + 2(P_s V - P_f C)C + (P_s V - P_f C)^2\right]$ ……………….. (B14)

$\sigma_s{}^2 = P_f P_s{}^2[V + C]^2$ ……………………………………………………….……… (B15)

$\sigma_s = P_s P_f{}^{1/2}[V + C]$ ……………………………………………………………….. (B16)

$\sigma_{sup}{}^2 = P_s[V^2 - 2\mu V + \mu^2] = P_s\left[V^2 - 2(P_s V - P_f C)V + (P_s V - P_f C)^2\right]$ …………... (B17)

$\sigma_{sup}{}^2 = P_s P_f{}^2[V + C]^2$ ………………………………………………………..… (B18)

$\sigma_{sup} = P_f P_s{}^{1/2}[V + C]$ …………….. (B19)

Ratio of the Upside or Opportunity Standard Deviation to the Downside Standard deviation:

$\sigma_{sup}/\sigma_s = \frac{P_f P_s{}^{1/2}[V+C]}{P_s P_f{}^{1/2}[V+C]} = \left[\frac{P_f}{P_s}\right]^{1/2}$ ……………………………………………………….. (B20)

Numerical Example

The numerical example is taken from Lerche and Mackay (page 18). The Mean-Variance and Mean-Semi Variance analysis is shown below.

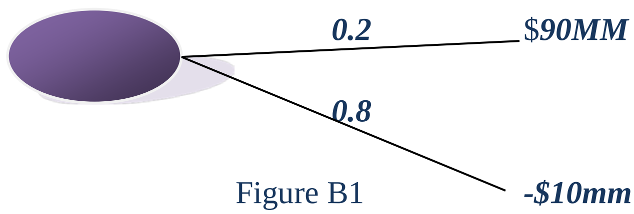


Figure B1

Expected Present Value of this Prospect:

$$EPV = P_s V - P_f C = (0.2)(90) + (0.8)(-10) = \$10MM$$

Total Variance, $\sigma^2 = (V + C)^2 (P_s P_f) = (90 + 10)^2 (0.2x0.8) = 1600$

Standard Deviation, $\sigma = (V + C)(P_s P_f)^{1/2} = 40$

Volatility, $v = \frac{\sigma}{\mu} = \frac{40}{10} = 4$

Downside Semi-Variance,

$\sigma_s{}^2 = P_f P_s{}^2 [V + C]^2 = 320$

Downside Standard Deviation,

$\sigma_s = P_s P_f{}^{1/2} [V + C] = 17.89$

Downside Volatility, = 17.89/10 = 1.789

Upside Semi-Variance,

$\sigma_{sup}{}^2 = P_s P_f{}^2 [V + C]^2 = (0.2x0.8)^2(90+10)^2 = 1280$

Upside Standard Deviation,

$\sigma_{sup} = P_f P_s{}^{1/2} [V + C] = (0.2x0.8)^{1/2}(90+10) = 35.78$

Upside Volatility, = 35.78/10 = 3.578

Ratio of Upside SD/Downside SD = $\sigma_{sup}/\sigma_s = \frac{35.78}{17.89} = 2.0$

The total variance (1600) is obtained by the addition of the downside and upside variances. It is readily observed that the upside standard deviation is twice that of the downside. There is, more opportunity for large rewards in this prospect than downside risk, which is also shown by the volatilities. The interesting observation from (B20) is that the ratio of upside standard deviation to that of the downside is solely dependent on the success and failure probabilities (P_s, P_f) and not on the success or failure values (V, C).

Equation (B20) shows that the Opportunity to Risk ratio, η, depends only on the success and failure probabilities only and independent of the success and failure values, V and C respectively. The implication of this is that regardless of the values of V and C, η will stay the same (as shown in the numerical example below). This is because changes in the parameters of Ps, Pf, V and C change the expected value (mean). The upside and downside variance changes but their ratio remains constant. If a particular target value or threshold expected worth is set by the Company or investor, the estimation of Downside and Upside variances varies slightly. Making zero (0) the "target" or "threshold" expected worth the following are the relationships for upside and downside semi-standard deviations, σ_s, σ_{sup} respectively:

$$\sigma_s{}^2 = \sum P_i[Min(x_i - 0), 0]^2 = P_s[Min(V-0), 0]^2 + P_f[Min(-C-0), 0]^2 \quad \text{(B21)}$$

$$\sigma_s{}^2 = P_s[0]^2 + P_f[(-C-0)]^2 = P_f C^2 \quad \text{(B22)}$$

$$\sigma_s = P_f{}^{1/2} C \quad \text{(B23)}$$

Similarly

$$\sigma_{sup}{}^2 = \sum P_i[Max(x_i - 0), 0]^2 = P_s[Max(V-0), 0]^2 + P_f[Max(-C-0), 0]^2 \quad \text{(B24)}$$

$$\sigma_{sup}{}^2 = P_s[(V-0)]^2 + P_f[0]^2 = P_s V^2 \quad \text{(B25)}$$

$$\sigma_{sup} = P_s{}^{1/2} V \quad \text{(B26)}$$

The ratio of the upside to the downside standard deviations τ is now given by the following relationship:

$$\tau = {}^{\sigma_{sup}}/_{\sigma_s} = \left(\frac{P_s}{P_f}\right)^{1/2} \left[\frac{V}{C}\right]$$ --------------------------- (B27),

The Opportunity to Risk Ratio, τ not only depends on the values of P_s and P_f, but also on the success and failure values, *V* and *C*. This is more intuitive – the higher the value of V (keeping C constant), the higher should be the Opportunity to Risk ratio, τ. The two prospect example is reworked to demonstrate the problem of the "moving mean" and the correction for this problem through the choice of a "static" threshold or target expected worth of zero (0).

Table 1A shows the 2-prospect analyzed for total variance, upside and downside variances and hence Opportunity to Risk ratio with a change in the value of success value. The values of V, C, Ps and Pf are kept the same as previously given and then the value of V is changed from $90 million to $200 million keeping all the other values the same. Intuitively, since only the success value is improved, the opportunity to risk ratio should improve. When the analysis is done using the mean to decompose the semi variances, the opportunity to risk ratio stays the same (τ = 2.0), showing the problem of the "moving" mean. However, when "static" threshold is used (in this case 0), the Opportunity to Risk Ratio, τ improves from 4.5 to 10, in agreement with intuition. When returns on a risky prospect are more than expected, the opportunity to risk should improve accordingly too.

Table B1

Success Probability, Ps =	0.2	0.2		0.2	0.2
Failure Probability, Pf =	0.8	0.8		0.8	0.8
Success Value, V =	90	200		90	200
Failure Value, C=	-10	-10		-10	-10
Expected Value, μ =	10.00	32.00		10.00	32.00
Variance, σ^2 =	1,024	5,776		1,700	8,080
Sd, σ =	32.00	76.00		41.23	89.89
Volatility =	3.20	2.38		4.12	2.81

Downside Semi-Variance, =	204.8	1155.2		80	80
Downside Semi-Deviation, =	14.31	33.99		8.94	8.94
Downside Volatility =	***1.43***	***1.06***		***0.89***	***0.28***
Upside Semi-Variance, =	819.2	4620.8		1620	8000
Upside Semi-Deviation, =	28.62	67.98		40.25	89.44
Upside Volatility =	***2.86***	***2.12***		***4.02***	***2.80***
Ratio σ_{sup}/σ_s =	2.00	2.00		4.50	10.00

Table B2 demonstrates the "moving" mean problem on the downside. The variances are estimated by holding all the values of V, P_s and P_f the same and changing the cost C from \$10 million to \$30 million. In this case, we should expect the opportunity to risk ratio to get worse (Expected value has decreased from \$10 Million to a negative \$6 Million). Choosing a "static" threshold agrees with this intuition (τ changes from 4.5 to 1.5), while estimating the variances using the mean (or expected value) keeps the opportunity to risk ratio, τ the same at 2.0 (as correctly predicted by equation B20).

Table B2

Success Probability, Ps =	0.2	0.2		0.2	0.2
Failure Probability, Pf =	0.8	0.8		0.8	0.8
Success Value, V =	90	90		90	90
Failure Value, C=	-10	-30		-10	-30
Expected Value, μ =	10.00	(6.00)		10.00	(6.00)
Variance, σ^2 =	1,024	576		1,700	2,340
Sd, σ =	32.00	24.00		41.23	48.37
Volatility =	3.20	(4.00)		4.12	(8.06)

Downside Semi-Variance, =	204.8	115.2		80	720
Downside Semi-Deviation, =	14.31	10.73		8.94	26.83
Downside Volatility =	***1.43***	***(1.79)***		***0.89***	***(4.47)***
Upside Semi-Variance, =	819.2	460.8		1620	1620
Upside Semi-Deviation, =	28.62	21.47		40.25	40.25
Upside Volatility =	***2.86***	***(3.58)***		***4.02***	***(6.71)***
Ratio σ_{sup}/σ_s =	2.00	2.00		4.50	1.50

APPENDIX C

RAV Analysis- Cozzolino Approach- Use of the Exponential Utility Function

Use of Exponential Utility Function:

$U(x) = e^{-rx}$ ………………………………………………………………………….....…(C1)

Where x = terminal wealth and r = risk aversion level =1/millionths

We also use the two outcome prospect shown in Figure B1

The Risk Adjusted Value can be represented by the general relationship:

$RAV = -\left(1/_r\right)\ln(\sum P_i e^{-rV_i})$ ………………………………………………………..….. (C2)

Risk Tolerance (Millions)

$RT = 1/_r$ .... (C3)

The Expected Utility (EU) of the prospect in Figure B1 can be expressed by the following:

$EU(x) = P_s e^{-\frac{WV}{RT}} + P_f e^{\frac{WC}{RT}}$ ………………………………………………………..…. (C4)

The Certainty Equivalent (CE) of this expected utility is the Risk Adjusted value and if we assume it is also of the exponential form can be expressed by:

$e^{-\frac{RAV}{RT}} = P_s e^{-\frac{WV}{RT}} + P_f e^{\frac{WC}{RT}}$ …………………………………………………………..…C5)

$RAV = -RT\ln\left[P_s e^{-WI*V/RT} + P_f e^{WI*C/RT}\right]$ ………………………………………….. (C6)

RAV is a non-linear function of Working Interest (WI) – thus there is a value of working Interest that maximizes Risk Adjusted Value, RAV

$W_{opt} = \frac{RT}{V+C}\ln\frac{P_s V}{P_f C}$ ……………………………………….…………………..…. (C7)

The constraint in (C7) is $P_s V > P_f C$

From (C7), Optimum working Interest, W_{opt} is a linear function of Risk Tolerance for specific V, C and P_s

Risk Adjusted Value at Optimum Working Interest:

$RAV_{Wopt} = -RT\left[P_s\left(\frac{P_f C}{P_s V}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_s V}{P_f C}\right)^{\frac{C}{V+C}}\right]$ …………………………………………… (C8)

The “Grossed Up” Risk Adjusted Value or Total Risk Adjusted Value if 100% of prospect is Optimum is simply the Risk Adjusted value at optimum working interest divided by the optimum working interest fraction, W_{opt}

$$RAV_{Gross} = \frac{RAV_{Wopt}}{W_{opt}} = \frac{-RT\left[P_s\left(\frac{P_fC}{P_sV}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_sV}{P_fC}\right)^{\frac{C}{V+C}}\right]}{\frac{RT}{V+C}\ln\frac{P_sV}{P_fC}} = \frac{-(V+C)\left[P_s\left(\frac{P_fC}{P_sV}\right)^{\frac{V}{V+C}} + P_f\left(\frac{P_sV}{P_fC}\right)^{\frac{C}{V+C}}\right]}{\ln\frac{P_sV}{P_fC}}$$

……………… (C9)

Equation C9 shows that the Gross Risk Adjusted Value of a prospect does not depend on the Risk Tolerance, but solely on the magnitude of reward V, loss, C and success probability, P_s.

From C7 the Risk Tolerance,

$$RT = \frac{W_{opt}(V+C)}{\ln\frac{P_sV}{P_fC}}$$

If historical working interests are known, and considered as a Company's optimum working interests, then the value of RT estimated can be regarded as the apparent risk tolerance of the Company (ART).

Recall from EPV analysis (B4), variance is given by $\sigma = (V+C)\left(P_sP_f\right)^{1/2}$

$(V+C) = \sigma/\left(P_sP_f\right)^{1/2}$ ………………………………………………………………… (C10)

Therefore $W_{opt} = \frac{RT\left(P_sP_f\right)^{1/2}}{\sigma}\ln\frac{P_sV}{P_fC}$ ……………………………… ……………………… (C11)

From C11, it is obvious that the Optimum Working Interest is an inverse function of the square root of the variance (Standard Deviation). An increase in variance can occur due to increasing values of reward, V or increasing value of costs, C. Increasing reward or "a high gain situation" is "good risk while increasing losses is "bad risk". It is the latter that the investor seeks to avoid because it can potentially lead to bankruptcy or "gambler's ruin". In "high gain" situations, the investor should take more of the good fortune and invest more (increase working interest). However, the optimum working interest peaks at a certain value and then decreases.

The paradox of aversion to reward has led some investigators to propose a downside risk model to evaluate uncertainty in prospect analysis. The downside risk model or semi variance analysis is premised on assessing returns below the mean or a specified benchmark e.g. a particular level of return – let's say 5% return on investment.

APPENDIX D

Risk Adjusted Value Analysis- Lerche and Mackay

Use of Hyperbolic Utility Function form:

$U(x) = 1 - tanh(x)$ ………………………………………………………………………. (D1)

Where x = terminal wealth and r = risk aversion level =1/millionths

We also use the two outcome prospect shown in Figure B1

The Expected Utility (EU) of the prospect in Figure B1 can be expressed by the following:

$$EU(x) = P_s\left[1 - \tanh\left(\frac{\mathrm{WV}}{\mathrm{RT}}\right)\right] + P_f\left[1 - tanh\left(\frac{-WC}{RT}\right)\right] \text{ .... (D2)}$$

$$EU(x) = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \text{ .... (D3)}$$

The Certainty Equivalent (CE) of this expected utility is the Risk Adjusted value and Lerche and Mackay assumed it is also of the exponential form and expressed it as:

$$e^{-\frac{RAV}{RT}} = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \text{ .... (D4)}$$

$$RAV = -RT\ ln\left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \text{ .... (D5)}$$

The Risk Adjusted Value RAV) is also a nonlinear function of the Working Interest, W. Differentiating RAV with respect to W and equating to zero, RAV has maximum value at Working Interest expressed implicitly by:

$$cosh\left(\frac{W_{opt}C}{RT}\right) = \left(\frac{P_f C}{P_s V}\right)^{1/2} cosh\left(\frac{W_{opt}V}{RT}\right) \text{ .... (D6)}$$

APPENDIX E

Risk Adjusted Value Analysis- Departure from Lerche and Mackay

Use of Hyperbolic Utility Function form:

$$U(x) = 1 - tanh(x) \text{ .... (E1)}$$

Where x = terminal wealth and r = risk aversion level =1/millionths

We also use the two outcome prospect shown in Figure B1

The Expected Utility (EU) of the prospect in Figure B1 can be expressed by the following:

$$EU(x) = P_s\left[1 - \tanh\left(\frac{\mathrm{WV}}{\mathrm{RT}}\right)\right] + P_f\left[1 - tanh\left(\frac{-WC}{RT}\right)\right] \ldots\ldots \text{(E2)}$$

$$EU(x) = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots \text{(E3)}$$

The Certainty Equivalent (CE) of this expected utility is the Risk Adjusted value and we will explore the option that Certainty Equivalent or Risk Adjusted Value also takes the hyperbolic form assumed and expressed as:

$$1 - \tanh\left(\frac{RAV}{RT}\right) = \left[1 - P_s tanh\left(\frac{WV}{RT}\right) + P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots \text{(E4)}$$

$$\tanh\left(\frac{RAV}{RT}\right) = \left[P_s tanh\left(\frac{WV}{RT}\right) - P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots \text{(E5)}$$

$$RAV = RT tanh^{-1}\left[P_s tanh\left(\frac{WV}{RT}\right) - P_f tanh\left(\frac{WC}{RT}\right)\right] \ldots\ldots \text{(E6)}$$

APPENDIX F

Tables of Results of Numerical Example

Table F1

Working Interest, WI	Exponential Model, RAV	Hyperbolic Model, RAV
0%	-	-
5%	928	999
10%	1,715	1,994
15%	2,364	2,977
20%	2,876	3,943
25%	3,254	4,887

30%	3,501	5,801
35%	3,619	6,681
40%	3,612	7,520
45%	3,481	8,314
50%	3,228	9,058
55%	2,857	9,746
60%	2,370	10,374
65%	1,769	10,937
70%	1,056	11,431
75%	235	11,854
80%	(693)	12,200
85%	(1,726)	12,468
90%	(2,861)	12,653
95%	(4,096)	12,755
100%	(5,428)	12,770

Table F2

RAV Sensitivity to Success Value, V- Exponential Model

Working Interest	**V=$0.5MM**	**V=$0.6MM**	**V=$0.7MM**	**V=$0.8MM**	**V=$0.9MM**	**V=$1 MM**
0%	-	-	-	-	-	-
5%	928	1,903	2,873	3,839	4,802	5,761
10%	1,715	3,613	5,496	7,364	9,216	11,053
15%	2,364	5,136	7,876	10,581	13,254	15,894
20%	2,876	6,476	10,017	13,501	16,927	20,297
25%	3,254	7,635	11,927	16,131	20,248	24,280
30%	3,501	8,619	13,612	18,481	23,228	27,857
35%	3,619	9,432	15,076	20,557	25,878	31,044
40%	3,612	10,076	16,327	22,370	28,210	33,854

45%	3,481	10,557	17,370	23,926	30,235	36,303
50%	3,228	10,878	18,210	25,235	31,963	38,405
55%	2,857	11,044	18,854	26,303	33,405	40,174
60%	2,370	11,056	19,307	27,139	34,572	41,623
65%	1,769	10,921	19,574	27,750	35,473	42,765
70%	1,056	10,641	19,660	28,144	36,120	43,614
75%	235	10,219	19,572	28,328	36,520	44,181
80%	(693)	9,660	19,314	28,309	36,685	44,479
85%	(1,726)	8,967	18,891	28,094	36,622	44,520
90%	(2,861)	8,144	18,309	27,690	36,341	44,314
95%	(4,096)	7,194	17,572	27,104	35,851	43,873
100%	(5,428)	6,120	16,685	26,341	35,160	43,208

Table F3

RAV Sensitivity to Success Value, V- Hyperbolic Model

Working Interest	**V=$0.5MM**	**V=$0.6MM**	**V=$0.7MM**	**V=$0.8MM**	**V=$0.9MM**	**V=$1.0MM**
0%	-	-	-	-	-	-
5%	999	2,000	3,002	4,004	5,006	6,010
10%	1,994	3,994	5,995	7,998	10,002	12,006
15%	2,977	5,970	8,964	11,958	14,949	17,938
20%	3,943	7,919	11,891	15,857	19,813	23,757
25%	4,887	9,829	14,760	19,671	24,559	29,416
30%	5,801	11,691	17,554	23,377	29,153	34,871
35%	6,681	13,495	20,258	26,952	33,564	40,079
40%	7,520	15,232	22,857	30,375	37,764	45,005

45%	8,314	16,891	25,338	33,626	41,727	49,615
50%	9,058	18,465	27,688	36,688	45,430	53,883
55%	9,746	19,946	29,895	39,545	48,854	57,786
60%	10,374	21,326	31,949	42,184	51,983	61,309
65%	10,937	22,599	33,839	44,593	54,806	64,439
70%	11,431	23,757	35,559	46,763	57,313	67,171
75%	11,854	24,797	37,102	48,688	59,500	69,502
80%	12,200	25,713	38,462	50,363	61,363	71,435
85%	12,468	26,500	39,635	51,785	62,903	72,976
90%	12,653	27,157	40,618	52,953	64,125	74,135
95%	12,755	27,679	41,410	53,868	65,032	74,922
100%	12,770	28,066	42,009	54,533	65,633	75,354

Tabl4 F4

RAV Sensitivity to Success Probability, Ps at different Working Interest Levels

Exponential Model

Working Interest	Ps=10%	Ps=20%	Ps=30%	Ps=40%	Ps=50%	Ps=60%	Ps=70%	Ps=80%
0%	-	-	-	-	-	-	-	-
5%	(2,040)	928	3,906	6,892	9,888	12,892	15,905	18,928
10%	(4,159)	1,715	7,625	13,570	19,550	25,566	31,619	37,709
15%	(6,356)	2,364	11,160	20,034	28,988	38,022	47,139	56,340
20%	(8,628)	2,876	14,513	26,287	38,201	50,259	62,464	74,820
25%	(10,973)	3,254	17,686	32,329	47,190	62,275	77,591	93,146
30%	(13,390)	3,501	20,682	38,163	55,955	74,070	92,519	111,315
35%	(15,877)	3,619	23,503	43,790	64,498	85,643	107,245	129,324
40%	(18,432)	3,612	26,152	49,213	72,817	96,993	121,767	147,171

45%	(21,054)	3,481	28,632	54,432	80,915	108,119	136,084	164,853
50%	(23,740)	3,228	30,944	59,450	88,792	119,021	150,193	182,368
55%	(26,490)	2,857	33,091	64,268	96,449	129,699	164,093	199,713
60%	(29,301)	2,370	35,077	68,890	103,887	140,153	177,783	216,886
65%	(32,173)	1,769	36,903	73,317	111,107	150,381	191,261	233,884
70%	(35,103)	1,056	38,572	77,551	118,110	160,385	204,525	250,705
75%	(38,090)	235	40,087	81,594	124,898	170,163	217,574	267,346
80%	(41,133)	(693)	41,450	85,449	131,472	179,717	230,407	283,805
85%	(44,230)	(1,726)	42,664	89,117	137,834	189,046	243,022	300,080
90%	(47,379)	(2,861)	43,732	92,602	143,984	198,150	255,419	316,167
95%	(50,580)	(4,096)	44,656	95,906	149,926	207,031	267,596	332,066
100%	(53,832)	(5,428)	45,438	99,031	155,659	215,688	279,552	347,774

Table F6

RAV Sensitivity to Success Value, V at different Working Interest Levels

Proposed Hybrid Model- Exponential/EV

Working Interest	V=$0.5 Million	V=$0.6 Million	V=$0.7 Million	V=$0.8 Million	V=$0.9 Million	V=$1.0 Million	V=$1.1 Million	V=$1.2 Million	V=$1.3 Million	V=$1.4 Million	V=$1.5 Million
0%	-	-	-	-	-	-	-	-	-	-	-
10%	1,438	1,530	1,596	1,645	1,684	1,715	1,741	1,762	1,781	1,796	1,810
20%	20,000	2,156	2,410	2,603	2,754	2,876	2,976	3,059	3,130	3,191	3,244
30%	20,000	20,000	20,000	20,000	3,235	3,501	3,720	3,905	4,061	4,196	4,313
40%	20,000	20,000	20,000	20,000	20,000	20,000	3,993	4,313	4,586	4,821	5,026

50%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	5,076	5,391
60%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
70%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
80%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
90%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
100%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000

Table F7
RAV Sensitivity to Success Value, V at different Working Interest Levels
Proposed Hybrid Model - Hyperbolic/EV

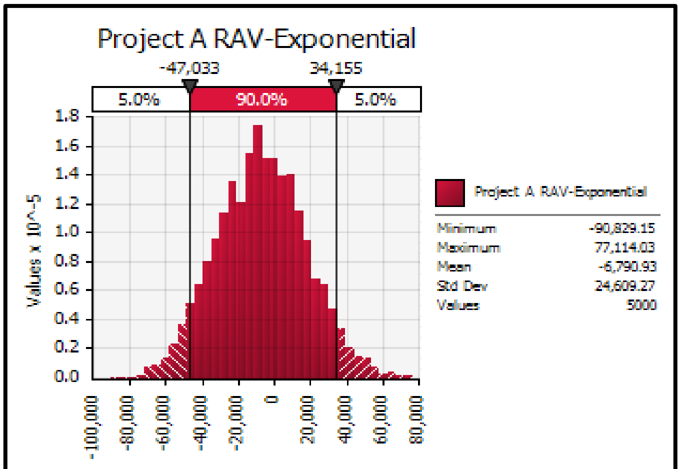
Working Interest	V=$0.5 Million	V=$0.6 Million	V=$0.7 Million	V=$0.8 Million	V=$0.9 Million	V=$1.0 Million	V=$1.1 Million	V=$1.2 Million	V=$1.3 Million	V=$1.4 Million	V=$1.5 Million
0%	-	-	-	-	-	-	-	-	-	-	-
10%	1,972	1,981	1,986	1,990	1,992	1,994	1,995	1,996	1,997	1,997	1,998
20%	20,000	3,835	3,880	3,909	3,929	3,943	3,954	3,962	3,968	3,973	3,977
30%	20,000	20,000	20,000	20,000	5,753	5,801	5,837	5,864	5,885	5,902	5,915
40%	20,000	20,000	20,000	20,000	20,000	20,000	7,605	7,670	7,720	7,760	7,792

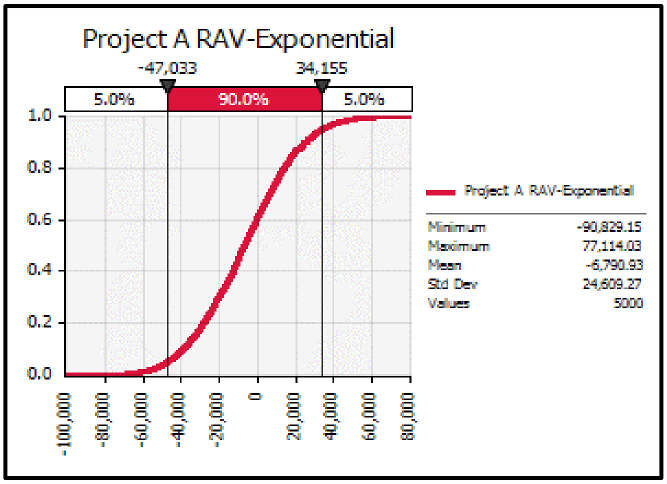
50%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	9,525	9,588
60%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
70%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
80%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
90%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000
100%	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000	20,000

Figure F1

Simulation Results for Projects A and B

@RISK Output Report for Project A RAV-Exponential





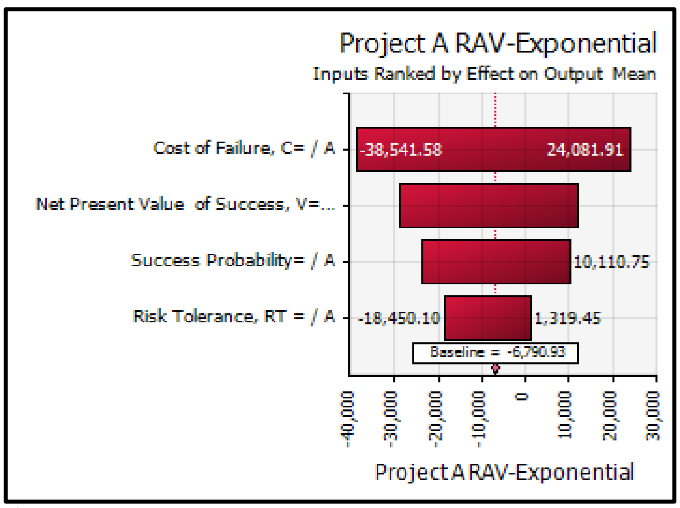
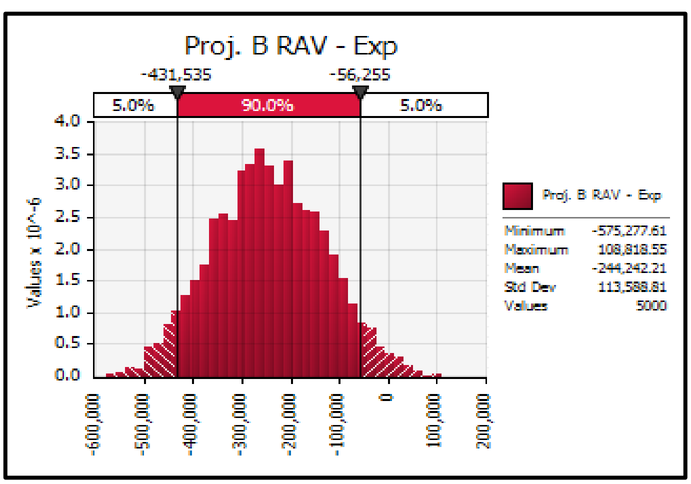
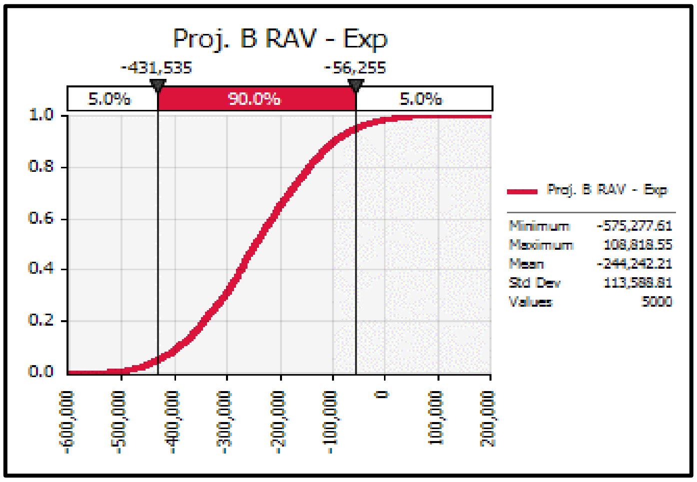


Figure F2

@RISK Output Report for Project B RAV - Exponential





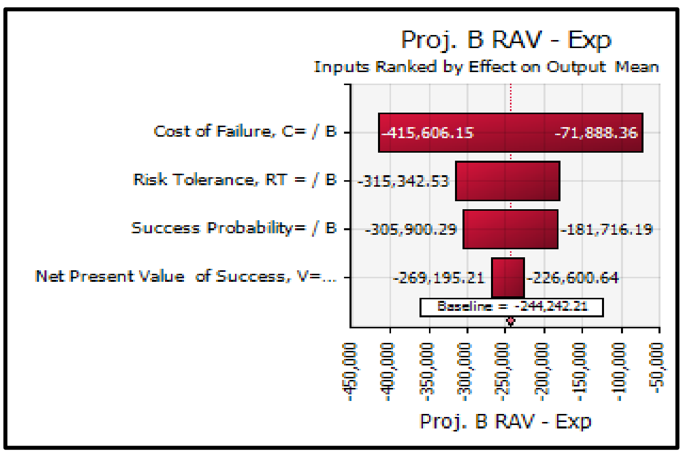
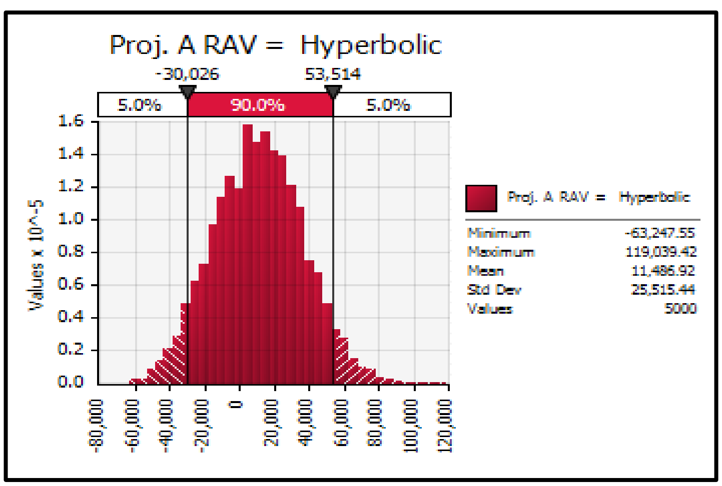
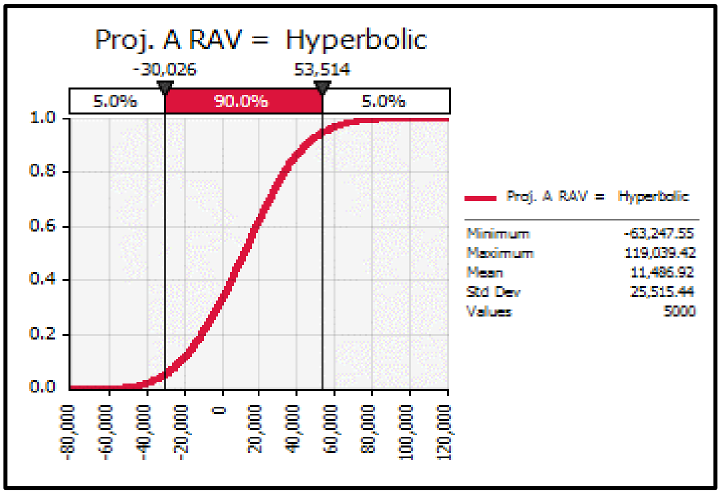


Figure F3

@RISK Output Report for Project A RAV = Hyperbolic





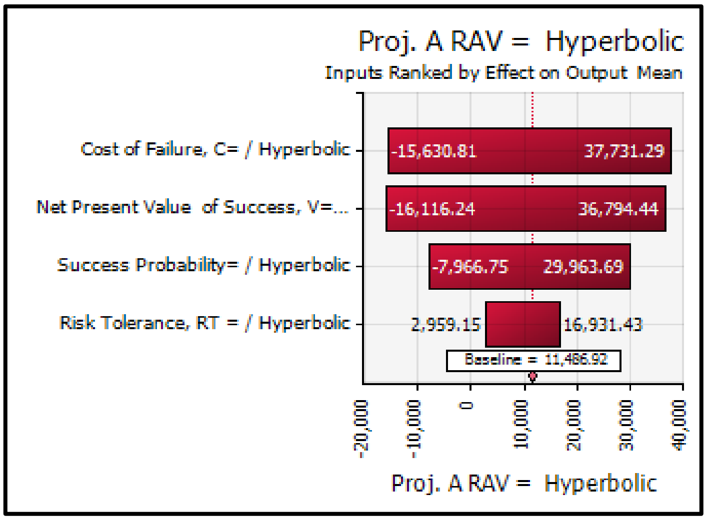
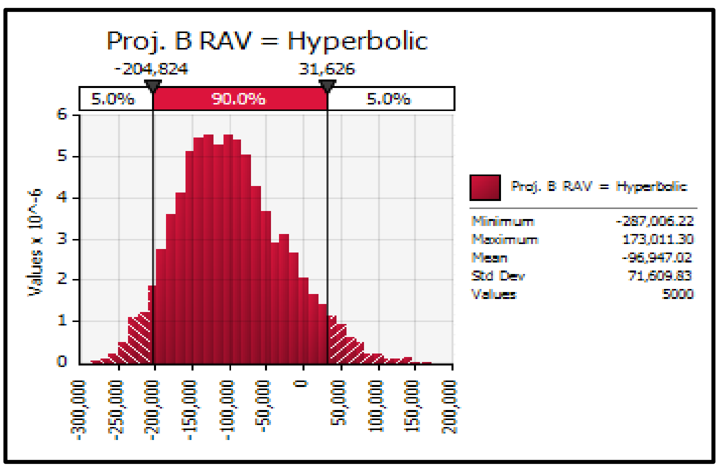
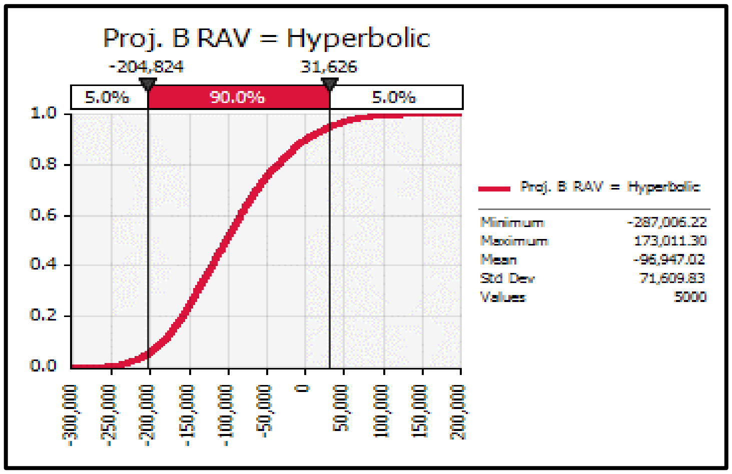
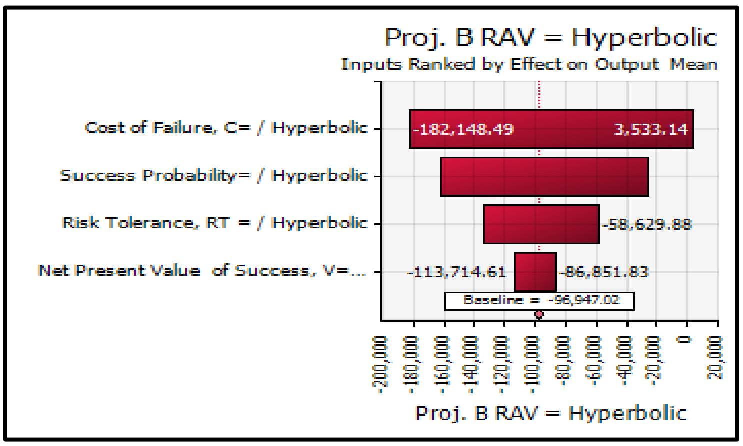


Figure F4

@RISK Output Report for Project B RAV = Hyperbolic




Proj. B RAV = Hyperbolic
-204,824
31,626
5.0%
90.0%
5.0%
1.0
0.8
0.6
0.4
0.2
0.0
-300,000
-250,000
-200,000
-150,000
-100,000
-50,000
0
50,000
100,000
150,000
200,000
Proj. B RAV = Hyperbolic
Minimum -287,006.22
Maximum 173,011.30
Mean -96,947.02
Std Dev 71,609.83
Values 5000


Proj. B RAV = Hyperbolic
Inputs Ranked by Effect on Output Mean
Cost of Failure, C= / Hyperbolic
Success Probability= / Hyperbolic
Risk Tolerance, RT = / Hyperbolic
Net Present Value of Success, V=...
-182,148.49
3,533.14
-58,629.88
-113,714.61
-86,851.83
Baseline = -96,947.02
-200,000
-180,000
-160,000
-140,000
-120,000
-100,000
-80,000
-60,000
-40,000
-20,000
0
20,000
Proj. B RAV = Hyperbolic