CONSTRUCTION AND USE OF MULTIDIMENSIONAL STUDENT MATHEMATICS ENGAGEMENT SCALE IN PREDICTING MATHEMATICS ACHIEVEMENT AMONG SENIOR SECONDARY SCHOOL STUDENTS IN EKITI STATE, NIGERIA

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A Thesis in the International Centre for Educational Evaluation, Institute of Education,
Submitted to the Institute of Education,
in partial fulfillment of the requirement for the degree of

DOCTOR OF PHILOSOPHY

of the

UNIVERSITY OF IBADAN

APRIL, 2019

ABSTRACT

A recurrent argument among educational psychologists globally is the number of dimensions that Student Mathematics Engagement Scale (SMES) should have. Some authors argued that it should be one dimension, while others posited two or three dimensions. Currently available SMES are either 1-dimensional or 2-dimensional and rarely more. With the introduction of more robust statistical methods, it is possible to have a multidimensional SMES; however this has not been fully explored as extant literature shows. This study was, therefore, designed to construct a reliable 6-dimensional SMES and to examine its predictive power on mathematics achievement among secondary school students in Ekiti State, Nigeria.

The study adopted a survey design. Three phases and three sets of samples were involved. Phase 1: Pilot testing of the initial pool of 100 items measuring SMES. Phase II: Calibration and selection of SMES items and Phase III: Usage of the SMES. All the three senatorial districts in Ekiti State were sampled, and four Local Government Areas (LGAs) were randomly selected from each senatorial district. For Phases 1 and III, three each of private and public Senior Secondary Schools (SSS) were randomly selected from each of the sampled LGAs. The sample sizes were 1008 and 1032 SSS2 students for Phases 1 and III respectively. For phase II, four each of private and public SSS were randomly selected from each of the sample LGAs. Sampling without replacement was adopted to avoid selection of same school twice. A total of 1600 SSS2 students participated in phase II. A 50-item Mathematics Achievement Test was constructed with a reliability index of 0.83 (KR-20 formula). Data were subjected to Exploratory Factor Analysis (EFA), Parallel Analysis (PA), Confirmatory Factor Analysis (CFA), Polytomous Graded Response Model (PGRM) and Multiple regression analysis.

Twenty-four factors, comprising 64 SMEI, were extracted through EFA. The 64 items from EFA were reduced to 45 items through PA and further reduced to 37 items through CFA. The 37 items were subjected to PGRM for item calibration and reduced to 35 items. Dimensionality analysis of CFA and PGRM showed that the 35 items loaded on six factors and denoted sub-scales. These factors were:Personal Agency Engagement, Positive Affective Engagement,Negative Affective Engagement, Positive Behavioural Engagement, Negative Behavioural Engagement and Cognitive Engagement. The reliability index of the6-dimensional SMES was 0.90, while the reliability index of each of the sub-scales of the SMES ranged from 0.68 to 0.87. Regression analysis of the sub-scales showed that only Negative Behavioural Engagement predicted students' achievement in Mathematics ($\beta = -0.12$, $t = -2.952$, $p<0.05$). This implies that students who exhibited negative behavioural engagement tend to perform poorly in Mathematics.

A robust 6-dimensional Students Mathematics Engagement scale was constructed, and its sub-scales were used to predict students' achievement in Mathematics. Mathematics teachers should be encouraged to use this scale for measuringsecondary school students' level of engagement in the subject.

Keywords: Student mathematics engagement scale, Dimensionality of scales, Prediction of mathematics achievement

Word count: 473

CERTIFICATION

I certify that this work was carried out by Mrs. J. Y. Adegbuyi in the International Centre for Educational Evaluation, Institute of Education, University of Ibadan, Ibadan.

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DEDICATION

This work is dedicated to the Almighty God, the author and the finisher of my faith, for his infinite mercies, provision, protection and guidance. To 'GOD' bethe glory. Also, to my husband, a doctor of pharmacology, Dr. A.T. Adegbuyi, and my children: Temitopeoluwa, Oluwatosin, Oluwalolade, Toluwani and Jesutofunmi.

ACKNOWLEDGEMENT

I give all glory to God who has been my help from the beginning of my existence. Honour, glory and adoration are unto his holy name. My God, the Alpha and Omega spared my life from the inception of this programme till today and made completion of this work a possibility. His mercies kept me throughout the journey. May his name be praised forever.

I am particularly indebted to my amiable, courageous and hardworking supervisor, Dr. B.A. Adegoke who supervised the work thoroughly,giving much support, encouragement, patience, perseverance, tolerance, and whose guidance has helped me in the organization and successful completion of this work. Thank you so much, sir. You really worked hard to make this work a success. The good God will reward you abundantly and you will attain greater height in your entire endeavours, sir.

I want to thank my Director, Professor Folajogun V. Falaye, for her support and encouragement; you are a mother indeed and also a woman of honour and integrity. God of heaven will continue to lift you higher, ma. I am very grateful, ma. I also wish to register my profound gratitude to our mothers in the Institute of Education, Prof. E. Adenike Emeke and Prof. E.A Okwilagwe. They exhibited motherly love and cared for me immensely during the course of this study. God of heaven will reward you abundantly, mas.

I sincerely appreciate my daddy, the head of ICEE, Institute of Education, Prof. J.G. Adewale, for the fatherly role he played during the course of this study. God of heaven will reward you and continue to be with you, sir. I am greatly indebted to all my lecturers in the Institute of Education, University of Ibadan. They are; Emeritus Prof. P.A.I Obanya, Prof. J.A. Adegbile, Prof. T W Yoloye, Prof. P.N. Okpala, Prof. C.O Onocha, Prof. Adams O.U Onuka, Dr. M.M. Osokoya, Dr. Dr. F. Ibode, Dr. J.A. Adeleke, Dr. S. F. Akorede, Dr. Monica M. Odinko, Dr. I Junaid, Dr. E. Babatunde, Dr. J.A. Abijo and Dr. Michael Metibemu. You are all blessing to all the students in the Institute of Education. God will continue to bless your memories in Jesus name.

I am grateful to all my pastors that kept praying for me while this programme lasted. They are; Pastor Amos Ifedayo of Deeper Christian life Ministry, Akure zone (a great and anointed man of God), Pastor and mummy Alebiosu, Pastor Oluwarotimi, Pastor Ogunleye, Pastor Bada, Pastor David Olajide,Pastor Amosun, Pastor Oyundoyin John, Pastor Moruwawon, Pastor Onipede, Pastor Olanlokun, and other brothers and sisters of Deeper Christian life

Ministry nationwide. God of heaven will continue to water your vineyard. I equally want to appreciate all my siblings for their support and prayers. They are; Mr Joseph Kolawole, Mrs Olaiya Mary, Mr Emmanuel Durojaye, Mrs Bolanle Funmilayo, Mrs Oluwole Folasade and Mr and Mrs Akerele. God will bless you all.

I also remember the roleplayed by Dr.Lasbrey Anochiwa and Miss Blessing in the editorial work of this thesis. God will bless you and your families. My sincere gratitude goes to Mr Dare Peter and Miss Durojaye Funmilola. Both of them worked tirelessly with me during the data collection and collation process. My heart prays for them; excellence is their portion. My appreciation goes to my mother in-law for her motherly support, Mrs Felicia Adegbuyi. Also, my mentors, daddy and mummy Afolabi, and my sister in-law, Mrs Salako Modupe are well acknowledged. God will be with you all.

My special appreciation goes to my husband, Dr. A.T Adegbuyi, a man of God, a pillar inmy life, a loving and caring husband. Thank you very much for your patience, endurance and prayers. Our source of joy will never run dry in Jesus name. Amen!To end it, my sincere appreciation goes to my children; Temitopeoluwa, Oluwalolade, Oluwatosin, Toluwani and more importantly, my little girl, Jesutofunmi, for their support, sacrifices and the understanding they showed in making this dream a reality. The God of heaven will continue to shine his glory upon you all, abide in your lives,and you will all succeed in life in Jesus name.

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INTRODUCTION

1.1 Background to the Problem

Mathematics is the science that describes the lucidity of shape, quantity, and arrangement. It is one of the core subjects that act as an axis on which the growth and technological progress of any countryare shaped. Expertise in Mathematics learning is vigorousto every individual's perception and innovativelife,because its knowledge is required for daily living. Science and technology are heavily dependent on Mathematics.In spite of the importance of Mathematics, the level of performance of students in senior school certificate examination is not encouraging enough. For example, the yearly discharge of senior secondary school certificate examination can attest to this fact. The results show the level of secondary school students' attainment in Mathematics. For instance, Table 1.1 displays the analysis of Nigeria students' performance in Mathematics from the year 2007 to 2016.

From the table, out of 1,249,028 candidates who sat for Mathematics examination in the year 2007, only 583,921, representing 46.75 % scored between A1 to C6 in the subject while 333,740 (26.72%) of the candidates scored between D7 and E8, and these were the only set of candidates that could use the result for admission into tertiary institutions, provided they had between A1 and C6 in other subjects as relevant to their proposed course of study. Also, the same year 2007, about 302,764 (24.24%) had F9. The last group (F9) had no opportunity of gaining admission into any tertiary institution to read any science course, because a minimum of E8 in Mathematics may be required for admission into some other Nigeria tertiary institutions like Colleges of Education or Polytechnics. Also, no significant improvement was recorded from 2010 to 2014 except in 2008, 2015 and 2016 when 52.27%, 57.02% and 70.23%, respectively, of the candidates made at least a credit pass in Mathematics. This can still be improved upon.

Table 1.1: Analysis of May/June of Senior Secondary School Certificate Examination (WAEC) Result in Mathematics between 2007 and 2016 in Nigeria.Source: Statistics Office,

Year	Total Sat	Credit (A1-C6)	Credit (%)	Pass (D7-E8)	Pass (%)	Fail (F9)	Fail (%)
2007	1,249,028	583,921	46.75	333,740	26.72	302,764	24.24
2008	1,292,890	726,398	52.27	302,266	23.83	218,618	17.23
2009	1,373,009	634,382	47.04	344,635	25.56	315,738	23.41
2010	1,306,535	548,065	41.95	363,920	26.85	355,382	27.20
2011	1,508,965	608,866	40.40	474,664	31.50	421,412	27.90
2012	1,550,224	723,024	46.64	445,224	28.72	380,425	24.54
2013	1,399,178	618,996	44.24	371,202	26.53	406,181	29.03
2014	1,547,140	621,950	40.20	427,342	30.53	451,301	29.17
2015	1,581,420	901845	57.02	425628	26.91	253947	16.06
2016	1,469,585	1032175	70.23	248676	19.37	188734	12.84

West African Examination Council, Lagos, Nigeria

Over the years, it has been observed that one of the learner's characteristics which has a high probability of affecting learning outcomes and achievement in Mathematics is students' engagement. This denotes students' vigorous obsession in learning activities (Reschly,Wylie & Christenson, 2012). It is a multiple dimensional concept comprisingfour discrete dimensions namely; behavioural, affective, cognitive and agentic dimension (Sinatra, Heddy and Lombardi 2015; Reeve and Tseng, 2011; Taps, 2016; Veiga, 2013). These four dimensions are inter-correlated. Behavioural engagement describes the level of students' involvement in teaching and learning activities in terms of devotion and effort (Reeve, 2013), students' level of performance during class, the way students participate in solving academic problems during class, students' abilities to complete homework, and students' response levels to teachers during class (Davis, Chang, Andrzejewski, and Poirier, 2010).Affective engagement refers to students' expressive responses in the classroom. These are; the presence of eagerness and curiosity or non-existence of annoyance, dryness, and worry (Reeve and Tseng, 2011). This dimension also involves students' negative or positive responses to schools, teachers, classmates, and the learning tasks, choice of flexible or tough tasks and whether or not learning is treasured (Davis, Chang, Andrzejewski, and Poirier, 2010).

Cognitive engagement describes students' level of investment in class activity, appreciation of the worth of learning and a readiness to go beyond the least requirements (Fredricks, McColskey, Meli, Mordica, Montrosse, and Mooney (2011). It also involves the level of students' persistence in solving academic problems, student's perceptions and beliefs about course materials,and the readiness to put on the energy needed tocomprehend difficult concepts andgrasp stimulatingtalents (Mahatmya, Lohman, Matjasko and Farb (2012). Agentic engagement or personal agency refers to students' deliberate, vigorous, and constructive influence to the drift of teaching they acquire through questioning, articulating of favourites, and students' demand for what they desire from the teacher (Reeve, 2013). In order to describe student agentic engagement, Reeve and Tseng (2011) presented some examples during the teaching and learning process. These examples are; Students suggesting relevant solution on how a particular problem can be solved in the classroom, recommendation of objective to follow by the student, students communicating their level of curiosity and asking for clarification on how problems can be solved in the class.

In learning Mathematics, engagement happens when students are systematically busy with the teacher inside the classroom, partake in solving problems, do the Mathematics, and grasp the opinion that having mathematical knowledge is meaningful and suitable, in the classroom and outside the classroom. Basically, the conception of engaging studentsisgrounded on the trust that their knowledge increases when they are snooping, fascinated, or encouraged. But their knowledge decrease when they are uninterested, calm, dissatisfied, or otherwise disengaged in learning activities (Attard, 2011). Students who are disengaged from Mathematics deny themselves the opportunity of studying any course that requires the basic concept of Mathematics in the higher institution. In addition, any student who refuses to learn Mathematics hinders himself orherself from gaining the ability to comprehend lifetime skills through a mathematical perception (Findlay, 2013).

Academically engaged students are dedicated, tenacious, and fascinated in their academic pursuit (Furrer and Skinner, 2009). Seeley (2004) noted that students'engagement brings about success and that commitment could have the utmost influence on fairness during Mathematics class. Seeley also noted that students' engagement makes students to study. To corroborateSeeley'sview, Akpan and Umobong (2013) carried out a research work onstudents' achievement, students' Inspiration and students' level of engagement during teaching and learning process in Nigeria. Their results showed thatstudents' success, students' performance, and students' achievement have a positive correlation with engagement. These researchers said that before a child can achieve successin his or her academics, he or she must affianced maximally in academic work, as academic commitment is essential for a child to be successful in the classroom.

However, despite the significance of engagement of students to their successin the classroom, the level of students' engagement in some of their subjectsis low, most especially in Mathematics. Conner and Pope (2013) explained that between 40% and 60% of students in secondary school remain persistently unengaged. They said that students fail to pay attention most of the times in the class. They also said that students do not do their classwork as well as their assignment given to them by their teachers. Furthermore, students often complain that classes are boring during lessons.Not only that, Dudley (2010) explained that the rate at which students are dropping out of school is as a result of their disengagement from their academic work. Dudley noted that this students' detachment from their academic work is a worldwide problem, which often resultin an increase in the dropout rates in many nations. However, in order to reduce the dropout rates, teachers and school administrators need to identify these set of disengaged students and occupy them in meaningful teaching and learning process, for high level performance in their classrooms. Flanigan and La Roche (2013) stressed that teachers should ensure that they engage students meaningfully;otherwise, their teaching would not be productive.

More so, Flanigan and La Roche (2013) suggested that teachers and educators should use any opportunity at their disposal to selectsome resourcesthat can engage students in critical thinking during lessons for them to solve the problem of disengaged students. Similarly, Silver and Perini (2010) affirmed that it will be impossible for students to understand what teachers are teaching them and be vigorous learners if teachers fail to present teaching that will engage students in critical thinking.Moreover, Conner and Pop (2013) observed the same feeling and advisededucationalists and teachers to be close to their students, and select necessary measure that will enable them deliver interestingteaching that can increase the level of students' engagement during their lesson. Basically, assessment of students' level of engagement is highly necessary in their subject areas, most especially in Mathematics. With valid and reliable student mathematics engagement questionnaire,researchers and teachers can investigate the feelings, views, and beliefswhichstudents have about Mathematics and how an adjustment can be made to improve students' level of comprehension during the teaching of Mathematics, among secondary school students in Nigeria.

However, for a teacher or researcherto use valid and reliable student mathematics engagement questionnaire, it is imperative he or she pays careful attention to the construction and validation of student mathematics engagement scale so as to ensure that its psychometric properties are good. The psychometric properties of student engagement construct are characterized by its good reliability and validity. Psychometric properties of student engagement construct are vital because the degree and superiority of engagement of students is a solid predictor of students' knowledge, attainment, and academic advancement (Ladd and Dinella, 2009; Jang, Kim, and Reeve, 2012).

Some student engagement scale developers have conceptualised a single engagement indicator into different dimensions due to the techniques used during the validation processes (Veiga, Reeve, Wentzel and Robu, 2014).For instance, Handelsman, Briggs, Sullivan, and Towler constructed students' engagement scale in 2005 titled, "Student Course Engagement Questionnaire (SCEQ)" with 23 items. During the validation process, these researchers decided to use Exploratory Factor Analysis (EFA) alone instead of using EFAand confirmatory Factor Analysis (CFA) with Parallel Analysis(PA) to decide the amount of factor to retain; and more importantly,Polytomous Graded Response Model (PGRM) of Items Response Theory framework for the final selection of the items. During the validation process, the result of exploratory factor analysis yielded two factors, namely: Engagement indicators and Engagement outcomes. The Engagement indicators were used to assess participation and emotionality, while Engagement outcomes were used to assess skills and performance. Moreover, it was discovered that certain discrepancies occurred during the conceptualisation of these engagement items (Veiga, et al. 2014). For instance, "participation" was seen by different scale developers as an item under the cognitive, behavioural and the agentic dimension ((Fredricks, McColskey, Meli, Mordica, Montrosse, and Mooney, 2011; Reeve, 2013).

Also, Student Engagement Questionnaire (SEQ) was constructed by Kember and Leung in 2009. This scale contains 27 items. However, during the cause of validating SEQ by these researchers, they made use of CFA without the use of EFA and parallel analysis as well as polytomous graded response model of IRT framework for the final selection of the items. Due to this shortcoming in the choice of method of analysis used by these researchers, they only succeeded in identifying students' engagement as behavioural engagement alone. Furthermore,Students engagement in school Four-dimensional Scale (SES-4DS)is a 20-item scale that was constructed and validated by Veiga, 2016 in Portugal. Veiga used both EFA and CFA with the criteria of Eigen value greater than one to establish the amountof latent variables, instead of using PA to decide the amount of latent variablesto keep as well as polytomous graded response modelof IRT framework for final segment of the items.

The result of CFA of SES-4DSidentified four dimensions: the cognitive, the affective, the agentic and the behavioural dimensions. Moreover, the four-factor model (the four-dimensions) was separately estimated and tested in Angolan males and females' data by Gutiérrez, Tomás, Chireac, Sanch and Romero (2016). These researchers reported that the fit statistics of this four-factor model was not significant on another samples of Angola students until after the removal of two items from affective dimension which further reduced the reliability of affective dimension from 0.82 to 0.60. Gutiérrez, Tomás, Chireac, Sanch and Romero now suggested further research on the dimensionality of student engagement construct as well as its reliability and validity.

These misperceptions in the number of dimensions in student engagement construct are not limited to those mentioned.Some educators and engagement philosophers view student engagement construct as one dimensional, while others consider it as two dimensional or three dimensional. For instance, (Veiga, Reeve, Wentzel and Robu (2014) and Veiga, (2013) gathered some items together termed multidimensional of student engagement. These researchers divided theseitems into behaviouraland cognitive engagement based on face validity alone. Not only that, in Nigeria, the few researchers that worked on students' engagement based their argument on 3-dimensional alone (Chika, 2012; Akpan and Umobong, 2013; Saliu, 2014). Out of these researchers, Saliu happened to be the only one who constructed a 36-item student academic engagement scale, with only three dimensions. The result of these dimensions is not unconnected with the procedure used during the validation process. In the cause of validation of this scale, Saliu used principal component analysis (PCA) instead of EFA and the Kaiser's rule for the resolution of amount of latent variables,rather than using parallel analysis to decide the amount of latent variables to keep as well as CFA to check out for the fitness of the model extracted from exploratory factor analysis, and polytomous graded response model of IRT framework for item calibration and final selection of the items.

Among psychometricians and mathematics' researchers, the dimensionality of student engagement construct has constituted a problem. Most of the instruments currently available to assess students' level of engagement, most especially in Mathematics in Nigeria and outside Nigeria, are validated by not very effective psychometric procedures. However, for researchers to assess the extent of students' involvement in Mathematics, so as to improve students' learning outcome and achievement, there is a need to develop and validate a multidimensional student mathematics engagement scale with sophisticated statistical tool, that is, Exploratory Factor Analysis(EFA), Parallel Analysis(PA), Confirmatory Factor Analysis(CFA), and Polytomous Graded Response Model(PGRM) of IRT frame work that will yield valid, reliable and robust multidimensional student mathematics engagement instrument in Nigeria.

The need for EFA, PA, CFA, and PGRM of IRT in this study is borne out of the fact that EFA is employed when a researcher is interested in developing a new scale, and when the researcher does not know the connection between an item and its corresponding factor (Kline, 2013). The EFA method explains how and to what degree the measure variables (items) are connected to their unobserved variables. In brief, EFA apprehends the clusters of measure variables (items) that are constantly walking together. In this wise, the constant arrangements of measure variables are recognized by the extraction and rotation of factors. In view of this, exploratory factor analysis is a device that helps researchers to toss a loop round groups of interrelated items, to differentiate among groups, and to isolate and remove irrelevant items (Jtneill, 2008).

In addition, CFAis a multivariate statistical techniquewhich is basically for hypothesis testing. The hypothesis deals with a latent pattern of factors behind the items. The CFA tested the extent to which a group of items represent the number of factors that have been generated through EFA (Statistics Solution, 2013). In CFA, the extracted factors must be identified by the researcher prior the analyses of data. Confirmatory factor analysis examined how well the measure of construct is reliable with the assessment of researchersconcerning the pattern of the latent variable. With this, part of the objectives of CFA is to verify how well the data fit the hypothesized model. This postulated model is grounded on a concept or prior logical study (Preedy and Watson, 2009).

Besides, CFA gives a more reasonable approach to a researcher during the evaluation of construct validity (Atkinson, 2010). With CFA, theories regarding the arrangement of factors in a set of data were clearly tested by the researcher due to having a determined model which specifies factors' number. After the confirmation of factor's number, the confirmatory approach try to maximally tie the measure variable and the hypothetical factor configurations for the confirmation of "goodness of fit" of the determined hypothesized model (Chequer, 2014). In Confirmatory Factor Analysis, "goodness of fit'' measures are employed to assess the degree to which the predicted latent variable by the researcher apprehended the correlation between all the items in the construct. An hypothesized construct will have good fit indices, if the control imposed by a researcher on the construct is consistent with the hypothesized measure, if not, the hypothesized construct will specify that the model is not fit,hence, will not be accepted. If the model did not have good model fit indices, this could be as a result of some items loading under two or more factors. The poor fit might also be that some items in a factor may also relate to each other than other items within that factor. In order to solve this problem, the overall model can be assessed to know what part of the model is wrong by inspecting the normalized residuals, to know which correlations are not well fit or the modification parameters indices that are fixed that need to be free.

Although, both the exploratory and confirmatory methods look out for considerably amount of differencein a set of items with a lesser amount of shared latent variables, EFA is mainly suitable for the development of a new scale when a researcher did not have any ideal of the patterns and quantity of common latent variables to keep (Kline, 2013). Therefore, one of the most serious decisions researchers should take when EFA statistical tools is used is the amount of latent variables to keep in mind. The choice concerning the amount of latent variables to keep in mind is essential for two reasons. First, the EFA requires that clarification should be made between mare items reduction and adequate representation of the relationship that exist within a group of items, since it is essential to depend on differentiating unimportant factors from important ones (Hayton, Allen, and Scarpello, 2004). Secondly, research has shown that overestimation and underestimation of the extracted factors resulted to wrong factor loading configuration and clarification (Velicer, Eaton, andFava, 2000).

In spite of the significance of decision regarding the amount of latent variables to consider and different researches that were carried out on the number of factors to keep during

EFA, no agreement has been reached concerning the right method to make use of. Different methods have been put in place to help these assessments, but none of them normally give the same result (Zientek and Thompson, 2007). However, Garrido, Abad and Ponsoda, 2012; Ruscio, and Roche, 2012; Henson and Roberts, 2006; Hayton et al., 2004 showed that parallel analysis happens to be accurate and a robust method which work better than the commonly used Kaiser's rule, screeplot test and maximum likelihood procedure (Timmerman and Lorenzo-Seva, 2011; Patil, McPherson, and Friesner, 2010; Henson and Roberts, 2006).

Courtney (2013) carried out an investigation on the best approach to use when deciding on the amount of latent variables to keep in EFA out of Parallel Analysis (PA), Scree test, Kaiser rule of eigenvalue greater than one, Minimum Average Partial correlation technique and Simple Configuration Measure using various settings; i.e number of participants, items' number, factors' number and items' number and their corresponding factor, and established that PA was dependable with real statistics used to decide the factors' numbers with 76.42% correctness, while Scree test tend to over factor. Courtney also established that Kaiser rule utterly overrate the factors' number and was only correct 8.77% of the times.Warne and Larsen (2014) collaborated this trend and reported that parallel analysis method perform better than traditional method of eigen value greater than one.

Velicer, Eaton and Fava (2000) examined the difference between Parallel Analysis (PA), Kaisar-Guttman criterion (KI) and Minimum Average Partial (MAP) test, and resolved that PA method gives the best resultnext toMAP, whereas the value of eigenvalue more than 1 method was exceptionally incorrect. However, for a researcher to construct reliable, valid and stable scale, the use of robust statistical tools is not limited to those mentioned only (that is, exploratory factor analysis, confirmatory factor analysis and parallel analysis). Polytomous graded response model of item response theory framework is another statistical technique and tool that is needed for item calibration and selection. Item response theory describes the correlation amonglatent variables that an instrument intends to measure, the properties of the measure variables in an instrument and test taker responseto different measure variables.

Item Response Theory (IRT) models are usuallyemployed during the analysis of data stemming from the responses of respondents in a scale that comprise variables with dichotomous or polytomous responses. Dichotomous responses arecommonly labelled as true or false, right or wrong, yes or no, whereas polytomous responsescorrespond to more than two options (Bacci, Bartolucci, and Gnaldi, 2012). With IRT model, the psychometric properties and the performances of scale(s)with dichotomously or polytomouslyresponses variables can be evaluated to remove irrelevant items, hence, producing accurate, usable, and moderately brief scale(s) that can be used in educational sectors (Edelen and Reeve, 2007).

However, the main problems encountered when evaluating the level of students' engagement in Mathematics is unavailability of multidimensional measure that possess a robustpsychometric properties (Kindermann, Furrer and Skinner, 2008; Wang, Eccles, and Willet 2011) for the purpose of measuring the level of students' engagement in Mathematics. The construct (Student engagement) has been conceptualised through 3-dimensional (affective, behavioural, and cognitive) in Nigeria (Chika, 2012; Saliu, 2014). Recently, Reeve and Tseng (2011) and Veiga, (2013) suggested that agentic or personal agency engagement could be added as an additional dimension (Sinatra, Heddy and Lombardi 2015; Taps, (2016).

There is a problem in the conceptualization of the number of dimensions of student mathematics engagement construct. Also, there are lapses in the technique used (that is, the use of the criteria of eigenvalue above 1 for the determination of factors' number (dimensions) instead of parallel analysis criteria, and the inability of some researchers to use confirmatory factor analysis. Finally, those engagement scale developers that did not use graded response model of IRT frame work for the final selection of student engagement items during the construction and validation of student mathematics engagement instrument that exist in the literature. So, there was need for improvement in the construction of valid and reliable multidimensional student mathematics engagement instrument with robust statistical tools,which can be made use of to investigate the level of students' engagement in Mathematics, for the purpose of improving students' learning outcome and achievement in Mathematics.

1.2 Statement of the Problem

Science and technology lean heavily on Mathematics. In spite of the importance of Mathematics, the level of performance of students in Senior School Certificate Examination is not too encouraging in Nigeria. One of the learner's characteristics that tend to have an obvious causal relationship with learning outcome and achievement in Mathematics is students' engagement. Yet, the dimensionality of this construct has constituted a problem. Some educators and engagement philosophers in other countries and in Nigeria view student engagement construct as one-dimension while others view it as two-dimension or three-dimension (cognitive, affective and behavioural).

However, research has shown recently that agentic or personal agency engagement could be an additional dimension, and understanding the pattern of factors that uphold student multidimensional engagement construct in Mathematics remains critically important. Though, most of the currently available instruments to measure this construct (student engagement), both in Nigeria and outside Nigeria, are validated without the use of effective psychometric procedures. More in-depth statistical methods, such as exploratory factor analysis (EFA), parallel analysis (PA), confirmatory factor analysis (CFA) and polytomous graded response model of IRT framework, are needed to produce a more robust student mathematics engagement scale in Nigeria. If such scale is constructed in Nigeria, the scale can be used to measure the level of students' engagement in Mathematics, so as to see how an adjustment can be made to improve the level of students' engagement in Mathematics for the purpose of improving learning outcome and achievement in Mathematics.

In view of these reasons, in this study, more robust statistical methods such as EFA, PA, CFA and polytomous graded response model of IRT framework, were adopted to construct valid and reliable six-dimensional student mathematics engagement instrument (that is, Personal Agency engagement, Positive affective engagement, Negative affective engagement, Positivebehavioural engagement, Negative behavioural engagement and cognitive engagement). Also, the relative influence of each of the dimensions of the developed student mathematics engagement scale on Mathematics achievement test was examined.

1.3 Research Questions

1a. How many items and factors are extracted from the initial draft of 100 items of studentsmathematics engagement scale?

1b. What are the appropriate numbers of factors to retain in students Mathematics Engagement Scale?

2a. Do the retained factors of students Mathematics Engagement scale show good model fit indices?

2b. Do the students Mathematics Engagement items show convergent validity?

3a. Are items of each of the dimensions of Students Mathematics Engagement scale unidimensional?

3b. To what extent are the Students Mathematics engagement items locally-independentof one another?

3c. How many items were selected as good items using Polytomous graded response model of IRT framework during calibration process.

4. What are the discriminant validity indices of the identified factors of studentmathematics engagement scale?

5. Is there any relationship between the identified factors of student Mathematics Engagement scale?

6a. Is the student mathematics engagement scale reliable?

6b. How reliable are each of the sub-scale of student mathematics engagement scale?

7. Which of the sub-scales of student mathematics engagement scale is the best predictor of mathematics achievement?

1.4 Scope of the study

This research work was limited to Senior Secondary School 2 in both public and privateSecondary schools which were selected from 12 LGAs out of the 16 LGAs in Ekiti-states, Nigeria. The study focused on the construction of a pool of 100-items of student mathematics engagement scale which were generated from three sources.The items pool of students' statements about their engagement in Mathematics which was collected by the researcher through an open-ended questionnaire from the representative sample of the target population.Statements of other senior secondary schools Mathematics teachers were collected through interviews.Statements of the researcher, based on her experience as a secondary school Mathematics teacher were also use.

1.5 Significance of the study

Secondary schoolteachers and school administratorscan use the validated scale forstudents' assessment.The validated scale wouldlikewiseassist the Mathematics teachers, school managers, parents, examining bodies and the governmentto investigate the attitudes, perceptions, and beliefs of students about Mathematics and how anadjustment can be made to improve the standard of Mathematics among secondary school students in Nigeria.The scale could also help to inspire teachers to ask more open-ended questions in Mathematics during class discussion, so that students' engagement in critical thinking could be improved.Also, the scale can strengthen the connection between teachers and studentsin Mathematics class.Furthermore, researchers, theministry of education officials and other stakeholders in educational measurement and evaluation who may be interested in measuring the level of students' engagement in Mathematics in secondary school can make use of the validated scale.The research work will also add to the array of literature on scale construction and validation in Nigeria.

1.6 Definition of Terms

1.6.1 Conceptual definitionof terms

Communality: This represents the amount of shared variance present in a test item. That is, the amount of variance that each variable has in common withother variables.

Uniqueness: This is the variance that is 'unique' to an item and not shared with other items. It is equal to 1 – communality.

Item calibration: This is amethod by which the parameters of large amounts of items can be assessed.

Item parameters: This refers to difficulty and discrimination indices of an item on a scale.

Affective engagement: This refers to students' negative or positive reactions to schools, teachers, and classmates,the learning tasks, choice of easy or hard tasks and whether or not learning is valued.

Behavioural engagement:This refers to students' level of performance during class; the way students participate in solving academic problems during class, students' abilities to complete homework, and students' response levels to teacher during class.

Cognitive engagement: Cognitive engagement describes student's level of investiment in class activity, appreciation of the worth of learning and a readiness to go beyond the least requirements

Agentic engagement or personal agency: This refers to students' deliberate, vigorous, and constructive influence to the drift of teaching they acquire through questioning, articulating favourites, and students demand for what they desire from the teacher.

Parallel Data: This is an artificial data set which contains the same number of variables with the same number of observation as the researcher's original data, but all variables included in this "parallel data" (i.e permutations of the original raw data set) are random variables.

Testlet: Sub-set of a test or a group of items that are measuring the same construct.

Instrumentation: This is the course of action (the process of developing, testing, and using a device.

Measured variable: This refers to the observed variables (Students Mathematics Engagement items).

1.6.2 Operational definition of terms

Multicollinearity: Multicollinearity is a statistical phenomenon in which multiple independent variables show high correlationbetween each other ($r = .9$ and above).

Singularity: Singularity occurs when variables are perfectly correlated in a data ($r = 1$)

Redundant variables: These are items which are highly correlated ($0.9 \geq r \geq 0.8$)

Irrelevant variables: These are items which have low loadings($r < 0.3$)

Unclear variables: These are items which don't load clearly on a single factor,that is, cross loading (for example, if item 1 load on both factor one and factor two).

CHAPTER TWO
REVIEW OF LITERATURE

2.1 Theoretical Background

Reeve Self-Determination Theory of Students' Engagement

Reeve happens to be the one who propounded Self-Determination Theory (SDT) in (2012). The theory stated that irrespective of students' race or nationality, gender, social context, economic status and age, they have inborn development trends, such as; inner inspiration, interest, and mental ability that afford them the opportunity to possess great level of engagement in the classroom and consequentlydevelop positive attitude to schooling (Reeve, Ryan and Deci, 2004; Ryan and Deci, 2002; Vansteenkiste et al., 2010). This theory on identification of students' internal motivational possessions made recommendations to teachers on how they can cherish and bolster these possessions among students in the classroom, so that their levels of engagement can be improved.(Niemiec and Ryan, 2009). The theory explains that engagement of studentsin school should center on the level of students' involvement during the learning process.

The stress on "learning process" is due to the fact that learning activity laid emphases on engagement as a mission. Learning activity occurs when students concentrate, pay attention, make effort and tenacious in learning (behavioural engagement), interest or distress (affective engagement), the use of refined instead of superficial approaches by students, that is, deliberate learning approaches, and active self-regulation (cognitive engagement). Student Engagement within SDTfocused only on these three dimensions (behavioural, affective and cognitive). For instance, Skinner (2009) noted that independent stimulus resulted to affective and behavioural engagement, while Vansteenkiste (2005) revealed that independent stimulus resulted to deeper learning instead of superficial learning (cognitive engagement). However, Reeve, 2012 explained that these components of students' engagement are incomplete because they only address teacher centered approach during the teaching and learning process.

For instance, a teacher may come to the class and give some Mathematics questions to students to solve, and students will react to the teacher's demand with certain level of cognitive, behavioural and affective engagement. In this scenario, students did not have the opportunity to contribute constructively to what the teacher was trying to pass across to them in the classroom. To give the students the opportunity to contribute positively to the teaching they receive from the teacher, Reeve, suggested the idea of agentic engagement or personal agency as the fourth dimension that was different but also highly correlated with the remaining three by means of EFA and CFA. In this wise, Reeve defined Agentic engagement as the students deliberate, vigorous, and productive involvement in the teaching they receive from the teacher in the classroom.

Reeve and Tseng, (2011) gives a classroom base example of agentically engaged student, like student's offered suggestion on how problems can be solve in the class, student asked for clarification, student ask for an example, etc. This Agentic or personal agency engagement can be assessed through a self-report questionnaire with the following items: I normally sit at the front for me to see the board clearly during Mathematics class, I ask my teacher questions for clarification during Mathematics class, I suggested to my teacher on how difficult questions can be solve during Mathematics class. So, in order to conceptualize students' engagement in the right perspectives, Reeve suggested adding agentic engagement by constructing valid and reliable four inter-correlated construct of student engagement instrument that will contain all the indicators of student engagement as explained in the students' engagement framework of STD.

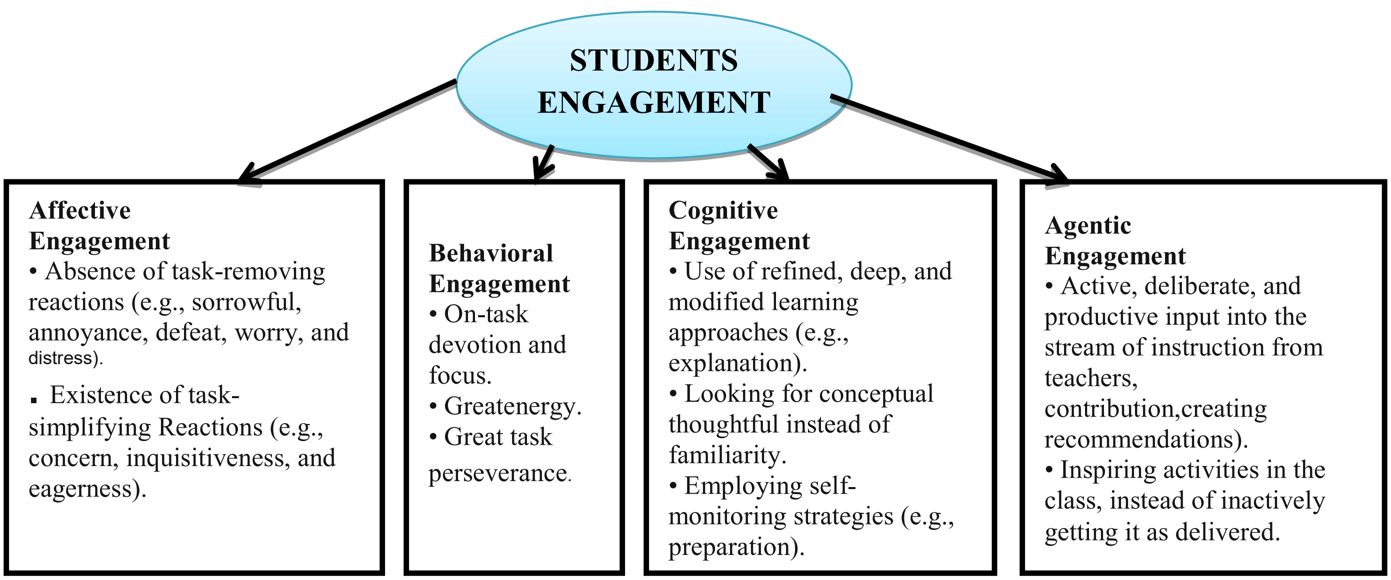

Figure 2.1: Self-determination Theory Perspective of Student Engagement.

2.2 Sequences for Instrument Development

Survey Instruments are found to be commonly used during the collection of data in research involving evaluation as well as education. Survey Instruments help researchers to collect facts on students' Engagement, students' attitudes, students' beliefs and students' behaviours to mention a few. Radhakrishna, Francisco, and Baggett, 2003 noted that out of 748 studies carried out by researchers in agricultural and extension education, only 64% out of the total studies used survey instrument. They also found that 31% and 33% of the researchers respectively did not ascertain the method used to carry out the validity and reliability of their instruments. However, it is highly essential to construct a valid and reliable instrument,so that measurement error can be avoided. Stanley (2011) gives the definition of measurement error as the extent to which a measured quantity varies from its real value.In order to construct a valid and reliable scale, several steps have to be put in place with substantial amount of time. Radhakrishna, (2014) explains that, researchers need to go through five steps during the construction and analysis of Survey Instruments that educators normally use for data collection. Therefore, the theoretical frame work that was used in this studyinvolved these five steps. Figure 2.2 describes the five steps.

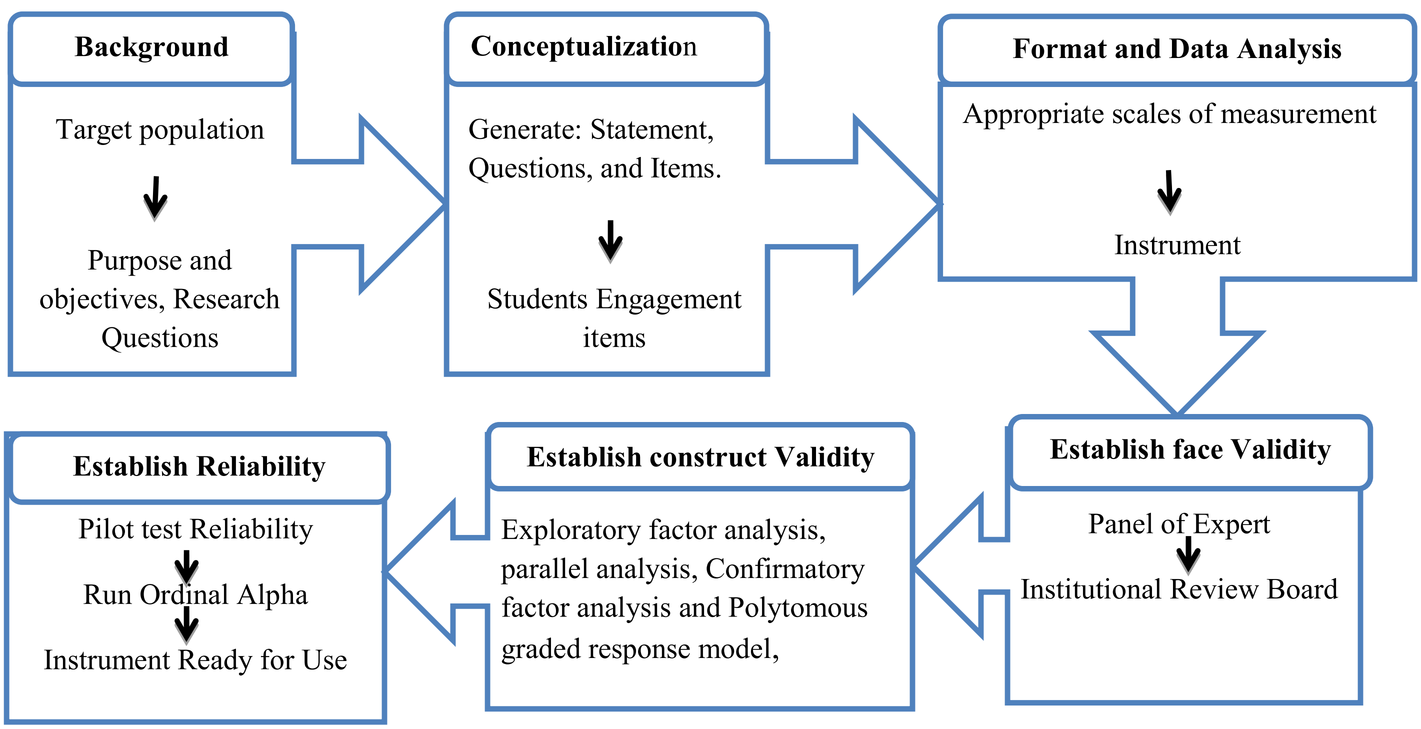

Figure 2.2: Orders of Scale Development (Designed by the researcher)

Stage One: Background

At the first stage, the main objectives and purpose of the studies are examined, the target population and research questions of the study must be determined. Also, the educational background of the participant especially their readability levels must be ensured. Plan on how the participant can be reached must be determined at this stage, the sampling procedure must be appropriate. An in-depth knowledge of the purpose of the study must be determined through literature search. Proper planning and knowledge of the first stage gives way to stage two.

Stage two: Instrument Conceptualization

With the adequate knowledge of the study, the next stage is to generate items for the students' mathematics engagement instrument. At this stage, contents (the statement of the students with the same characteristics as the main sample, the statements of other mathematics teachers and researcher's experience) are changed into items/questions. Additionally, there must be a link between the research purpose and what the item statements are addressing. For instance, what the instrument is measuring must be specified by the researcher.

Stage three: Measurement Format

At this stage, the generated items/questions are written down and the appropriate scale of measurement is chosen. Questions will be arranged and front size will be determined. At this stage, appropriate data analysis must be carried out; for instance, if factor analysis is the chosen method of data analysis, the data generated from the target population must be measured on an ordinal scale with likert type response mode (e.g. strongly agree to strongly disagree).

Stage four: Establishing Face Validity

After stage one to three, what is next is ensuring the appropriateness of the drafted questions by establishing face validity. Validity can be defined as the degree to which an item assesses what is expected to assess, and this can be carried out by expert reviewed. The kind of validity to use can be determined by the purpose of the research. At this stage, some questions are addressed.

i. Does the question measure what it proposed to measure?

ii. Did the test signify the content?

iii. Does the test suitable for the participants?

iv. Can the content of the test have the ability to extract all that is needed to carry out the study?

The ability of experts to address these questions will enhance the instrument validity. After the expert review, the next step is to collect the letter of permission from the department/faculty. After the approval from the department, expert opinion will be taken into consideration and changes will be made when necessary. After that the items will be pilot testing for EFA.

Stage five: Establishing Construct Validity

At this stage, the items will be validated through EFA to show the arrangement of the relationship among the item and their corresponding factor. The next is the PA to decide the amount of factor to keep, while CFA is meant to assess how fit is the retained factors. Also, the polytomous graded response model is for further confirmation of the numbers of the retained items.

Stage six: Establishing Reliability

At this stage, the reliability of all the items in the final scale will be determined. Miller, 2015 define reliability of a scale to mean the degree to which a scale gives the similar result over and over again. Different types of reliability measure are available to carry out the reliability of a scale. For instance, the appropriate measure of reliability of items with ordinal data is ordinal alpha coefficient. (Radhakrishna, 2014).

2.3 Concept of Engagement

Differentdescriptions and debates on engagement presented by researchers reveal that the term denotes different meaning. Kuh, Cruce, Shoup, and Kinzie (2008) explain the concept of engagement as “the vigor and period students devote in academically focused events”, these areconcentration, attempting assignment, and displayingexcitement for school activitythrough asking questions, contribution tocluster and individual learning activities and helping peers. Also, Trowler (2010) refers to engagement of studentsas the communication among the period, energy and other important possessions devoted by students and their schools with the aim of optimizing the students’ experience, enhancing their understanding and also developing students’ performance. Nako (2015) also views students' engagement as a multidimensional variable which includes: behaviours, emotions, and psychological orientation that are affected by the gratification of the elementarymental need of kinship which affects student'sattention in learning, indelight in challenges, and perseverance in accomplishments of goals.

Gutiérrez et al. (2016) defined student engagement as the involvement of students in educationalattainment. They viewed it as an integrative construct, or macro construct which consist of many dimensions. According to them, most recurrent work identifies Cognitive, behavioural, and affective dimension and in recent times, a fourth dimension called personal agency, which echoes students' productive input into the stream of the teaching they obtain (Reeve and Tseng, 2011; Sinatra, Heddy and Lombardi 2015; Veiga, 2016; Taps, 2016). Furthermore, Krause and Coates (2008) also defined students' engagement in term of the worth of energy a student devoted to academically determined events that give a desire result. In addition to the above definitions, Chen, Gonyea and Kuh (2008) defined student engagement as the level of participation of students in their academic work. They said that student engagement positively influence students' performance as well as student gratification and determination.

Abbort (2014) in its own submission characterised student engagement as the amount of care, inquisitiveness, attention, confidence, and desire that students should showed in order to have a deep knowledge of what they have been taught in the classroom as well as their proper school functioning. Highly engaged students participated maximally in the classroom and school activities, they are vigorously engaged in their schooling, do the essential in school for their good performance, while the somehow engaged students are indifferent, inattentive and lazy. Engagement occurs when students invest in learning. They do their best to learn what the school has to offer. They are interested not only in scoring good marks, but in having the proper knowledge of what they are been thought in the class and integrating or adopting it as ways of life. Horstmanshof and Zimitat (2007) established that increase in the level of students' engagement can be influenced by the visions they have toward their studies, which will eventually lead to the possibility that they would continue with their educations for longer periods. Taken this into consideration, engagement can be liken to students' perseverance in schools. They also affirm that the kind, occurrence and level of students' engagement have been shown to have influenced their knowledge and perseverance and also developtheir educational capability" (Reason, Terenzini and Domingto, 2006).

The main view in the study of engagement; is the ability, participation, and energy that students put into learning,which influenced their educational attainment**.** Nako, 2015 noted that academically engaged students are energetic, supportive, participate in inspiring educational events, originate class discussion with the teacher, participating in inspiring scholastic skills, and having interest in educational resources which in turn influenced their educational attainment.

Willms et al. (2012) explained that the students' level of involvement in class activities can be an indicator of engagement in their learning. They are engaged when they are involved in class discussions, attentive during instruction, and affianced in productive and cooperative group work among peers. In the light of this level of students' involvement, Willms et al. claim that students' involvement is one of the major requirements of schooling, for instance, attendance in the classes and doing assignments can be indicators of engagement.

Also, Dunleavy et al. (2012) in their own view said that attendance, effort and completing assignment are the three indicators of student engagement that can be linked togood grades in Mathematics. Engagement of students in school can be seen as a psychological and social aspect of student learning and progress. Different researches have been carried out in the pass to assist this construct (Kuh, 2008; Coates, 2007). Research indicates that disengagement of students in the school can resulted to students in ability to complete their education (Hupfeld, 2010). They may display unruly habit which can lead to their low performance. Sharma et al. (2013) carried outresearch on the engagement of students among Indian B-schools and concluded that pupils are found to be more activein term of engagement when they are reasonably engaged relatively to commitment and enthusiasm. Student engagement is beneficial to both scholars and experts. Experts would like to investigate it often and often and use it to develop scholastic performance. But unfortunately, some of the available instruments used to measure this construct (students' Engagement) in Nigeria and outside Nigeria examined 1-dimension, 2-dimension or 3-dimension of students' engagement alone, and majority of these instruments lack good psychometric procedure. This was due to the inability of the engagement developers to use appropriate statistical tools with the right technique during validation process.

As one example, the Student Engagement Questionnaire (SEQ) was develop and validated by Coates, 2010, the scale includes vigorous knowledge, students and staff communications, inspiring scholastic skills, educational task, convenience learning atmosphere, profession inclination and effort assimilated knowledge. However, Veiga, et al. 2014 confirmed that this survey instrument show low sign of validity, most especially, the national survey of student engagement.Another instrument was constructed by Wang, Willet, and Eccles, (2011). It is called school Engagement measure. This scale contains 23 indicators for the assessment of behaviour (example, I pay attention to my teacher in the class), affective (I love to be in mathematics class), cognitive (I plan on my course work ahead of my teacher for me to pass). This instrument contains 5 response modes that measures students' engagement in school. The scale contains three aspects, they are: behavioural commitment which includes: devotion and conformism in the class), affective commitment (which includes sense of appreciation and interest in schooling), and cognitive commitment (which includes self-controlled and usage of intellectual approaches). But this scale focuses on 3-dimensions alone.

Another scale named Motivation and Engagement Scale (MES) was constructed and validated by Martin, (2009). This scale contains 11 others sub-scales, of which some measure items of engagement while others measure items of students' inspiration which include: self-assurance, attention, perseverance, preparation, learning organization, apathy, self-disruption, nervousness, disappointment prevention, and tentative mechanism. The instrument also measures behavioural (perseverance), affective (apathy, worry), and cognitive engagement (preparation, learning control). Every sub-scale contains 4 indicators. For example, I don't have interest in school. However, the information about the dimensionality of these sub-scales is limited.

Also, Tufeanu (2013) used student engagement in school 4-dimension scale to measure the connection between educational underattainment students and their academic engagement on a different sample of 254 grades 9^{th} and 10^{th} Romanian's students to verify how valid the scale is using EFA, the Romanian version of student engagement in school 4-dimension scale showed that the underline latent variable of the scale explained 55.29% of the shared variance among items. Also, the result showed that the scores of underperformance students considerably lesser than high-performance students in their mental ability, behavioural, and their engagement in the class. In this work, suggestions were made for further research regarding the validity of the 4-dimensional scale (Veiga, 2014)

However, some of the literatures reviewed on students' engagement recommend the necessity for additional verification on the number of dimension students' engagement scale should have with the evidence of validity and reliability. In some analyses that involved the construction and validation of student engagement, researchers assembleditems together to construct a scale builtonly on the researcher's view of which of the items are measuring the same construct. (Fredricks, et al. 2011; Veiga, 2016; and Gutiérrez, et al. 2016).However, no indication of any empirical study that shows that these items are related and are also measuring the same construct without further evidence of validity and evidence of a scale's internal consistency (Veiga, 2014). In view of this, and given the multidimensional character of student engagement construct, there is need for in-depth verification of the number of dimension students engagement construct should have with their respective items for the assurance of validity and reliability.

2.4 Constituents of Engagement

2.4.1 Affective Engagement

Affective engagement refers to students' expressive responses in the classroom. These are: the presence of eagerness and curiosity or non-existence of annoyance, dryness, and worry (Reeve and Tseng, 2011). This dimension also involves students' negative or positive responses to schools, teachers, classmates, and the learning tasks, choice of flexible or tough tasks and whether or not learning is treasured (Davis, Chang, Andrzejewski, and Poirier, 2010**).** Affective engagement can also be seen as attitudes and feelings as exhibit by students (Saliu, 2014). Blumenfeld et al., (2005) see affective engagement as the degree of confident and undesirable responses to tutors, peers and the learning task.Constructive affective engagement is assumed to form learner links to school which in turn inspire learners' readiness to learn. Affective engagement addresses emotional reactions of student to learning task, for example, students feeling happy in the classroomnervous, articulating curiosity and pleasure, showing amusing and enthusiasm, feeling secure, devising helpful interactions with classmates and teachers, devising personal sustenance for education, articulating the spirits of acceptance, and treasuring schooling (Reeve and Tseng, 2014; Fredricks et al.,2011).

The following are the items in Affective engagement:

I pretended to be working during Mathematics class.

I appreciate acquiring fresh knowledge during Mathematics lesson.

I feel unhappy any time my teacher introduce new topic in Mathematics class.

Knowledge of Mathematics is important to me.

I find Mathematics fun and exciting

I am no longer have interest in Mathematics.

I get in trouble during Mathematics class

2.4.2 Behavioural Engagement

Behavioural engagement can be defined as the level of students' involvement in learning activity in terms of devotion and effort (Reeve, 2013), students' level of performance during class, the way students participate in solving academic problems during class, students' abilities to complete homework, and students' response levels to teacher during class (Davis, Chang, Andrzejewski, and Poirier, 2010). Also Behavioural engagement focus on the time students spend on homework, their attendance in the class, attention during class work, readiness for classwork, involvement in class activity, attentiveness, obeying class rules and regulation and threat behaviours (like absenteeism), ((Reeve,

(2013). Items in Behavioral engagement are:

I often come to Mathematics class unprepared.

I put more energy in my Mathematics classwork.

I made a presentation during Mathematics class.

I listen to my Mathematics teacher during lesson.

I do my Mathematics assignment late or not at all.

2.4.3 Cognitive Engagement

Cognitive engagement denotes student's level of venture into learning, appreciation of the worth of learning and a readiness to go beyond the least requirements (Fredricks, McColskey, Meli, Mordica, Montrosse, and Mooney (2011). It also, involves the level of students' persistence in solving academic problems. Student's perceptions and beliefs about course materials and readiness to apply the effort needed tounderstand difficult concepts and master challenging skills (Mahatmya, Lohman, Matjasko and Farb (2012). Features of cognitive policy can be liken to the requests concerning the practice of superficial or profound approaches to learning, recall, and comprehend what the school has to offer. This approach asks student question on the benefit education and prospect. The items include the following:

I work to go ahead of my teacher in Mathematics class.

Before Mathematics test or examination, I work hard for me to pass.

I normally plan ahead on how I can solve questions on new topics in Mathematics..
The exams I do in Mathematics are able to assess my ability in Mathematics
Doing Mathematics, gives me the hope of brighter future.

2.4.4 Agentic Engagement

Agentic engagement or personal agency refers to students' deliberate, vigorous, and constructive influence to the drift of teaching they acquire through questioning, articulating favorites, and students demanding for what they desire from the teacher (Reeve, 2013). In other to describe students' agentic engagement, Reeve and Tseng (2011) presented some examples during the teaching and learning process. These examples are: Students suggesting relevant solution on how a particular problem can be solved in the classroom, recommendation of objective to follow by the student, students communicating their level of curiosity and asking for clarification on how problems can be solved in the class. The items include the following:
During Mathematics class, I request an answer to difficult problems for me to learn.
During Mathematics class, I cheer my views and ideas.
I suggested to my teacher on how difficult questions can be solved during Mathematics class.
I ask my teacher to allow me to sit at the front for me to see the board clearly in Mathematics classroom.
I ask my teacher to allow me to do the correction of the assignment given to us for others to learn.

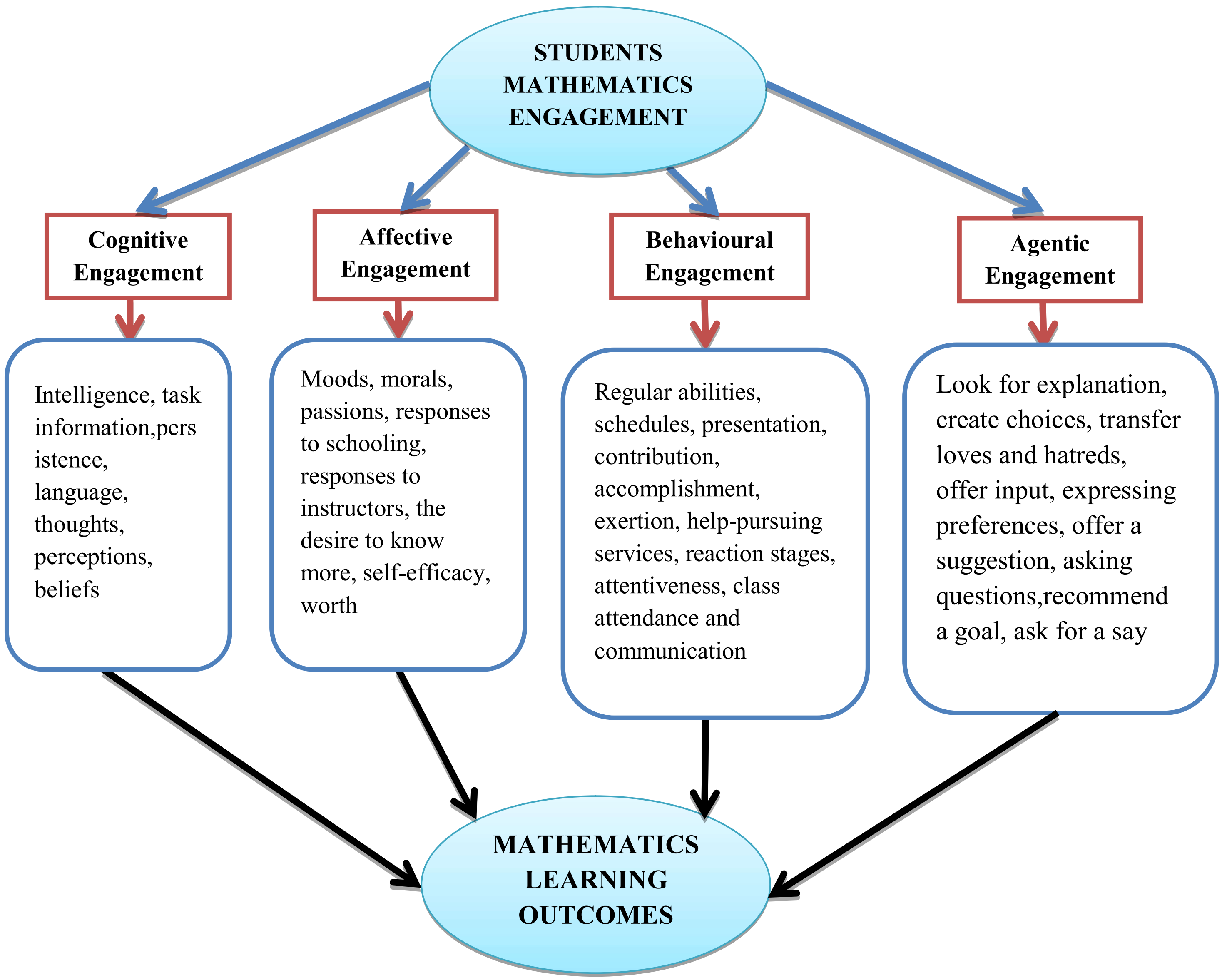

Figure 2.3: A conceptual framework of Student MathematicsEngagement

Student Mathematics Engagements Model (SMEM) is planned to offer a completetacticon how engagement occur among senior secondary school students. The model comprised of four components: The Cognitive, Affective, Behavioural and Agentics Engagement. Cognitive Engagement in the model, tells us about students' perception about learning Mathematics, their belief about Mathematics, what they think about Mathematics, the information available to them on how mathematics problems can be solved, their persistence in solving Mathematics problems, the level of their intelligence when dealing with Mathematics problems and how well they are able to link ideas in order to solve Mathematics problems. Behavioural engagement in the model, tells us about students' level of involvement in learning Mathematics, their effort, Natural skills, actions, performance and participation during Mathematics class. Completion of mathematics assignment, seeking help from teachers and peer during and outside Mathematics class, students' response levels, their attentiveness in the class, their class attendance and how they communicate with the teacher during Mathematics class.

Affective engagement in the model involves students' emotional reactions in Mathematics classroom, such as student's Feelings about Mathematics, how they value Mathematics, their emotions during Mathematics class, their reactions to Mathematics tasks, reactions to teachers and peers during Mathematics class, their desire to know more and self-efficacy. Agentic engagement in the model denotes pupils' deliberate, active, and productive input to the stream of training they get during Mathematics class. For example, students' asking questions during Mathematics class, expressing preferences, Look for explanation, create choices, transfer loves and hatreds, offer input, offer a suggestion, recommend a goal, and ask for a say during Mathematics class. The arrows show that all the four components resulted to learning outcome in Mathematics.

2.5 Empirical studies on students Engagement

2.5.1 Student Engagement for meaningful learning

The focus of Student engagement in school is to promote students' learning. Schlechty (2011) noted thatthe objective of engagement should focus on how meaningful learning canoccur sincestudent reluctant and partialobediencecan result tolowknowledge,also, lack of cooperation and insurgencewill definitely yield to students' failure. In the classroom, teacher should try as much as possible to understand the dimension of students' engagement in order to modify their set of courses and teaching that willcater for the needs of students for their high quality performance. Also, Silver and Perini (2010) stated that when students engaged deeply in their academic work in their classes they learn better. Silver and Perini said that when teacher engage students in academic work during lesson they tend to have high quality attainment. Furthermore, these researchers noted that when teachers introduceadditionalattractiveteachingsin their classroom, they witness lesser or no behavioural difficulties among the students. Not only that, research has also established that when students engage with the teacher during lesson, they areattentive and focus. They are also motivated to involvein advanced level ofcritical thinking which will eventuallystimulatestheir meaningful learning capabilities.

Teachers who embrace a student-centered method ofteachingintensifychances for student engagement that will help every student to effectivelyattain the course learning objectives. Unfortunately, despite the important of students' engagement to their academic performance, the levels of students' engagement in some of their subject's area are low, most especially in Mathematics. Conner and Pope (2013) noted that between 40% and 60% of studentsin secondary schools remain persistently not engaged; These students are not attentive, they applysmallenergy, and they are not thoroughas they remain indifference to both their class work and home work.Dudley (2010) noted that, any students with this type of indifferent attitude to their academic often turn out to be a drop out in many of the nation. However, in order to reduce the dropout rates, teachers and school administrators need to engage students with this type of habit in meaningful teaching and learning process for their high level performance in the classroom.

Moreover, to reduce the dropout rate, La Roche and Flanigan (2013) submitted that teachers and educators should try all their possible best to use different teaching aids with more enriching lesson that can engaged students maximally in their class, and hence tackle the problems of students' disengagement during lesson. Not only that, Silver and Perini (2010) stated that when teacher failed to build enriching lessons that can increase the level of students' engagement in learning, such teacher cannot expect those students to learn and understand what the teacher is teaching them. Conner and Pope (2013) also have the same notion, so, they suggested that educationalists and teachers should relate with their students in a meaningful way and also select appropriate tools that can arose the interest of students in learning and there by solve the problems of students' disengagement in the classroom. However, before teachers can solve the problems of disengagement in school, there is need for teachers to assess students' level of engagement in their subject areas, most especially in Mathematics with valid and reliable students' engagement questionnaire. If teachers and educationalist can have the knowledge of the level of student engagement in their subject areas, such knowledge will be able to help them to review and reproduce school curriculum that will cater for students' needs in the classroom.

2.5.2 Students Engagement and Learning Outcomes

Research has established that student higher level of engagement in academics work is aninfluentialpedal to increaseknowledge and guide instruction (Friesen, 2009). Highly academically engaged student seek to inspire and form the skill that can help them to achieve high quality work. Students are assessed base on their ability and skill by their teacher. The teacher gives them descriptive feedback that shows what good work looks like, and how they can improve their academic work and also definevalues for variousstages of performance.

This level of student performance and quality of students' work shows how much material a set of students have learned. That is their learning outcome. There are behaviours that students exhibit that can also influence their learning outcome, such behaviours are; student's participation in class activities, attendance in class, on-time delivery of assignments e.t.c. Parents frequently share the opinion that obedienceto these standards of behaviour will imparttasksthat is vital in once life.

Elsewhere, researchers have also documented that societal, school and academic engagement jointlygiverise to significantdevelopingeffectsonteenagestudents(Dunleavy, Milton and Crawford 2010). Engaging school, for instance, offers learners the right direction to follow by developingthem withnoble work and individualaccountability(Dunleavy, Willms, Friesen and Milton, 2012). However, for student to be focus and be accountable, thoughtful teachers should be put in place in the classrooms to help them acquire these skills, as well as motivate them during classwork. A review of the literature suggests that being interested and engaged do help students achieve higher levels of understanding in Mathematics. Many studies reviewed that if students involved substantively and behaviourally, acquiring skills and achievement gains follow (Akey, 2006). Brown (2009) argued that students' emotional and behavioural engagement in school and class activities is of critical importance. Wiliam (2011) summarized that "maximumengagement in the classroom appears to have a substantialinfluence on students'attainment". Cooper (2011) stated that "engaged students learn, disengaged students don't". Brown (2009) claimed that when students involved substantively they perform better. Also, Finn and Zimmer (2012) reported that when students engaged with the teacher in the class and in the school activities they have positive attainment results.

2.5.3 Students Engagement and Mathematics Achievement

Educationalistfrequentlyemphases on motivational modelslike engagement, self-efficacy, issues that bother on the value of training in colleges like technique of instruction and the classroom environmentand their effects on achievement (Zabihi, Newsha, and Mansouri, (2012), in Education, the connection between the engagements of students and their educational attainment as relate to Mathematics is of critical important. Basically, educational commitment gives rise to additional strength and involvement in educational events which eventually upgrade the learning outcomes. Zabihi, Newsha and Mansouri (2012), explained that engagement comprises of three constituents: the behavioural, cognitive and affective and recently agentic engagement (Reeve and Tseng (2011). Behavioural engagement denotes a range of noticeable behaviour in relations to commitment to assignment and of sustaining student's ability in relation to school activity, and since these set of students look for assistance from peers in order to attend to their assignment and also participate in class activities, they tend to have high achievement (Finn, and Rock, in Blank, (2016). Furthermore, Scholars submit that behavioural engagement contributes positively to educational attainment of students most especially in Mathematics (Dupeyrat and Marian, (2005).

Abolmaali, et al. (2014) refers to cognitive engagement as a diversity of information dispensation tactics that students practice to acquire knowledge. They said that the connection between knowledge and academic success of students is facilitated by cognitive engagement. The unit of cognitive engagement has been explored to be a moderator item that has influence on learning tactics. The results of Wolters, (2004) showed that when prosperous secondary school students involve in doing Mathematics assignment, they exploit more cognitive approaches. The affective engagement comprises of distinct items (inner items that can be linked to personality) and social trait (like noble interactions, classroom environment and domestic backing). These items can also influence student educational attainment (Larocque, 2008, Sungur and Gungoren, 2009). More recently, agentic engagement, like behavioural, emotional, and cognitive engagement also influence students achievement positively, so an agentically engaged student is active, vibrant and constructive, which in turn lead to high achievement (Reeve, 2013). Consequently, examining the connection between engagement and attainment can demonstrate a significant part in understanding the growth of students' institution career and in predicting whether or not they manage to effectively finish their schooling in a given environment.

Willms et al., 2012; Hume, 2011; Dunleavy et al., (2012) explain that while providing an engaging and supportive learning environment, learning professionals need to guide the learners to be accountable for their personal knowledge and also to be intrinsically engaged for them to reach their academic prospective. However, students who do not believe they have the ability to learn Mathematics can be taught skills that can help them to have good result through their commitment and effort. Students who combine commitment with effort tend to become self-regulated learners and develop meta-cognitive skills to optimize their acquisition of knowledge (Schneider and Stern, 2010) and hence have a high level of achievement in Mathematics.

2.5.4 Students Engagement and Mathematics

The subject of student engagement in mathematics is a continuousissue of debate and worryin and outside the classroom and the school, hitherto, how much care is given to the engagement of instructors? Attard (2016) believes that one of the openingnecessities for engaging students in mathematics is aninstructor who is passionate, well-informed, self-confident, and zealous about mathematics training, that is, a teacher who is engaged in mathematics. Study has established that the majorstimulus on engagement of student in mathematics is the teacher and the instructional interactions and practices that are established and applied in day to day teachings (Attard, 2013).

Brown (2009)carried out a studyon theviews, attitudes,and the beliefs that students have in relation to their engagement in Mathematics. The participants were 11th and 12th grade secondary school students selected from one of the local community in the southeast of United States. The result of the analysis showed that engagement of students can be experimental. Also, their opinion and practices can be examined. Their engagement designs can be discovered. Educators and researchers can make suggestion on how secondary school students can have high level of engagement during Mathematics class. The researcher found out that, teachers is expected to know that; concentration, contribution, and help-seeking assistances contributed to lively commitment during class. Therefore, there is a need to conduct training that will educate teachers of secondary school during Mathematics class on how to create enabling environments that will encourage concentration and participation of students.

Also, the researcher found out that, it is out of place to discourse about student engagement if the student is deprived of contributing and involving in the conversation. The result also showed that individual learner has an exclusive perception and their personal ways of reacting to issue inside and outside of classroom which in turn disturbs the level of their engagement. Therefore Since the level of student engagement influences their achievement in Mathematics at our Secondary schools (Akey, 2006), further investigation of the construct is essential during Mathematics class so that such instrument can be recommended for use during Mathematics class.

Furthermore, Kong, Wong, and Lam, (2003) developed a scale title Student Engagement in Mathematics Classroom Scale (SEMCS), The instrument was constructed to assess lack of engagement among primary five students. The scale contains 57 items with five response mode that ranges fromtotal disagreement (1) to total agreement (5). The items in the scale assessed three sub division of engagement with ten items each that addressed engagement in Mathematics. Examples of such items are: affective engagement (curiosity, achievement bearings, worry, defeat); and behaviouralengagement (devotion, energy, time consumed) cognitive engagement (surfacetactic, profoundtactic, belief).

These researchers discovered that, those students that were engaged were found to be hard-working,attentive, and eager to devotemore time for mathematics learning both within and outside the classroom. They also found out that engaged students were eager to obey the teacher's directives, they used substantial time in problems solving, and theyspent reasonable time to learning mathematics. While disengaged students desist from studying mathematics or refuse to learn. They noted that the way and manner of students' engagement are different from each other which will eventually result to different learning outcomes. For example, student who do mere memorization of the course materials, hardworking, and use to following the teacher directives which can be termed as shallow learning approaches can have instanteffects (like getting high mark in the test) but these can also lead to anxiety and frustration (Kong, Wong, and Lam).

Also, Singh et al. (2016) used four 5th-grade pupils from a bond school who have been diagnosed by their paediatricians (children's doctor) to haveanyof the mainlydistracted or collective subtype of Attention Shortage/Hyperactivity Disorder (ADHD). Themajordifficulty associated with pupils with ADHD isinattention, and this adverselydisturbs their livelyparticipation in schoolevents and may result to behaviouralproblems in the classroom (for example, putting on apathybehaviour during classwork, out-of-seat, disturbing class, being unruly). Students who exhibit these behaviourstend to have low educationalattainment when likened to their mates without ADHD (Kamphaus and Frick,1996). In this study, each of the students was provided with secular meditation training as well as Samatha meditation training separately, and the teachermodified the orders and rehearsal to the wants of everypupil in relations toverbal and manifestation. The exercise was immediately followed by mathematics lessons, and the pupils were trained to carry onby their regulardeliberationtraining and to customize their ruminationabilities to carry their attentions back to their trainingsin the classroom. The result of the analysis plainlyrevealed a statistically significant growth in all the four pupils'vigorous engagement in mathematics nextto training in Samatha rumination. The result also revealed a statistically significant rise in the percentage of mathematics difficultiesresolved by all the four pupils.

2.6Measure of Mathematics Engagement scale

2.6.1 Reliability

Steadiness in theresponse of the respondents to an instrument on different occasions is the reliability of that instrument. So, the extent to which a scale is consistent in measuring student behaviour regardless of who is administering it is the reliability of the scale. Miller, (2015) distinct reliability as the level to which a survey instrument, question, opinion or any measurement methodyieldssimilaroutcomes on recurring trials. Miller said further that, reliability is seen as the stability in the scores of two or more rater that are carrying out observation at the same time.Also, Chapman, (2003) defined reliability as the degree to which an instrument is repeatable withsimilaroutcomes. Chapman affirms that ‘’an instrument may be reliable and not valid’’. However, if an instrument is valid, then it is also reliable and if it is not reliable, then it cannot be valid (Callans, 2012). Chapman explained further that one of the ways to demonstrate reliability is to display stability by reiteratingthe assessment with similaroutcomes.

2.6.1.1 Test of Stability

Reliability estimations are used to assess (1) the stability of items given at different periods to the same persons (test–retest reliability) or (2) the equivalence of groups of variables fromsimilarassessment (internal consistency) (Kimberlin and Winterstein 2008). Stability of an instrument can be assessed by giving a question to the same person at two occassion and establish the relationshipbetween the two groups of marks (Miller, 2015). So, test-retest technique is to ensurethe stability of a scale for a longer periods. (Martyn, 2009).Basically, thereliability of an instrument can be assessed through four major ways; these areinternal consistency, test-retest (stability), inter-rater andequivalent or parallelform reliability (Bolarinwa, 2015).

2.6.1.2 Test-retest reliability (stability)

Test-retest relationshipoffers asuggestion of steadiness over a certain period of time (Wong, Ong and Kuek 2012; Deniz and Alsaffar, 2013; Pedisic, Bennie, Timperio, Crawford, Dunstan and Bauman 2014; Wells, 2015). Test-reteststability orreliability arises when similar marks are gottenfrom same group of respondents at different time of testing(Wong, Ong, and Kuek, 2012; Deniz and Alsaffar, 2013; Liang, Laua, Huang, Maddison and Baranowski 2014; Erdvik, Øverby and Haugen 2015). Say differently, the scores are stable at one time or the other. Stability is verified by means of test-retest method that comprisesof giving the same test or questionnaire to an individualover and overunder the same situationsfor certain periodsof time (Bolariwa, 2015). Test-retest reliability occurs when a test is given to a group of people at one time, and also give it again on the same group of people at another time, and then checkthe test-retest connection between the two sets of scores obtain at the two different occasion. If the test is reliable, the scores of studentsat two different testing should be related. That is, one would presumethat the correlation between the first and second testing to be positive.

But unfortunately, most of the time, this method is not practicable since the response of the respondentmight be pretentious by recurrentassessment (Venkitachalam, 2015). For instance,respondentscould be familiar with the examination and therebyget a greater marksome other time in the examination (Martyn, 2009). This scenario is refers to as carry-over effect. Therefore, cautiousapplicationof test-retest method is powerfullysuggested (Yu, 2005). Also, William, (2006) says that the length of time is a critical issue in test-retest approach. William says that opinion of people may change due to intervening factors. For instance, theinfluence of parent,teacher and even pairs may change the response mode of the testee.

Williams explains further that the correlation between a testadministers to a testee at two different timesdepend on in part by how much time elapses between the two measurements. He says that the lengthier the time gap, the lesser the correlation and also the smaller the time gap, the greater the correlation.It happens this way because the longer the time between the administrations of the tests the more influence the factors that contribute to anerror we have on the reliability of the test. Also, Venkitachalam (2015) explains that the stability of a test given at two or more times cannot be ascertained if the testee has learned before the administration of the second, third or more times, so in order to verified the stability and consistency of a measurement instrument, researchers should undergo series of validation stages to establish validity and reliability of a test or construct (Wright, 2014).

2.6.1.3 Parallel Forms Reliability

Another step a researchershould take to avoid the effect of respondents knowledge of the test administer to them at two different occasion is to administer two different set of tests that are measuring the same construct to the same group of respondent at different time. Thereafter, the parallel forms reliability of the scores obtained from the two parallel tests will becalculated. In this wise, one will expect that the score obtained from these two set of score tests should have a positive correlation if the two tests are parallel.

2.6.1.4 Internal Consistency

A scale is said to have internal consistencyif theresponses of respondents across the items in the scale are constant.In fact, all the items present in that scale should measure a single construct, so respondents'marks on the items in the scale should be related. Like other test of reliability, consistency can only be measured by analyzing the data collected. One methodof analysis is split-half, whichrequired the division ofscores of items in a scale into two equal parts (scores of odd and even-numbered items).After the division, each half of the test scores will be computed and the correlation between the two scores will be inspected. Consider this scenario: test takers were requested to state their level of engagement in Mathematics survey questionnaire. One question is: "I feel very sad during Mathematics class." Another statement is: "I enjoy staying in Mathematics class." Respondents thatdisagree with the second declaration should agree with the firstdeclaration and in the revise case. If the answer of respondents to the two declarations is low or high amongstparticipants, then, the responses of participant to questions in the scale are said to be unreliable.

2.6.1.5 Interrater Reliability

Interrater reliability is the degree to which diverseviewers are steady in their findings. For instance, if you want to assess the level of engagement of secondary school students in Mathematics, you can video them as they are engagedin Mathematics during Mathematics class. Then two or more viewerscan watch the videos and the level of engagement of each student can be evaluated and rated accordingly. In this case, the ratings of different ratersshould beinterrelated with + 0.80 as the minimum value of internal consistency.Thoughreliability is necessary butit is not enough to establish validity. So, for an instrument to be useful, it must both be reliable and valid, and once a test is valid then it is also reliable (Callans, 2012).

2.6.1.6 Ordinal alpha coefficient and Cronbach's coefficient alpha as a measure of reliability

The calculation of alpha coefficient comprises the matrix of relationshipsbetween all available items in a scale. Examples of such coefficient alpha are Ordinal alpha coefficient and Cronbach's alpha coefficient. Basically, one can say that Cronbach's alpha is theoreticallyequal to Ordinal alpha because the computation of both Ordinal alpha coefficient and Cronbach's alpha coefficient are based on correlations or covariances matrix. However, the two are differ from each other in that Cronbach's alpha is based on Pearson covariance matrix while ordinal alpha is centered on polychoric correlation matrix (Zumbo, (2012).Theoretically people belief that the usage of Pearson correlation matrix comes to stay when dealing with continuous data, and if this hypothesis is sullied, the Pearson correlation matrix can be fundamentally misleading (Flora and Curran, 2004). In many of the social sciences research, Likert-type response mode are commonly use with ordered response categories, for example, four categories format that ranges from 'totally agree' to 'totally disagree'.). In this wise, the data collected from a questionnaire with ordered response mode are ordinal, not continuous; however, often time, some researchers' often classified them as continuous data that have been measure on an interval scale.

Moreover, Gadermann, Guhn and Zumbo (2012) noted that the use of Cronbach's alpha coefficient in any research that involved ordinal data (Likert type) and Nominal measurement tend to underestimate the reliability of the construct , as Cronbach's alpha coefficient does not relate to the internal consistency or unidimensionality of the construct (Sijtsma, 2009). Gadermann, Guhn and Zumbo said further that the calculation of ordinal alpha coefficient should be used for the reliability of any data with ordered response categories during factor analysis or coefficient ordinal theta should be used during principal component analysis.

2.6.2 Validity

One of the criteria that are essential to having a good engagement indicator or selecting an instrument for use is in the area of validity of the instrument.Validity of an instrument is ascertaining ifthe instrument assesswhat it meant to assess. In years back, validity of a measure has been well-definedto refer to the suitability, relevance, and utility of the preciseconclusionsmade by researchers through data collection (Theresa, 2006). The researcher said that it is possible to have ahighly reliable instrument that is useless. For an instrument to be useful, the reliability and validity of the instrument must be ascertained. Four major ways of determining validity are generally recognized: criterion-related (predictive and concurrent) validity,construct (discriminant and convergent) validity, content validity and face validity (Marsh et al. 2013).

2.6.2.1 Construct validity

Construct validity represents the degree to which aquality or trait is assessed by a questionnaire. Construct validity explains the extent to which questionnaire or test assesses a theoretical construct. Alumran et al. (2014) see it as a means of identifying the nature of latent variables amongst a set of items. For instance, the construct validity of an engagement instrument is confirmed by verifyingwhether the questions or variablescomprise a component variable (trait) do load meaningfullyunto theirrespective factor.

Thus, factor analysis is a way of defining construct validity. Since factor analysis depends exclusively on statistics verification, it is categorized as a numericalmethod of creating the validity of an engagement instrument. Agreeing to Garson (2010), such a statistical method is value-free and also has theability to regulateunimportantitems. Factor analysis, in fact, allows you to see patterns (factors) in the construct and each of them can be studied. It gives useful detailed information. Furthermore, poorly worded questions can be easily detected and rectified during analysis. There is also room for empirical criterion keying in the construction. So, for researcher to assess the reliability and validity of a construct, researchers should take cognizant of consistent moving together of measured variables.

Consequently, to recognizethe presence of such variables, exploratory factor analysis(EFA)should be utilized. In EFA the constant movements of measured variables are recognized through Promax with oblique rotation and usage Principal axis factoring (PAF) as method of extraction to explore the relationships among these measured variables and their latent variables (factors). The numbers of latent constructs to keep from EFA will now be suggested by Parallel analysis.Following the EFA, and decision regarding the usage of Parallel analysis for the numbers of factors to keep, confirmatory factor analysis will be preparedtoenquire the construct validity of the retained Latent variables. In other to display the goodness of fit of the retained Latent variables (factors), it is necessary to go through confirmatory methods (Cokluk, Sekcercioglu and Buyukozturk, 2010).

During this method of analysis, several model fit indices and their criteria will be assessed to test the goodness-of-fit of the latent construct.These indices comprise: χ^2 and its successive ratio with degrees of freedom (χ^2/df); comparative fit index (CFI) goodness-of-fit index (GFI);root mean square error of approximation (RMSEA) and adjusted goodness-of-fit index (AGFI); The chi-square valueis verified to evaluate the fit among the hypothesized model and the regularmeasured variables.Chi-square statistics explainsthe similarity of the detected and estimated matrices. The model fit is acceptedif the chi-square probability is equal to or greater than 0.05. However, a statistically significant chi-square test will suggest that the model did not fit the data (Suhr, 2006). Adding to chi-square measurements, there are other statistical fit indiceslike goodness-of-fit index (GFI) which explainthe fitness of a set of experimental data.The GFI demonstrates the extent to whichthe hypothesized model explained the relationship and similarity among items in a data.GFI value ranges from 0 to 1. If the value is 1, itspecifiesthat thefit is perfect (Suhr, 2006).

Comparative fit index (CFI) relates to null model'sfit (that is, when latent variables are unconnected) with the researcher model's fit (Babyak and Green (2010). When the value of CFIis more than 0.90, itindicates asuitable fit to a given data (Tabachnick B, Fidell (2007). Another quantitative value that explains the level to which a model fits anexperimental data is Root mean square error of approximation (RMSEA). For RMSEA to indicate a good fit, the value must be less than 0.05.After the confirmation and proof of goodness-of-fit of the retained factors, the next step is to evaluate the construct validity that is, the Convergent and discriminant validityof the latent variable.Agreeing to the norm of convergent validity, scores of hypotheticallyrelatedconstructs must be significantly inter-correlated. Kimberlin and Winterstein (2008) note that, for a researcher to institute convergent validity, the relevant connectionsamong the measured variables and their latent construct must be meaningfully differ from zero and adequatelybig.

Also, conferring to the standard of discriminant validity, scores of hypotheticallydiverse but interrelatedlatent variablesmust notrelateexceedinglyamong themselves. Tothis conclusion, the inter-factor relationships will be inspected as well as the degree of simple configuration for justification. Also, anotherimportant way by which the discriminant validity of a construct can be determined is to calculate the average variance extracted (AVE) for each construct. Average variance extracted is the variance that is unique to a construct and not shares with other construct in a model. McAdam et al. (2010) suggested that for a researcher to establishthe discriminant validity of a construct, the AVEof that construct (within construct variance) must belarger than the squared correlation of that construct (variance between that construct and another).Squaredcorrelation is the square of standardized regression weight. Also, squared correlation can be defined as a unique contribution of independent variable to the prediction of the dependent variable when the variance explained by all other variables in a model is controlled for (Pallant, 2010).In summary, Convergent and discriminant validity of any construct depend on the assessmentsof the constraint data. For example, if thecorrelation between two factors is greater or equal to 0.80 it specify lack of good discriminant validity index (Brown, 2006). Also, high loadings factor that do not cross-loadspecify good convergent validity. Basically, factor loadings less than 0.40 are not strong and factor loadings greater or equal to 0.60 are recommended foraccepted convergent validity (Garson, 2010).

2.6.2.2 Criterion-related validity

Criterion-related validity refers to the extent to which a set of items candetermine an outcome due to the availableevidence from other items (known as criteria). This canbe attained if a set of items from a questionnairecan be related to a behaviouralstandardapproved by educationalist. The best approach to this bothered on thedegree to which a mark from the assessment of somebody'snature can foretellhis or her future behaviour or performance.

Two types of Criterion-related validity are available, they are; predictive or concurrent validity. The term "concurrent validity" can be defined as the extent to which the score obtain from two different instruments that are measuring the same constructwhich are given to respondents around the same period relatestogether. In Standardized Educational & Psychological Tests, ‘we ask’, ‘do the usage of the testgives the same value as other measure the construct in the same way that others have measured it?’ While Predictive validity is defined as the degree to which the value obtain from a questionnaire or test can be used to determine the expected value from other related instruments.That is, does the questionnaire or test scores relate to the scores of other form of achievement tests? So, to establishCriterion-related validity of a scale, the connectionamongst the sub-scale of student Mathematics engagement and achievement test in Mathematics should be established.

2.6.2.3 Content Validity and Face validity

Content validity is the extent to which a questionnaire or test symbolizes all features of a given model. That is, do the Mathematics academics engagement questions cover all possible facets of the model?For example, face validity did not require any analysis but we only depend on expert’s consultation for the evaluation of relevance and noticeable view of the context and a multidisciplinary grouptactic to item collation and item examination. On the other hand,the ability of a test on its face value to measure what it supposes to measure. In a real sense, face validity does not refer to what is actually being measured rather what it appears to verify. So, to determinethe face validity of a given items, such items should be verify by certain number of experts to ascertain that the items represent the modelfor which it is meant for. (i.e. Mathematics academic engagement items)

What it takes to construct a good test

2.7Factor Analysis

Factor analysis (FA) is a statistical techniqueemployed to findthe connectionsbetweentwo or more items. This permitsvarious inter-correlated items to be reduced into smaller numberof sizes, called factors. In this work, items represent the amount of contractwithin a number of words about behavioural, emotional,cognitiveand agentic tendencies towards Mathematics. Through inter-correlation of these items, clusters of related engagement components form factor. Tabachnick, and Fidell (2007) definedFA as a statistical methodthatisolatea group of related items. Factor analysis belongs to a group of methods for identification of related items. The objective of FA is to isolate a clusterof items that have things in common.This cluster of inter-correlated items isterm factor.

Also, Tavakol et al (2011) describe factor analysis as one of theinfluential statistical method for scrutinizing the relationship between items and their corresponding latent variable.In this wise, scholarsexamined the inter-correlated differencesbetween a groups of measured for the purpose of gathering information from their unobservedconstruct.Henson and Roberts (2006) explained that FAmoderatesa huge set of facts,like survey information, to describecorrelatedresults in form of a lesser number of underlying factors. According to them, FAassiststhe removalof redundant items (highly correlated items), unclear items (cross-loading items) and irrelevant items (low loadings items).

Purposes of factor analysis

Factor analysis performs two major functions, one: to lessen the quantity of items, and also to identifythe pattern of relationship among items.

Conceptual models of Factor analysis

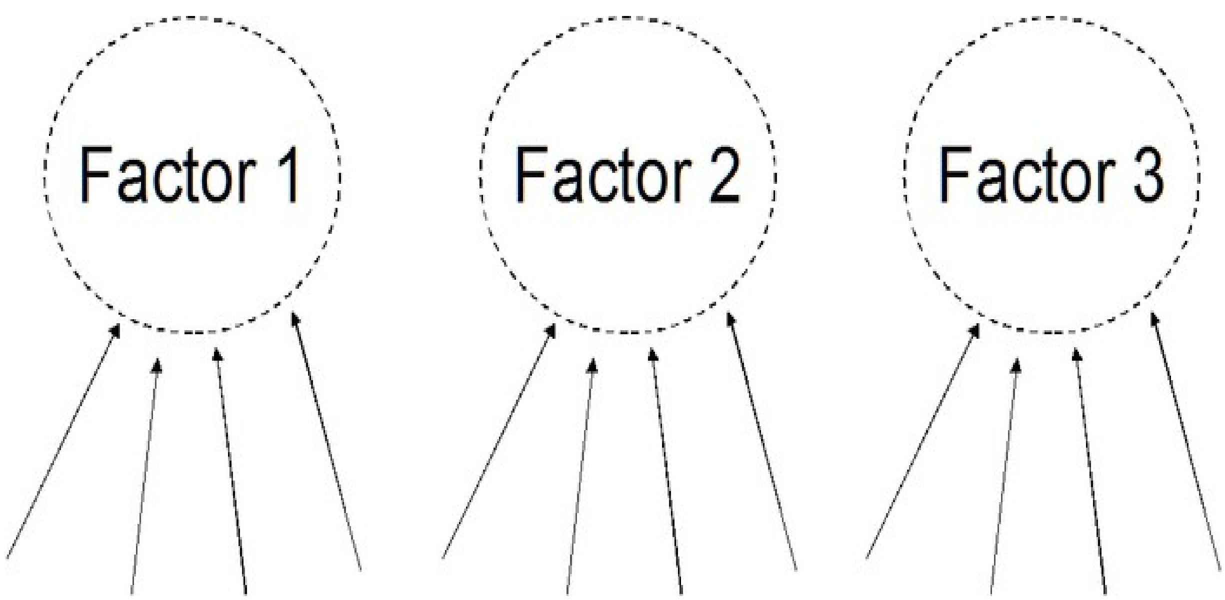

Figure 2.4: Hierarchic (Ledesma and Valero-Mora, 2007)

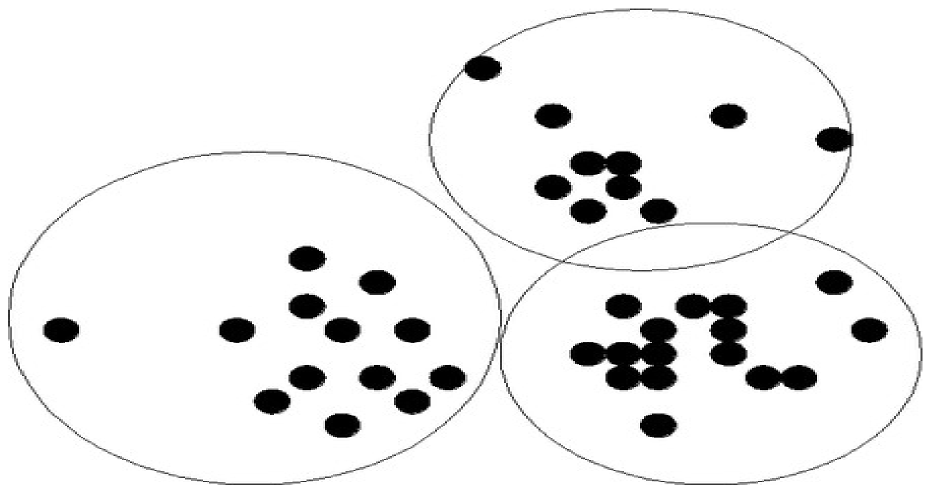

Figure 2.5:Cluster(Ledesma and Valero-Mora, 2007)
A simple factor analytic model e.g., 39 items testing might actually tap only 3 underlying factors.

During the process of factor analysis the inter-correlations betweenvariables are identified to form a group of items. However, exploratory factor analysis is a numerical method thatassists aninvestigatorto ascertainthe relationship between clusters of interrelatedvariables, to differentiateamonggroups, and to detect and removeunrelated or unclearvariables(Jtneill, 2008).

2.7.1 Types of factor analysis

Two types of factor analysis exist, they are: Exploratory and Confirmatory Factor Analysis.

2.7.1.1 Exploratory factor analysis (EFA)

Exploratory factor analysis (EFA) is a method of analysis employed to change a bigamount of items to a lesseramount of items called factors or components.In this case, these groups of items have a common goal. In EFA the relationshipbetweenclusters of items are recognized and changed to smalleramount of interrelated factors. In short, EFA arrestsclusters of items which are constantlywalking together. In this development, the constantarrangementsof items are recognizedby means of factor removal and factor revolution. Therefore, EFA is a valuabledevice for exploring the relationshipsbetweenitems and a trivialamount of fundamentalelements (Noor,Naziruddin and Ilham, 2016).

Exploratory Factor Analysis is used when a researcher is interested in developing a new scale and the researcher did not know thepattern of structure that exist between the measure and underlying variables, the EFA methodexplain the extent to which the items are correlatedwith their underlying constructs(Kline, 2013). The amount of underlying construct that were extracted from a group of variables through this approach is referred to as model. For instance, if a research has produced threeunderling constructs, the model denotes 3-factor model.For Confirmatory Factor Analysis (CFA), it is not so, CFA is employed when a researcher have pre-knowledge understandingof the nature of latent variables that have been generated through EFA. CFA seek to approvethe number of latent variables that have been generated through EFA. The EFA method is a data-determinedmethodthrough which a model is generated while CFA is a model determinedmethod where a model is confirmed(Tavakol et al., 2011).

2.7.1.2 Confirmatory Factor Analysis

Confirmatory factor analysis(CFA) is basically hypothesis testing. The hypothesis deals with a latent pattern of factors behind the observed variables. It tells the extent to which the measured variables signify the amount of factorsthat have been generated through Exploratory Factor Analysis (EFA) (Statistics Solutions, 2013). In CFA, researchers need to know the amount of factorsbefore confirming the factor. CFA verify whether the information gather by the researcher through EFA is true or not. So, one of the goals of CFA is to verify whether the factors that have been generated through EFA have a good model fit indices. (Preedy and Watson 2009).

Adding to this, CFAsuggestsa more reasonable approach to researchers for theassessment of construct validity (Stapleton, 1997). This approach gives researchersthe opportunity to test for the assumptionsregarding the pattern of factorsdue to having pre knowledge of the number of factor to keep. CFA approaches, after knowing the number of factors tend to maximally tie the measure variables to their corresponding factors to ascertain the goodness of fit of the retained factors.(Stapleton). In usingCFA, researchersneed to develop a theoryconcerning the number of factors representing the observe variables. (For example "personal agency" can be a factor representing contribution, preference, ask question) and the researcher canenforcecontrols on the construct due to its previous theory. For illustration, if a researcher discovered that there are two factors responsible for the interaction that exist in a scale, but the two underlying factors are uncorrelated, the researcher can now force the connection between the two factors to be zero by creating a model.

With this constrain the assessment of the Model fit indices might be found to verify how fit the anticipated model apprehended the relationshipamong all the variables in the model. If the model fit is poor, it implies that the controlsenforced by the researcher on the sample data in the model are unreliable and the model will not be accepted. If there is apoor fit in the model, this may be as a result of some variablesthat are quantifyingmany factors.It might also be that some variables within a factor are more interrelated to each other.In other to solve this problem, the overall model can be diagnosed to known what part of the model is wrong by inspecting the normalized residuals to known which correlations are not well fit or the modification parameters indices that are fixed that need to be relaxed (Jöreskog, 1996).

2.7.1.3 Differences between Exploratory Factor Analysis and Confirmatory

Basically, EFA and CFA aremeantfor the explanationofvariability that the observe items shared with their corresponding factor or latent construct. But in spite of this common attribute, both of them are still different from each other in that,EFA is used to classify latent constructon the basis ofinformation collected and also capitalize on the extent of the clarified variance (Suhr, 2006).In EFA, researcher does not need any definitetheoriesto know the possible number of factor that will be extracted with their corresponding items during EFA**.** However, if suchtheories exist, EFA do not make use of these theories during analysis and this doesnot disturb the outcomes of the analysis.

In contrary to EFA, CFA assessesthe already stipulated number of factors , and ismainlydetermined by theory. In CFA,researcher is expected to know the amount of factor to keep beforethe confirmation of the retained factors. Researcher need to know whether the extracted and retained factors are interrelated,and also to know the items that load on each of the factor (Thompson, 2004).InEFA, the loading of items to their corresponding factor should be allowed to vary while CFA constraints the loadings of items to their factors to zero.

2.7.1.4Confirmatory factor analysis (CFA) and structural equation modeling (SEM)

Examples of SEMsoftware that are usually use for the analysis of CFA are LISREL, AMOS, EQS,Mplus, Starta and laavan package in R.CFA is usually use as the first step in the assessment of the suggested measurement model in SEM. CFA like otherSEM have some guiding rule for interpreting model fit and model modification during assessment.However, CFA is different from other SEMin that,under CFA, arrows are not direct among factors. That is, latent variables in CFA do not have direct influence on one another.In SEM specification are usually made for specificlatent variable and itemsthat will contribute to hypothesized model. Also, CFA in SEM usually refer to as “measurement model”whereasrelationshipsamongfactors that have direct arrowsis termed “structural model”.

General Purpose and procedure of confirmatory factor analysis (CFA)

Defining individual construct: Theinitial stageis todescribethe latent variables hypothetically. At this stage, the hypothesized model variables will be pilot tested to assess the hypothesized model items. To assess the model items, confirmatory assessmentwill be carried out on the hypothesized model with the use of CFA.Develop the generalhypothesized model concept: In the analysis involving CFA, effort should be made to ensure the principle ofunidimensionality between the latent variable and their corresponding items.Also, for a good research, a minimum of four factors with at least three variables on each factor must be extracted.

Design anoptimal research that willyielda realistic outcome: In this case, the hypothesized construct need to be indicated. For example, the estimated value of an item loading should be from only one construct, there shouldn’t be any cross loading. Assess the validity of hypothesized construct: Validity of the hypothesized construct is assessed through the contrastbetween the hypothetical model and the real model for theverification of fit indices of the measurement variables. To ascertain how validthe hypothesized construct is, the quantity of variablesassists the researcher. For illustration, the loading of variables to their corresponding factor orconstruct needto be morethan0.7. Some of model fits statistics that help researcher in ensuring the validity of a model or construct are: Normed Fit Index (NFI), root mean square residual (RMR), goodness of fit index (GFI), RootMean Square Error of Approximation(RMSEA) and test of Chi-square and soon.

2.7.1.5Model fit assessment

Considerable number ofstatistical procedures simply requiresa single statistical assessmentfor the determinationof significance level in some analyses. But, forconfirmatory factor analysis (CFA), determinations of fit indices of a given data are ensured through numerous statistical assessments (Suhr, 2006). However, when a hypothesized model fits its measurement data, it indicates an acceptable model (Schermelleh-Engel, Moosbrugger and Müller 2003). Also, after the identification of good model fit, researcher should take the following into consideration during the interpretation of the output of CFA; (i) the hypothesized model (ii) the alterationsensured(iii) identification of items that load under each factor (iv) relationshipamong factors(iv) and someadditional relevantmateriallikethe introduction of control or not (Jackson, Gillaspy and Purc-Stephenson 2009). Moreover, in reporting statistics model fit, researcher should avoid reporting only the estimation of statistics that have the best model fit. Kline (2010) acclaimsthat the following fit statistics should be reported, they are: comparative fit index (CFI), Absolute fit indices, test of Chi-square,standardized root mean square residual (SRMR) and root mean square error of approximation (RMSEA).

Comparative fit index (CFI)

The analyses of fit statistics of comparative fit index is carried out through the examination of the differences between the data and the measurement model and alsoregulatingtheproblems of sample size which was built in through the test ofchi-squareof fitstatistics (Gatignon, 2010).The range of value of comparative fit indexis 0 to 1, the more the value the better the fit. The acceptable model fit for CFI is 0.90 to 1(Hu and Bentler, 1999).

Absolute fit indices

Absolute fit indices (AFI)regulatethe extent to which hypothesized model fits the measurement data (McDonald and Ho, 2002). Some examples of absolute fit indices are: Chi-Square test, root mean square residual(RMR), standardized root mean square residual (SRMR)Root mean square error of approximation (RMSEA), Goodness of fit index (GFI), Adjusted goodness of fit index (AGFI) (Hooper, Coughlan, and Mullen, 2008).

Chi-square test

Chi-squareassessesthe assumption that the hypothesized model is reliable with the design of co-variation amongst the measured variables. For chi-square statistics, the number that is close to zero specify an enhancedfit indices; this imply that, the lesserthe difference between predictable and experiential covariance matrices the better the fit (Gatignon, 2010).The statistic of chi-square is been affected by the number sample, which make it unclear in some of the analyses whether the significance level of chi-square test is as a result of poor fit to the model or to the sample size (Gatignon, 2010). This doubt has introducedthe use of additional statistics to evaluategeneral model fit statistics (Stevens, 2012).

Standardized root mean square residual and root mean square residual and

The standardized root mean square residual (SRMR) and root mean square residual (RMR) show the square root of the differenceamong the sample correlation matrix and the constructcorrelation matrix (Hooper, Coughlan and Mullen 2008). The RMR may be somewhat difficult to interpret (Kline, 2010). However, the standardised root mean square residual eliminates this problem of interpretation, and the value is from 0 to 1, and a value of .08 or less shows asuitable model fit (Hu, and Bentler 1999).

Root mean square error of approximation

The root mean square error of approximation (RMSEA) deals with the problems of sample size through the analyses of the differencesamong the measurement model, by maximallyselectedthe limitestimations, and the population correlation matrix (Hooper, Coughlan, and Mullen, 2008). The value of RMSEA ranges from 0 to 1, and a lesser value signifying good fit indices. A value of .06 or lesserindicatesacceptable model fit (Hu and entler, 1999).

Goodness of fit index and adjusted goodness of fit index

Goodness of fit index (GFI) representsthe amount of fittingthat exist among the measurement model and the experimentalcorrelation matrix. The degree of fittingin GFI depends on the number of items present in a latent variable which grossly affect the result of the hypothesized model. In this wise, the adjusted goodness of fit index (AGFI) will be introduced to normalize the GFI. The acceptable value to indicate model fit for both GFI and AGFI is between 0.9 and 1 (Baumgartner& Homburg, 1996)

Relative fit indices

Relative fit indices or incremental fit indices (Tanaka, 1993) and comparative fit indices (Bentler, 1990) liken the baseline modelto chi-square for the postulated model (McDonald and Ho, 2002).However, Most of the time, thebaseline model do contains unrelated items, which resulted to a very large chi-square that suggested a poor fit. (Hooper, Coughlan and Mullen, 2008).The combination of normed fit index and comparative fit index is called Relative fit indices.

Normed fit index and non-normed fit index

The normed fit index (NFI) is used for the analysis of thedifferencethat existamong the chi-square value of the postulated model and the chi-square value of the unacceptable model (Bentler and Bonett, 1980) However, NFI tends to be undesirablyinfluenced (Bentler, 1990). The non-normed fit index (NNFI)was built in by Tucker and Lewis, in 1973 which can also be refer to as Tucker-Lewis index, as it was constructed on an index fashionedwhich resolves some of the problems of undesirable bias(Bentler, 1990). However,for NFI and NNFI to indicate a good fit the value should be between 0.95 and 1(Hu and Bentler, 1999).

Interpretation of Confirmatory Factor Analyses

One need to know that two or more hypothesized model can be found to fit a measurement data from the result of CFA during interpretation (Biddle and Marlin, 1987; Thompson and Borrello, 1989). In view of this, looking for model with good statistical fit can result into having more than one model as only one model may not givethe best interpretation for a given data. Adding to this, research has shown that multiple statistics fit indices should be simultaneously evaluatedand comparison should be made among these fit indices before researcher can conclude that a given data fit a hypothesized model in Confirmatory Factor Analysis(Campbell, Gillaspy, and Thompson, 1995)

However, if the hypothesized model does not fit the measurement data in CFA, the researcher can look for ways to recover the hypothesized model by discoveringthe constraintsthat need to be freed that was fixed and the constraints that was fixed that need to be freed. In order to get the accurate result, researcher can make use of computer sets to modifyconstraints, and this should be done individuallyso as to regulatethe changes that suggested the highestnumber of development in the model fit.

2.7.2 Main Approaches to Exploratory Factor Analysis (EFA)

There are two majortechniques to EFA,namely;principal component analysis (PCA)and principalaxis factoring (PAF). PCAis occasionallymisunderstood forPAF due to the high level of resemblance between the procedures used in carry out the two analyses. Both of them do reduce items to a smaller group of measuredvariables that tend to swing together(Garson, 2008).Both processes can be accomplished with SAS soft-ware and at times produce the same results.Nevertheless, there are certainvitaltheoreticaldissimilaritiesamongthe two that need to be put into consideration. PCA is more common and more practicable. It reduces items to lesseramount of latent variabledenoted by factor. It involves the analysis of variability that occurs in everyitem.PAF, on the other hand is more hypothetical,is used to reveal the arrangement of afundamental set of unique items (Garson, 2008).It's analysis lean emphasis on variance sharing. Perhaps Principal axis factoring explains the hypothesis that caters for the unobserved fundamentalconfiguration: Principal axis factoring accepts that the co-variation in the measured variables is as a result of theexistence of one or more hypothetical model that applyfundamentalstimulus on these measured variables. The figure below presented such fundamental configuration.

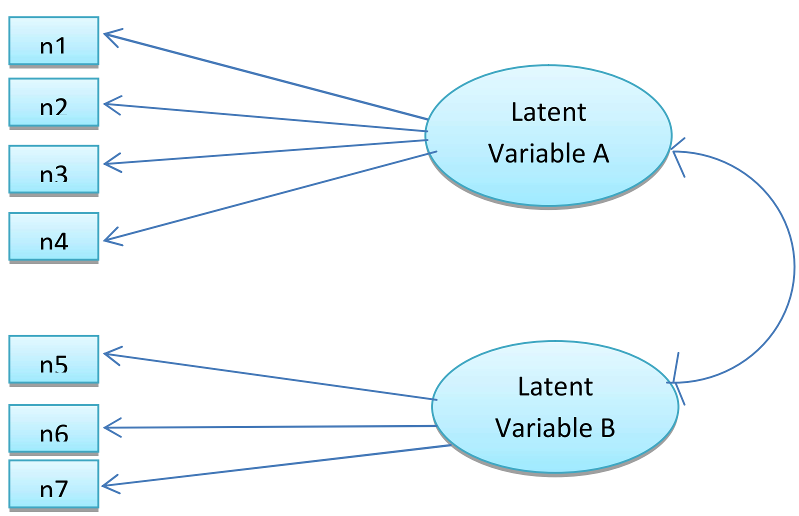

Figure 2.6: Fundamental Structure in principal axis factorin

The eclipses in Fig 2.7 signify the latent variables (unobserved variables) of “Mathematics Engagement scale.” These factors are called latent because they are presumed to trulyoccur in the students’ trust, but cannot directly be measured. But, they often employ an effecttowards students’ reactions to the seven variables that institute the Mathematics academics engagement question. (The seven variables represent the rectanglecategorizedas n1-n7 in the figure). It is shown that the “Latent variable A”employsimpact on variables n1-n4 while the Latent variable B employs impact on variables n5-n7.

When researchers have confidence that definite latent variable exists whichcauseunderlyingeffect on the items they are reviewing they make use of principal axis factoring.In addition to this future, PAFassist the researcher ascertain the amountof latent variable to retain while thePCAcreates no hypothesisconcerning the number of factor to retain. What it does is to reduce items into smaller amount of component called factor. (Matsunaga, M. (2010).

The diagrams in figure 2.8 illustrate the differences between Principal axis factoring and Principal component analysis.

Factor Analysis (Principal axis factoring) **Principal Component Analysis**

Observed variables are reduced into component

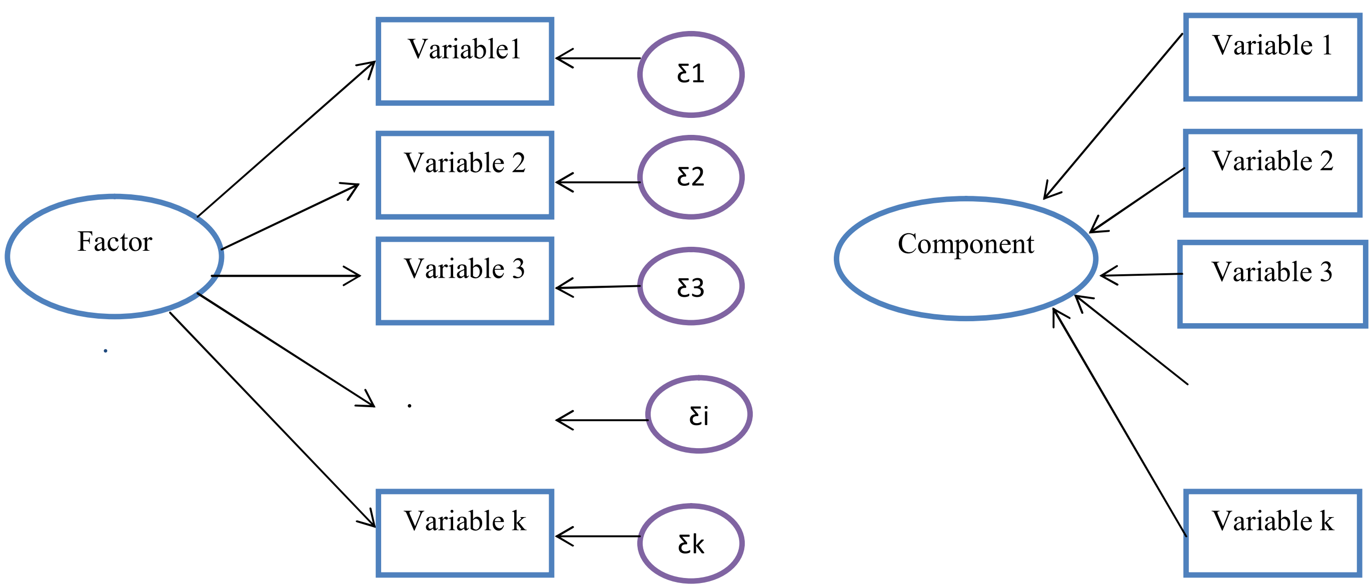

Figure 2.7

In summary, both principal component analysis and principal axis factoring performed significant duties in exploratory factor analysis, but their theoreticalfundamentalsfinding are different.

Terminologies Associated with Exploratory Factor Analysis

Communality

Communality signifies the percentageof variance that everyvariable shared with other variables in a data. It can also be defined as the percentage of variance in a measured variable that can be explained by the underlyinglatent variables. The higher the loading of an item to its corresponding factor, the larger the communality.The quantity of variance that an item did not share with other items in data is the total variance in that item minus communality. So, communality can be estimated from the squared multiple correlations of an item with all other items in a data.

Not only that, if the communality for item t is 0.562 it means that 56.2% out of the total variance in item t is shared or common variance. The communalityof an item is the square of the loading of each of the item on their corresponding factor and it varies from 0 to 1. When the communality is high (i.e. greater than .5), it shows that the extracted factors account for a substantial amount of variance in the retained items. However if the communality is low (i.e. less than .5, it means that many of the variance in the retained items are not accounted for by the extracted factor. In this case, more factors need to be extracted to account for the unexplained variance. (Hogarty, et al. 2005).

Eigenvalues

Eigenvalue indicates the totalpower of the connection between an unobserved and its corresponding item. That is, Eigenvalue is the sum of squared correlations for each factor. Put simply the eigenvaluesrelatedto an itemspecify the amount ofinfluenceits factor has for it. Successive eigenvalues have lower values. Eigenvalues over 1 are 'stable'. It is important to decide on the amount of factor to keep. Anindividualprocedure is to seek to expoundfull variance with little amount of factors. Kaiser recommends that researcher should keep factors that have Eigen Values that are above 1. Cattell disagrees on the bases of research carried out and suggest 0.7. Cattell said that interpretation of factor must be mindfully and theoretically sensible. He explained that researchers should aim at getting at least 50% of the variance explained as cut-off using factors that are 1/3 to 1/4 the number of thetest item, and that extracting of factors should stop when they cannot be meaningfullyrepresented by groups of items.

Scree plot

A graph of Eigenvaluesis scree plot. It represents the quantity of variance that can be accounted for by the retained factors. Costello and Osborne (2005) said that researchers should examine theangle at which a factor suddenly breaks away from other factors. At this stage, the first factor has most of the items loaded on it while the last factor has few items loaded on it. The loadings of item to a factor specify the relationship between an item and its factor.At the first extraction,every factor tends to capture allinexplicable variance. In this wise, most of the items will try to capture the first factor. In this case, factorcan be defined as a group of items that are consistently moving together.The first solution is obtained using un-rotated factor structure(Costello and Osborne, 2005).

Exploratory Factor Analysis Objectives

The maingoals of Exploratory Factor Analysisare to regulate

1. The amount of commonlatent variables that manipulate a group of observed variables.
2. The amount of connection between every latent variable and itmeasured variable.
3. Certainmutualaspect of Exploratory Factor Analysisthatclassifies the type of the latent variable that causes fundamentalreactionsamong items.
4. Regulatea group of items that swing together in a scale.
5. Establish the number of factor in a scale.
6. Determination of the highestsignificantaspect during the classificationof items.
7. Createvalue for latent variablesfor use in other analyses.

Step involved in exploratory Factor analysis(EFA)

Details of steps in EFA are outlined below:

- Test assumptions
- Verify Sample size
- Select type of analysis

- How will the factors be extracted? (Principal Components (PC) and Principal Axis Factoring (PAF)
- Selection of Rotational Method(Orthogonalvarimax/Oblique Promax) Rotation
- Determine amount of latent variables to keep (What criteria will help in choosing the amount of factors to keep)
- Isolatethe variable that hang undereverylatent variable
- Remove bad items and go through stages three and four 4
- Interpretation
- Factors' naming and definition.
- Examination ofrelationshipbetweenfactors and analysis ofinnerconsistency

2.8Assumptions Underlying Factor Analysis

There are assumptions underlying factor Analysis. They have to do with thelevel of measurement, normality, linearity, outliers, factorability of the correlation matrix.

The level of measurement (LOM)

All measured variables are essential to be appropriate for inter-correlational analysis, that is, they must have been measured on an ordinal scale (Likert scale). Measurement's level can be defined as the correlationbetween the valuesallotted to traits in a variable. For example, in figure 2.8"bash membership" as a variable have someamounts of traits that areassigned to it. For instance, in the figure, thevotingsettinghave "antiroyalist", "egalitarian", and "autonomous" as traits with the values 1, 2 and 3 which can be used for the purpose of analysis.

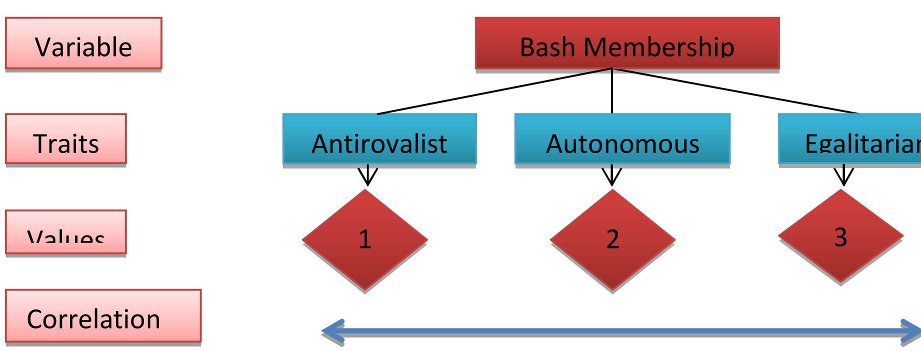

Figure 2.8

Measurement's levelrefers to the connectionbetween these three values. The values are used byresearchersto represent long statement in a scale. Higher value does not mean something more and lower value does not mean something less. For example, if thevalue of 2 is allotted to egalitariansand 1 is allocated to antiroyalist that does not mean that egalitarian is as twice asantiroyalists. Researchers just use the value as a short form of writing the trait.

Need for Level of Measurement

Measurement's levelassists researcher in the interpretation of result of a data during analysis. At first, fora nominal data,numerical values are assigned to traits to shorten the lengthy names. Secondly, measurement'slevel assiststhe researcher to chooseappropriate statistical procedure on the measured data. For nominal measure, researcherstake appropriate measure to avoid the use of average and t-test on the measured data.Four levels of measurement arerecognized. They are: Nominal, Ordinal,Interval and Ratio measurement.

Nominal measurement

In Nominal measurement numerical valueis assigned to attributeexclusively. There is no ordering of data. For instance, sport shirtfigures of footballer are identified with a numerical values. Footballer with number 22 is not special or greater thanfootballer with number 11 and is definitelydid not double footballer number 11.

Ordinal measurement

Ordinal datacan be ordered. In this case, a space between traits does not indicate anything. For instance, in a scale, you can assign values to level of education as follows: Primary school as 0;Secondary school as 1; College of Education or polytechnic as 3; University as 4. For this quota, greaterfiguresindicate highereducational attainment. But the interval amongthe figures cannot be interpreted in an ordinal data.

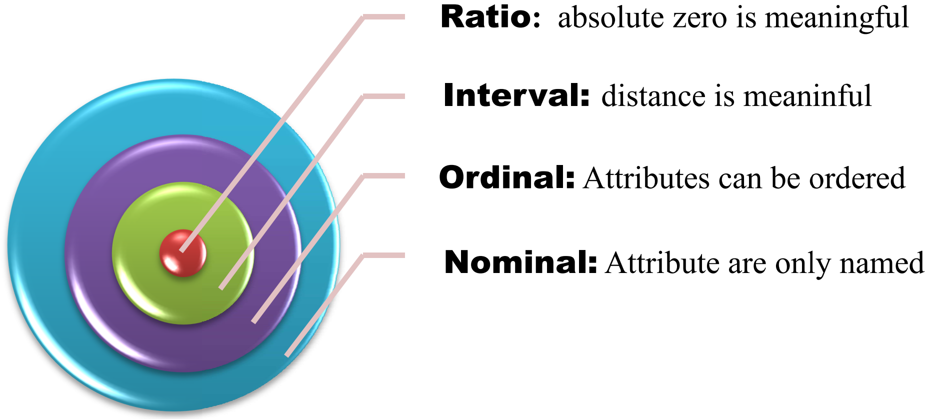

Figure 2.9: Order of Level of Measurements

Interval measurement

Distance between traitsin interval measurement can be meanifully interpreted. For instance, temperature is measured in degree centigrade so the distance between50 and 60 is equivalent to the distance between 90 and 100. So, interpretation of distance between values can be done. Forthis, it is sensible to calculatethe average of an interval data, while it is not right to calculate average in ordinal data. Similarly, it is out of point to calculate ratio in interval data.For instance, 60 degrees is not two times as hot as 30 degrees (though the traitworth is double).

Ratio measurement

Zero has meaning in ratio measurement. Meaning that fraction can be calculatedwith a ratio variable. Example of a ratio variable is Mass. In social sciences, many of the calculationare carried out with ratiovariables,for instance, the number of customersfor three weeks. How? Sinceit is possible to have no (zero)customers and since it has meaning to mention that our customer as today is double what we have in the last tree week.

However, one should know that measurement levels are in grading. Assumptions restriction and data analyses sensitivity depend on measurement level. Lesser level required lesser assumption with low level of sensitivity to data analysis.

Not only that, at each level of grading, the present level containsthe entirelypotentials of the one under it and enhancesthe quality by adding a new thing. More importantly it is require tomakeuse of upper level of measurement such as interval or ratio measurement rather than a lesser one such as nominal or ordinal measurement(William, 2006).

Normality

Factor Analysis is strong to normal distribution rules (for factor analysis to yield a good result, the items must be normally distributed.Assumption of normality involves therelationship among the exogenous and endogenous variables in a construct. This assumptionis often miscalculation by researchers. Researchers are often confusedinformation of normality. In multivariate analysis,regulardistribution of information isrequiredto solve the problem of abnormal situation.The problem of normality does not emaciate from exogenous variables as is frequentlythought. Possibly the misperceptionaround this hypothesisoriginatesfrom inability of researchers toknowwhere the problem is coming from. During analysis, everyinformationaboutthe number of participant usually takesa diverse arbitrary item thatincludesall the problems that cause changes in the measured and expected values as fashioned by a multivariate equation, and it is what is causing problem in the analysis that supposed to have anormaldistribution. In factor analysis, if Principal axis Factoring Analysis is employed, then expectationsconcerning normality are not necessary. Though, when variable havenormal disribution, it improve the result of analysis(Tabachnick and Fidell, 2007). But once the amounts of latent variable are ensured by means of statistical interpretation, the normality of multivariate should be expected.

Although, Multivariate normality can be violated even though the variables in a data are normally distributed.However, in order to avoid Multivariate normality violation, Mecklin and Mundfrom (2005) characterized MVN assessments into four clusters: correlational and Graphical methods (example, graph of chi-squared), kurtosis and Skewness methods (example is Mardia's tests of kurtosis and skewness), Consistent methods (example, Henze-Zirkler test making use of empirical characteristic function) and Goodness of fit methods (example, Shapiro-Wilk multivariate omnibus assessments and Anderson-Darling). Out of these available tests, Mecklin and Mundfrom (2005) suggested two of the test for variables that are not normally distributed. These are Royston's (1995) amendment of a goodness of fitmultivariate addition to the Shapiro-Wilks W test for samples that are small and the Henze-Zirkler (1990) reliable test for samples that are large. The first assesses how straight the normal quantile-quantile (Q-Q) probability plot is while the second assesses the space between the postulated MVN dispersal and the detected dispersal (Farrell, Salibian-Barrera and Naczk, 2006).

However, before a reasonable decision can be taken about normality, other MVN statistics test should be interpreted alongside with those suggested above. Moreover, for univariate normality (UVN), Srivastava and Hui (1987) suggested Shapiro-Wilk W-test as the most appropriate assessment. They noted that a single test cannot verify all the differences that do occur under normality. To support this assertion, Looney (1995) claimed that the decision concerning normal distribution should depend on the cumulative outcomes of a series of diverse assessments with moderately great control. Outlier is another statistical test that should be verified when assessing normality of a data.

Outliers

Exploratory Factor Analysis is sensitive to outlying cases. Therefore if there are outliers in the data, they have to be removed before factor analysis.

Causes of Outlier

i. wrong data entering
ii. Inability to identify and fixing omitted data

Outlier is identified from the value ofmahalanobis,any mahalanobis variable with the value less than critical value χ^2 is counted as anoutlier and should be deleted.

Linearity

Multivariate normality suggests linearity; therefore linearity between two variables is evaluated by inspecting the scatterplot (Tabachnick and Fidell, 2007). So to assess the linearity among the variables in a data, researcher needs to have both strong negative and positive skewness. However, to display a certain level of linearity among variables, the scatterplot must demonstrate a stable spread of marks. Agreeing to Tabachnick and Fidell, during the assessment of bivariate scatterplots, if the distribution is oval-shaped, it means that they are normally distributed and linearly related. However, since, Factor Analysis is built on relationships among items, it is essential to ensure that true relationship exist among the items.

Factorability

Interactions among different itemscan be ascertained through intercorrelation matrix when carrying out Exploratory Factor Analysis procedure. (Tabachnick and Fidell (2007) suggested that the intercorrelation matrix (which is refer to as Factorability of R) of more than .30 should be used in EFA. Hair *et al*. (2006) categorized loading of item on their factor as ±.30 to be minimal, ±.40 to mean important, and ±.50 as practically significant. If any of the loading is not more than .30, then the researcher should take another look into whether to use factor analysis procedure or not.

2.9 Sample Size

The number of participant to use is highly essential in factor analysis.Hogarty, et al. (2005) established that the reliability of FAdepend on the number of participants. They said that before FA can be carried out, large numbers of participants are needed. However, scholars have recommended two broad approaches to the smallestnumber of sample to make use of in FA. One approach is the consideration of the number of participants, and the other approach is the ratio of items to participant (Hogarty, et al. (2005). Field (2005) suggested above 300 samples with the shared variance between factors and the items that loaded on them after extraction to be 0.5 and above. Also, Comrey and Lee (1992) gave the summary of the number of participants in FA to include this range:50-200 as poor-fair, 300-500 as good to very good, while 1000 and above is termed asoutstanding.

Selection of Rotational Method: (Orthogonal/Oblique)

There are two types of rotation; these are orthogonal rotation and Oblique rotation.Orthogonal rotation with varimax method is used when a researcher has a course to believe that the latent variables are uncorrelated (Field, 2005). Oblique rotation with Promax method, on the other hand, is used when the researcher has cause to believe that the latent variablesare not independent of one another (they are correlated), therefore, in researches involving human behaviour, oblique rotation with Promax method is to be preferred since various components of individual are related (Schmitt and Sass, 2011).

2.10 Determination of number of factors to retain

Although both the exploratory and confirmatory methods look out for considerably amount of variance in a set of measured variables (items) with a lesser number of shared factors, EFA is mainly suitable for the development of a new scale when a researcher has little or no knowledge of the patterns and amount of common latent variables to keep (Kline, 2013). Therefore, one of the major serious decision researchers should take when using EFA is the amount of latent variables to keep in mind. The choice concerning the amount of latent variables to keep in mind is essential for two reasons. First, the EFA required that clarification should be made between mare items reduction and adequate representation of the relationship that exist within a group of items, since it is essential to depend on differentiating unimportant factors from important ones (Hayton, Allen, and Scarpello, 2004). Secondly, research has shown that over estimation and under estimation of the extracted factors resulted to inadequate factor-loading configuration and clarification (Velicer, Eaton, and Fava, 2000).

But in spite of the significance of decision concerning the amount of latent variable to consider and different research that were carried out on the number of factors to keep during EFA, no agreement have been reached concerning the right method to make use of. Different methods have been put in place to help these assessments, but none of them normally give the same result. (Zientek and Thompson, 2007). However, Garrido, Abad and Ponsoda, 2012; Ruscio, and Roche, 2012; Henson and Roberts, 2006; Hayton et al., 2004 show that parallel analysis happen to be accurate and robust method which work better than the commonly used Kaiser's rule, scree test and maximum likelihood procedure (Timmerman & Lorenzo-Seva, 2011; Patil, McPherson, and Friesner, 2010; Henson and Roberts, 2006).

Courtney (2013) carried out an investigation on the best approach to use in deciding the amount of latent variables to keep in EFA out of Parallel Analysis (PA), Scree test, eigenvalue greater-than one rule, Minimum Average Partial correlation technique and Simple Configuration Measure using various settings; that is, number of participants, items' number, factors' number and items' number and their corresponding factor and established that PA was dependable with real statistics used to decide the factors' numbers, with 76.42% correctness while Scree test tend to over factor. Courtney also established that eigenvalue greater-than one rule utterly overrate the factors' number and was only correct 8.77% of the time.Warne and Larsen (2014) collaborated this trend and reported that parallel analysis method perform better than traditional method of eigen value greater than one.Velicer, Eaton & Fava (2000) examined the different between

Parallel Analysis (PA), Kaisar-Guttman criterion (KI) and Minimum Average Partial (MAP) test, and resolved that PA happen to be the best approach, next toMAP, whereas the value of eigenvalue more than 1 method was exceptionally incorrect.

Furthermore, Cesar and Marisol (2013) revalidated a scale called Maslach Burnout Inventory-Human Services Survey (MBI-HSS) with 22 items developed by Cordoba et al. (2011) using both PA and CFA, the result of both parallel analysis and confirmatory factor analysis generated 3 factors as against the 7 factors reported by Córdoba et al. in their exploratory factor analysis with Kaiser's rule of Eigenvalue greater than 1 as the criteria for the amount of latent variable to keep. This result was also linked to Kaiser's criteriaof over-estimation of the number of factor to retain; a situation that plainlyhappens in the results reported by Cordoba et al. Evidence of the above can also be seen in the work of Noor, Naziruddin and Ilham (2016) in the development and validation of their marketing items, these researchers used Parallel analysis to retain three factors in their exploratory factor analysis they did notdepend on eigenvalue greater than one criterion that tends to over factor (Cesar and Marisol, 2013).

Other evidence can also be seen in the work of Atari and Jamail, (2016) in their survey instrument title: Dimensions of Women's Mate Preferences, Validation of a Mate Preference Scale in Iran used EFA along with PA and also CFA on different participant to certify the scale. The result of both EFA with PA and CFA yielded 5 factors while the criteria of scree plot and eigenvalue greater than 1 yielded 6 factors.Henrie-Barrus, *et al.* (2016)in the cause of validating their survey instrument title: Development and preliminary validation of the OpioidAbuse Risk Screener used EFA with PA as well as CFA with adequate sample size. The result of both PA and CFAyielded the same amount of latent variables. But despite the robustness in the use of parallel analysis, it is hardly used by researchers in the literature. According to Hayton et al. (2004),out of the 142 researches concededwith the use ofexploratory factor analysis between 1990 and 1999 inthe Academy of Management Journal and the Journal of Management,no oneout of themtestifiedthe use of PA.

Not only that, Adegbuyi, 2011;Ojo, 2013; and Saliu, (2014) in the development and validation of their survey instrument used EFA with PCA as a process ofextraction. They also used Scree plot, Kaizer rule of eigenvaluelarger than 1 and component matrix as the criteria for factor retention. None of these researchers use PA as criteria for determines the amount of latent variable to keep in their work.Also, Veiga, (2016) in the cause of developing students' engagement instrument title: Assessing student Engagement in School: Development and validation of a four-dimensional scale which contain 20 items used factor analysis with an eigenvalue greater than1 rule for the amount of latent variable to keep instead of PA. However, some reliability problems have emerged from the scale (Gutiérrez et al.; 2016).

2.11 Parallel analysis

Parallel analysis is based on random data reproduction to determine the number of factors to retain in Principal Component and Exploratory Factor Analysis using the Monte Carlo Simulation Technique, a random simulative (artificial) data set is generated besides the actual (real) data set and the estimated eigenvalues are calculated. When the method is employed, the number of factors at the point where the eigenvalue in the simulative data is equal to or greater than that of the actual data is considered significant (Uyar, 2012).

Basic procedure for parallel analysis

First, researcher will run an EFA on their original data and record their eigenvalues of the extracted factors; next, “parallel data” will be generated, this is artificial data set which contains the same number of variable with the same number of observation as the researcher’s original data, but all variable included in this “parallel data” (permutations of the original raw data set) are random variables; the “parallel data” are factor-analyzed and eigenvalues for factors are computed; this procedure of generating “parallel data” and factor analyzing is repeated usually 500-1000 times, and eigenvalue of each trial will be recorded.

Then the average of those eigenvalues will be compared to those factors extracted from the original data; if the eigenvalue of the original data’s factor is greater than the average of the eigenvalues of the “parallel factor” (factor of the same rank extracted from the parallel data), that factor is retained; if the eigenvalue of the original data’s factor is equal to or smaller than the average, that factor is considered no more significant than a random factor and therefore discarded. (Hayton et al., 2004).

2.12Item Response Theory (IRT)

Calibration of responses of participants to items in a survey instrument in scholastic evaluation is often carried out by Item Response Theory (IRT) models.IRT describes the correlationamongthe latent variables,the characteristicsquality possess by each itempresent in an instrument, and responses of the participant to each of the item. IRT models are usually used to analyze the information derived from the responses of participants to questionnaires that contained items with dichotomous or polytomous responses (which is also refer to as dichotomously-scored items (items that are scored in two categories) or polytomously-scored items (items that are scored in multiple ordered-categories). Dichotomous responses are usually labelled as true or false, right or wrong, yes or no, whereas polytomous responses correspond to more than two options. (Bacci, Bartolucci, and Gnaldi, 2012).

With IRT model, the psychometric properties and the performances of scale(s) with dichotomously or polytomously scored items can be evaluated to maximallyreduce the items to a great extent, and hence, produce accurate, valid, and moderately brief instruments that can be used in educational sectors (Edelen and Reeve, 2007). But to explain the performances of IRT models in a better way, theperceptionsof models with dichotomously scored items (i'e. model with dichotomous responses) can be liken to ordered polytomous responses model(for instance, Likert-type response). Number one attribute of IRT theory can be tracedto the estimation parameters.There are two major parameters estimate that are recognized in IRT models: A person parameter which denotes the latent trait and refers to the possibility of a testee choosing a test item at a specific trait level, and it is denoted by theta (θ) (Lee, 2016). The second one is the item parameter which can be classified into 3: the difficulty index, discriminating power and guessing.

All IRT models differ in the item parameter they utilized. The first IRTparameter is the difficulty index. The type of item parameter chooses by researchers will be based on the information available to the researcher. For instance, if the concern of the researcher is on the difficulty of the test alone, then the best thing is for the researcher to select the model that will contain thedifficulty parameterwithout the use of the remaining parameters. The hypothesized model in this work is students mathematics engagement construct, so the graded response model for ordered polytomous responses, which is an extension of two parameter logistic models (that is,item difficulty and item discrimination), will be used to assess the scale items.Item difficulty refers to the level of understanding of a respondent on a set of items at a given trait level. It is denoted by 'bi' and the value ranges between -3 to 3(Lee, 2016). The second parameter is discriminating power, this refers to the extent to which an item in a scale can differentiate between respondents' responses, and is often denoted by 'a_i'.

The value of 'a_i' can range from 0 to infinity, but theoretically this value should range from 0.5 to 3 (Baker, 2001; De Ayala, 2009 and Toland, 2013).However, a scoring method that is consistent with Polytomous item response theory (PIRT) models is summing across the number of measurement opportunities available within each task-based replication. Polytomous item response theory (PIRT) model specify an item response function (IRF) which is defined as the probability of the item confirmation associated with a specific trait level for each possible outcome (Lee, 2016). An item response function (IRF) i, specifies the probability of an outcome Yi as a function of the target trait. Unique to PIRT models is the transitional models that specify a wide range of item response function (IRFs) using some number of item parameters (Naumenko, 2014). Various Polytomous IRT models can be specified based on how step functions are defined and used to interpret the probabilityof a response category. These models are: Partial Credit Model (PCM; Masters, 1982), the Generalized Partial Credit Model (GPCM; Muraki, 1992; 1993), and the graded response model (Samejima, 2010).

Partial Credit Model

The partial credit model (PCM) developed by Masters (1982), is a unidimensional latent trait model for the analysis of responses scored in two or more ordered categories. In this sense, the model is designed for the same purpose as several other models in Polytomous IRT model including Graded response model (GRM: Samejima, 1969; 2010). The partial credit model (PCM) differs from Graded response model (GRM), in that, it belongs to the Rasch family of models which assess only the difficulties indices of items in a scale and so shares the distinguishing characteristics of that family. For example, If only two categories are present in a scale, e.g "right" and "wrong", "yes" and "no", the model is equivalent with the Rasch model for dichotomous items. Also, the desirable properties of the dichotomous model, such as separation of person and item parameters and the existence of sufficient statistics for both sets of parameters, are preserved in the partial credit model.

Not only that, the partial credit model (PCM) uses the Rasch model to specify the probability of success at kth step such that the item response function (IRF) for Yi = 0 has the form $p_{j0}(Q) = \frac{1}{1+\sum_{r=1}^{m}[\exp\sum_{k=1}^{r}(Q-b_{ik})]}$ and the IRF for Yi = j > 0 have the form

$$P_{ij}(Q) = \frac{\exp\sum_{k=1}^{r}(Q-b_{ik})}{1+\sum_{r=1}^{m}[\exp\sum_{k=1}^{r}(Q-b_{ik})]}$$

where step is denoted by r = 1, 2, 3, m and j represents the score category. Thus, for a set of n items there will be n x m item parameters. Furthermore, the Partial Credit Model (PCM) is quite popular in assessment contexts due to its parsimonious nature. Because the PCM allows for a relatively small number of estimates per set of items, sample sizes as small as 300 return stable item parameter and trait estimation (de Ayala, 2009).

Generalized Partial Credit Model

Unlike the PCM, the GPCM includes the item-level discrimination parameter and expresses the IRF for Yi = 0 as $p_{i0}(Q) = \frac{1}{1+\sum_{r=1}^{m}(\exp\sum_{k=1}^{r}[a(Q-b_{ik})])_{i}}$ and the IRF for Yi = j > 0 have the form $P_{ij}(Q) = \frac{\exp(\sum_{k=1}^{r}[a_i(Q-b_{ik})])}{1+\sum_{r=1}^{m}(\exp\sum_{k=1}^{r}[a_i(Q-b_{ik})])}$

where a_i is the item discrimination parameter common across all m steps, but unique to each item. For a set of n items, n (m+1) parameters are estimated. The GPCM is the most flexible of the three "divide-by-total" PIRT models; fixing the value of a_i to 1 across items reduces to the PCM. The GPCM is flexible in that it allows the possibility of identifying item response options that may be redundant with each other. For example, IRFs for some response options may be centered at the same ability estimate (Naumenko, 2014).

Another characteristic that is also unique to the generalized partial credit model (GPCM) is that the derived a-parameter (discrimination parameters) formula for the generalized partial credit model (GPCM) is quite different for three or more response categories. More specifically, it has been algebraically revealed that having more response categories for an item leads to lower a-parameter value in the GPCM (Ostini and Nering, 2005). Therefore both the Partial credit model and the generalized partial credit model will not be the model of choice for items calibration of students' Mathematics engagement items, rather, the Graded Response Model will be used due to the following reasons: the cumulative category response functions of GRM belong to the homogeneous cases and identical in shape (Samejima, 2010). Also, the item discrimination parameter of the graded response model does not depend on the number of response categories (Samejima, 2010).

The Graded Response Model

The graded response model (Samejima, 1996, 2010) is an extension of the 2-PL logistic model, is appropriate to use when item responses can be characterized as ordered categorical responses. In the graded response model, each item is described by a slope parameter (discrimination parameter) and between category threshold parameters (a set of m-1 threshold parameters that is, one less than the number of response categories). The item discrimination describes how well the item can distinguish between individuals with different levels of ability. For the graded response model, one operating characteristic curve needs to be estimated foreach between category thresholds. In the graded response model, items need not have the same number of response categories. Threshold parameters represent the trait level necessary to respond above threshold with 0.50 probabilities. Category response curves represent the probability of responding in a particular category conditional on trait level. Generally speaking, items with higher slope parameters provide more item information. The spread of the item information and where on the trait continuum information is peaked are determined by the between-category threshold parameters.

Also, the GRM is an indirect model which the probability of responding to each category is captured by obtaining the item response function (IRFs) from the difference between adjacent step functions. The *bik* are interpreted as the target trait value at which *Pi0* (*Q*) = .5, *bim* as the target trait value at which *Pim*(*Q*) = .5, and for values in between steps (*bik* + *bik* + 1)/2 corresponds to the modal point of the IRF for *Yi* = *k* (Penfield, 2014). The justification for using GRM, or any model based on ordered response categories, with testlet-based scores is that testlet-based scores can theoretically have an ordered quality if they "correspond to the extent of completeness of the examinee's reasoning process within a testlet" (Lee, 2016). That is, the more dichotomously-scored measurement opportunities within one testlet are answered correctly by an examinee, the more extensive is her ability.

2.12.1 Basic assumptions of polytomous IRT Model

One of the assumptions of polytomous IRT model is monotonicity: Monotonicity assumption is satisfies if the respondent responses to a set of items in a scale represent the true trait they exhibit. For example, items in student Mathematics Engagement scale (SMES) is said to satisfy monotonicity assumption, if the responses of students to SMESare their true level of engagement in Mathematics. So, the true relationship between the respondent responses to a set of items in a scale and the trait they exhibit is called their true monotonic relationship. The graphical representation of monotonicity is called item characteristic curve (ICC) with 'S' shape. Where the latent trait level is on the vertical-axis and respondents responses is on the horizontal axis.

Another assumption under IRT is invariance: For this notion, the assessment of items parameters and the latent trait are expected to be autonomous of the respondent features in a populace. For example, if the participants or respondents to a set of test items are drawn from a heterogeneous sample, the responses of every participant should not be affected by the individual characteristics. For example, "I ask questions during Mathematics class," should not differ by characteristics of students, such as age or gender. Thus, in IRT, the SMESshould assess a student level of engagement in Mathematics regardless of their age or gender.

Local independence is additional IRT assumption. Under this assumption, it is presumed that respondentsreaction to test items are not in any means relate to each other. Two things are considered under local independence; one, single latent attribute is measured; two, the response to an item is not through the information acquire from any of the remaining items. Problem associated with this is when an item failed to measure a single construct. For example, when a set of 8-items in Students' Mathematics Engagement Scale (SMES) that supposed to yield only one factor can yield two factors. In the IRT analysis, this problem occur, if for instance, 8-variables in SMES contains three variables that are tested in the opposite way of the remaining 5 SME variables, which can mislead the students who find it difficult to understand the test items (Carlson, Wilcox, Chou, Chang, Yang, and Blanchard et al., 2011).

The other difficulty is satisfying the assumption of local independence, if the reaction of a respondent to a question influences his or her reaction to another question. This happened when two variables or questions are alike in a scale. (for example, SMES questions that read 'I find mathematics class fun and exciting' and 'I enjoy mathematics class'). Responses of respondents to these set of questions will be similar, and this can affect the reliability and validity of the scale. To solve this problem, one of the items will be removed.Another IRT theory that matches the assumption of local independence is unidimensionality. This assumption supports the idea of a factor having a fixed number of variables or questions in an instrument. In this case, a set of related items or variables will only load on a single factor. There will be no cross loading of items into two or more factors. In other to achieve this, CFA will be employed for the determination of number of dimensions. And if CFA isolates only one dimension or factor, it means that the unidimensionality assumption issatisfied (Yang and Kao, 2014).

2.13 Appraisal of the Literature review

Literature reviewed showed that when students engaged maximally in any subject, they perform better, most especially in Mathematics. However, in the process of learning Mathematics, engagement happens when students are systematically busy with the teacher inside the classroom, partake in solving problems and do the Mathematics, and grasp the opinion that having Mathematical knowledge is meaningful, and suitable in the classroom and outside the classroom. Scholars explained that the concept of engaging students is grounded on the trust that, their knowledge increases when they are snooping, fascinated, or encouraged; but their knowledge decrease when they are uninterested, calm, dissatisfied, or otherwise disengaged in learning activities. They noted that students who disengaged themselves from Mathematics class denythemselves the opportunity to study any course that requires the basic concept of Mathematics in higher institution. Other scholar noted that, student who cease from learning Mathematics hinders himself or herself from gaining the ability to comprehend lifetime skills through a mathematical perception.

In the literature reviewed, scholars explained that between 40% and 60% of students in secondary school remain persistently unengaged; fail to pay attention most of the time in the class; and donot do their classwork as well as the assignment given to them by their teachers. Furthermore, students often complain that classes are boring during lessons. Not only that, they explained that the rate at which students are dropping out of school is as a result of their non-participation during academic work. The literature revealed that the detachment of students from their academic work is a worldwide problem, which often results in an increase in the dropout rates in many nations. However, in order to reduce the dropout rates, suggestions were made for teachers and school administrators to identify these set of disengaged students and occupy them in meaningful teaching and learning process for high level performance in their classrooms.

From the literature reviewed, other scholars stressed that teachers should ensure that they engage students meaningfully; otherwise, their teaching would not be productive. Furthermore, these scholars suggested that teachers and educators should use any opportunity at their disposal to select some resources that can engage students in critical thinking during lesson for them to solve the problem of disengaged students. Similarly, other researchers affirmed that it will be impossible for students to understand what teachers are teaching them and be vigorous learners if teachers fail to present teaching that will engage students in critical thinking. Moreover, some researchers observed the same feeling and advice educationalists and teachers to be close to their students, and select necessary measure that will enable them deliver interesting teaching that can increase the level of students' engagement during their lessons.

The literature reviewed showed that before teachers and educationalist can increase the level of students' engagement during the teaching and learning process for the purpose of improving learning outcome and achievement in mathematics, these set of teachers or educationalists need valid and reliable student mathematics engagement questionnaire that researchers and teachers can use to investigate the feeling, views, and beliefs which students have about Mathematics, and how an adjustment can be made to improve students' level of comprehension during the teaching of Mathematics among secondary school students in Nigeria. However, the review of literature showed thatfor a teacher to use valid and reliable student mathematics engagement questionnaire, it is imperative to pay careful attention to the construction and validation of student mathematics engagement scale so as to ensure that its psychometric properties are good.

But, the dimensionality of this construct has constituted a problem. Some authors argued that students Mathematics engagement scale should be one dimension, while others posited two or three dimensions. Currently available student mathematics engagement scale are either 1-dimensional or 2-dimensional and rarely more. Not only that, scholars have also showed that there are lapses in the technique used (that is, the use of the criteria of eigenvalue above 1 for the determination of number of dimensions, instead of parallel analysis criteria, inability of some researchers to use confirmatory factor analysis, and finally, those engagement scale developers that did not use graded response model of IRT frame work for the final selection of studentengagement items during the construction and validation of student mathematics engagement instruments that exist in the literature).

So, there was need for improvement in the construction of valid and reliable multidimensional student mathematics engagement instrument with more robust statistical method, such as; Exploratory Factor Analysis (EFA), Parallel Analysis (PA), Confirmatory Factor Analysis (CFA) and polytomous graded response model of IRT framework that test developers can make use of to investigate the level of students' engagement in Mathematics, for the purpose of improving students' learning outcome and achievement in Mathematics.

CHAPTER THREE

METHODOLOGY

This chapter discusses research design, population, sample and sampling technique, instrumentation, data collection procedure and data analysis procedure.

3.1 Research design.

This study adopted a survey design under Instrumentation research type. This type of research verifies large and small population by choosing and studying small samples from the population to ascertain the relative occurrence, sharing, and relationship between the latent variables and its corresponding measured variables. It involves validation of Instrument and subsequent collection of data to evaluate the level of students' engagement in Mathematics.

3.2 Population

The population for this study comprisedall Senior Secondary School 2 Students, both public and private schools in Ekiti State. Mathematics students of these classes were chosen because they have been exposed to Senior Secondary School 3 syllabus. They also have the sound understanding of what engagement in Mathematics is all about.

3.3Sample and Sampling technique

The sampling for this study was carried outinthree phases and three sets of samples were involved. Hence the total samples for this study comprised three thousands six hundred and sixteen(3616) senior secondary school two students**.**In each of the phases, a multi-stage sampling technique was used. These phases are outlined here:

Phase 1: Pilot testing of the initial pool of 100 items of student mathematics engagement scale after the field test for Exploratory Factor Analysis (EFA), and determination of numbers of latent variables to keep from EFA data and parallel datausing parallel analysis.

Phase 2: Involved the use of student mathematics engagement items extracted from Parallel Analysis (PA)on a larger sample that was different from the initial samples usedduring EFA for

Confirmatory Factor Analysis (CFA), and alsofor calibration and selection of the retained items from CFA.

Phase 3: Usage of the final scale of student mathematics engagement scale along with the Mathematics achievement test, to another large population.

3.3.1 Phase 1: Sampling Procedure for the pilot testing of the initial items pool of students' Mathematics engagement items for exploratory factor analysis and parallel analysis.

Multi-stage sampling procedure was used to select the sample at this phase.First, one Senatorial District was randomly selected from the existing three Senatorial Districts in Ekiti State.From the selected Senatorial District, there are five Local government areas out of which four LGAs wererandomly selected.From each of the selected Local government areas, simple random sampling was used to select three public senior secondary schools and three private senior secondary schools. Thus, the number of schools for this phase was12 public senior secondary schools and 12 private senior secondary schools respectively, making a total of 24 schools. Finally, 42 SS2 students were randomly selected from each school. So the sample size at this phasecomprised1008 students.

Table 3.3.1: Sampling Frame for Phase 1 according to State, Senatorial Districts,Local govt., Type of schools and number of students.

State	Senatorial District	Local Government Areas	Type of Schools	No of Schools	No of Students
Ekiti	Ekiti North	Ido-Osi LGA	public	3	126
			Private	3	126
		Ilejemeje LGA	public	3	126
			Private	3	126
		Ikole	public	3	126
			Private	3	126
		Moba	public	3	126
			Private	3	126
	TOTAL	4	8	24	1008

3.3.2 Phase 2: Sampling procedure for the usage of student mathematics engagement items extracted from exploratory factor analysis and parallel analysis on a larger population for Confirmatory factor analysis as well as item calibration and selection

Multi-stage sampling procedure was also used in this place. First, from the remaining two Senatorial Districts in Ekiti-State, one Senatorial District was selected. From the selected Senatorial District, there are six Local government areas out of which four LGAs wererandomly selected. From each of the selected Local government areas, simple random sampling was used to select four public senior secondary schools and four private senior secondary schools. Thus, the number of schools for this phase was16 public senior secondary schools and 16 private senior secondary schools respectively, making a total of 32 schools. Finally, 50 SS2 students were randomly selected from each school. So the sample size at this phase comprised1600 students.

Table 3.3.2: Sampling Frame for Phase 2 according to States, Senatorial Districts, Local govt., Type of schools, and a number of students.

State	Senatorial District	Local Government Areas	Type of Schools	No of Schools	No of Students
Ekiti	Ekiti Central	Ado LGA	public	4	200
			Private	4	200
		Irepodun/Ifelodun LGA	public	4	200
			Private	4	200
		Ijero LGA	public	4	200
			Private	4	200
		Ekiti-West LGA	public	4	200
			Private	4	200
Total	2	4	8	32	1600

3.3.3 Phase 3: Sampling procedure for the use of Mathematics achievement test and the final scale of student mathematics engagement scale.

The remaining senatorial district in Ekiti-State was used in this phase and multi-stage sampling procedure was also used. From the selected Senatorial District, there are five Local government areas out of which four LGAs wererandomly selected. From each of the selected Local government areas, simple random sampling was used to select three public senior secondary schools and three private senior secondary schools. Thus, the number of schools for this phase was12 public senior secondary schools and 12 private senior secondary schools respectively, making a total of 24 schools. Finally, 43 SS2 students were randomly selected from each school. So the sample size at this phase comprised1032students.

Table 3.3.3: Sampling Frame for Phase 3 according to States, Senatorial Districts, Local govt., Type of schools and number of students.

State	Senatorial Districts,	Local Government Areas	Type of Schools	No of Schools	No of Students
Ekiti	Ekiti South	Ikere LGA	public	3	129
			Private	3	129
		Ekiti-East LGA	public	3	129
			Private	3	129
		Gboyin LGA	public	3	129
			Private	3	129
		Ise-Orun LGA	public	3	129
			Private	3	129
	Total	4	8	24	1032

3.4 Instrumentation

Three instruments were used for this study to collect data. These were:

1. Initial items of student mathematics engagement scale: This can be seen in Appendix I
2. Mathematics achievement test items: This can be seen in Appendix II.
3. Student mathematics engagement Scale: This can be seen in Appendix III.

3.4.1. Initial items pool of student mathematics engagement instrument

Initial items pool of student mathematics engagement scale consisted of 100 items which were generated from three sources, viz:

1. Theitemspool of students' statements about their engagement in Mathematics was collected by the researcher through an open-ended questionnaire from the representative of the target population.
2. The statements of other secondary schools Mathematics teachers through interview.

3. The statement of the researcher based on her experience as a secondary school Mathematics teacher.

The details of the procedure for the construction of the 100 initial items of studentmathematics engagement scale are presented below.

3.4.1.1 Procedure for the Development of Student MathematicsEngagement items

Step one: Generation of items involved the initial writing of an open-ended questionnaire by the researcher; asking students about their engagement in Mathematics which was given to the descriptivesample of the real population of Senior Secondary School 2 students in Ekiti-State, (150 SS2 students were involved). The selection of the items was done through an empirical criterion key. Items were retained if more than twenty percent of the respondents listed it in their response. Also, items from other secondary schools Mathematics teachersthrough interview and items from the researcher based on her experience as a Secondary School Mathematics teacher were used. Finally, a total of 100 items from these three sources was collated for expert review.

Step two: Six experts including my supervisor from Institute of Educationreviewed the initial draft of the scale. The information provided on each of the test itembased on expert review wasused to re-write a pool of 96items.

Step three: A four point Likert type scale was developed using these items. A score of four indicate the maximum possible positive score for an item while a score of one wasassigned the minimum possible negative response.

3.4.2Mathematics Achievement test

The instrument contained 50 items with options A to D, which was adapted from WAEC multiple choices objectives questions in Mathematics. The items were selected from 2010 to 2015 WAEC objective questions in Mathematics using SS2 syllabus. Each correctly answered question attracted 1 mark such that possible range of scores on the test was 0 to 50 marks. The items covered the entire topics in the SS2 syllabus. The details of the procedure for the Construction of Mathematics Achievement test were presented below.

3.4.2.1 Procedure for the Construction of the adapted Mathematics Achievement test

Step one: A self-constructed Mathematics achievement test blueprint using SS 2 syllabus along with the SS2 Mathematics textbooks that were duly recommended by WAEC and WAEC Mathematics series was used to generate the Mathematics achievement items. Table 3.4 displays thetable of specification.

Table 3.4:Table of Specification for WAEC objectives questions in Mathematics using SS2 syllabus

S/N	Subject Matter Content	Levels Of Cognitive Domain				Total
		5 Items Knowledge 10%	**10 Items Comprehension 20%**	**20 Items Application 40%**	**15 Items Analysis 30%**	
1.	**Number and Numeration (32%)**	2 (1,2)	3 (3,4,5)	4 (6,7,8,9)	7 (10,11,12,13, 14,15,16)	**16**
2.	**Algebraic Process (16%)**	-	2 (17,18)	3 (19,20,21)	3 (22,23,24)	**8**
3	**Mensuration (14%)**	-	3 (25,26,27)	3 (28,29,30)	1 (31)	**7**
4.	**Geometry (12%)**	3 (32,33,34)	-	3 (35,36,37)	-	**6**
5.	**Trigonometry (6%)**	-	-	3 (38,39,40)	-	**3**
6.	**Statistics/ Probability (20%)**	-	2 (41,42)	4 (43,44,45,46)	4 (47,48,49,50)	**10**
	Total	**5**	**10**	**20**	**15**	**50**

Step two: The items were taken from WAEC Mathematics series of 2010 to 2015 using the table of specification.

Step three: The items were revalidated by seeking the opinion of experts concerning the appropriateness of the items.The test items were trial tested on a sample of 200that have similar characteristics with the intended population to ascertain the difficulty index of each item.Forty-eight out of the fifty items survived. The difficulty indices of the 48 survived items ranged from 0.20 to 0.91. The reliabilitycoefficientof the test itemsusing (Kuder-Richardson)KR-20 formula was 0.83.

3.4.3 Final scale of Students Mathematics engagement items

The initial items of students mathematics engagement scale was validated to get the final scale of students mathematics engagement scale using EFA, PA, CFA and PGRM for Item calibration and selection. Ordinal alpha coefficient was used to obtain the internal consistency of the final items

3.5 Method of Data Collection

The researcher collected anintroductoryletterfrom the director, International Centre for Educational Evaluation (ICEE), Institute of Education, University of Ibadan, to the State Ministry of Education. The researcher also collected a letter of permission from the State Ministry of Education and presented same to the selected school principals. Students were given one week to prepare for each of the examination.

The researcher personally visited all the selected schools in Ekiti State to collect the data with the help of 80 trained research assistants and two supervisors. Strongguidelines wereorganized for the participants.The participants were asked to read the general instructions carefully before responding to the test items. During the pilot testing of the test, the researcher herself guided the respondents properly. The respondent were asked to answer the questions as honest as possible. Also, the respondents were asked not to write their names and the names of their schools, so that they will be able to give the correct information about their engagement in Mathematics. The students were made to indicate their opinion by ticking in front of the statement they thought was best to represent their present practice. No timewasgiven to respondents to complete the student mathematics engagement questionnaires, but the mathematics achievement test lasted for $1\frac{1}{2}$ hours. The analysis of data was based on the response of the students'scores in the entire variables under study. The periods for data collection for the three phases and validation of the instrument lasted for 31 weeks.

3.6 Procedure for checking out the fitness of the students Mathematics Engagement items for Exploratory and Confirmatory Factor Analysis

1. Preliminary Analysis 1

This involved checking of the information collected from the students for assumptions of factor analysis

- Level of measurement (LMO): The scale was assessed for the level of measurement (LMO). It was measured on an ordinal scale (Likert scale), but converted to summated rating scale.
- Normality of the distribution: Histogramwas checked to assess the normality of the distribution. This was also reinforced byassessing the normal probability plots. A reasonable straight line suggested a normal distribution.
- Outlier: Boxplot was assessed to check for outlier cases. A small circle(s) with a number attached is termed as outliers. If these numbersspreadbeyond 1.5 from the edge of the box, the data was inspected and the outlier was removed before the main analysis.
- Linearity and factorability: Inter-item correlation was done to verify the level of the directconnection among variables.The value of 0.3 to 0.8 in the correlation matrix indicated the linear relationship among variables, hence, it is factorable.

2. Preliminary Analysis 2

This involved screening of the data and checking the adequacy of the sample size.

- Inter-item correlation (R-matrix) was done to check for the case of singularity and multicollinearity.
- The presence of such items was suggested by the value of the determinant of the R-matrix. A determinant of less than 0.00001 indicated the presence ofsingularity or multicollinearity.
- If the R-matrix determinant is less than 0.00001, the matrix was inspected to delete such items (i.e those having a correlation of .8 and above with other items).
- One of each pair in this category was dropped since it would be unnecessary duplication.
- KMO measure of sampling adequacy was done toassess sample size adequacy. The interpretation is as follows; a score close to one (1) indicate that the data is suitable for exploratory factor analysis. A benchmark of 0.5 and above was set as a benchmark for this study.

- Bartlett's test of sphericity was also done to check if the R- matrix is not an identity matrix.
- There wasanalysis based on the item-total correlation. Items that had a low correlation ($r < .3$) were removed, (Item measuring the same trait should correlate well).

3.7Item selection procedure

Exploratory Factor Analysis

Oblique rotation with Promax method was used to remove irrelevant, redundant and unclear itemsfor ease of correlation. Also, the primary loading between 0.5 to 0.6 with secondary loading of 0.2 to 0.3 were retained.

Parallel Analysis

Watkins (2006) Monte Carlo PCA for Parallel Analysis was used to decide the amount of latent variables to keep. In this analysis,parallel data was factor-analyzed 1000 times and eigenvalues from the extracted factors were computed.Then the averages of those eigenvalues from parallel data were compared to those from the real data.Thus, Factors that have minimum numbers of three items and also have a loading between 0.3 and 0.8 were retained.

Confirmatory Factors Analysis

The retained factors from parallel analysis were tested for model fit and the following criteria were strictly followed. Factors that did not meet these criteria were discarded.

RMSEA should be0.06 or lesser.

RMR and SRMR should be0.08 or lesser

GFI and AGFI should be over 0.9.

NFI and NNFIshould have a cutoff of .95 or larger

CFI should have 0.90 or larger indicate acceptable model fit.

Polytomous Graded Response Modelof IRT framework

The retained factor from CFA was subjected to item calibration. During the calibration exercise,the polytomous graded response model of item response theorywas used for items calibration.According to the classification rule (Baker, 2001; De Ayala, 2009 and Toland, 2013), the range of model fit for the slope parameter (discrimination parameters) and thresholdparameters (difficulty indices) for ordered polytomous graded response model of IRT are 0.5 to 3 and -3 to 3 respectively.

3.8 Method of Data Analysis

Analyses of the data were carried outwith the following statistical software packages: SPSS, Monte Carlo PCA softwareIRT PRO, AMOS and LISREL. The details of the analytical procedures for each question are shownin table 3.6.

Table 3.6 Method of Data Analysis

1a. How many items and factors are extracted from the initial draft of 100 items of students Mathematics Engagement Scale?	Exploratory Factor Analysis using principal Axis factoring
1b. What are the appropriate numbers of factors to retain in students Mathematics Engagement Scale?	Parallel analysis using Monte Carlo Principal Component Analysis technique
2a. Do the retained factors of student's Mathematics Engagement scale show good model fit indices?	Confirmatory factor analysis,LISREL package
2b. Do the students Mathematics Engagement items show convergent validity?	Confirmatory factor analysis,AMOS package
3a. Are items of each of the dimensions of Students Mathematics engagement scale unidimensional?	Confirmatory factor analysis,AMOS package
3b. To what extent do the Students Mathematics engagement itemslocally independent of one another?	Polychoric correlation using IRT PRO package
3c. How many items were selected as good items using Polytomous graded response model of IRT framework during calibration process.	Polytomous graded response model using IRT PRO package
4. What are the discriminate validity indices of the identified factors of student Mathematics Engagement scale?	Confirmatory factor analysis,AMOS package
5.Is there any relationship among the identified factors of Student's Mathematics Engagement scale?	Confirmatory factor analysis,AMOS package
6a.Is the students Mathematics Engagement scalereliable?	Reliability analysis using Ordinal Alpha coefficient analysis
6b.How reliable are each of the sub-scale of student Mathematics Engagement scale?	Reliability analysis using Ordinal Alpha coefficient analysis
7.Which of the sub-scales of student Mathematics engagement scale is the best predictor of Mathematics achievement?	Inferential statistics using multiple Regressions in AMOS package

CHAPTER FOUR

RESULTS AND DISCUSSION

Results and discussion of the study are presented in this chapter. The study constructed and used student mathematics engagement scale in predicting Mathematics achievement among senior secondary school students in Ekiti Statethrough the following procedure: Exploratory factor analysis, Parallel analysis, Confirmatory factor analysis, Polytomous graded response model, Polyserial test of close fit, Polychoric correlation analysis,Pearson Correlation Coefficientanalysis, Reliability analysis and multiple Regressions analysis.The results presented in this chapter centered on the stated research questions.

Results of Exploratory factor analysis

4.1 Research Question 1a

1a. How many items and factors are extracted from the initial draft of 100 items of students' Mathematics Engagement Scale? Table 4.1,Table 4.2 and Figure 4.1 present the results of exploratory factors analysis.

For Table 4.1, Exploratory Factor analysis extraction was done using principal axis factoring extraction with promax rotation to show the initial eigenvalue of student mathematics engagement construct. The Table shows the eigenvalues associated with each factor before extraction, after extraction and after rotation.

Table 4.1: The Eigen Value of the Original Data

Total Variance Explained

Factor	**Initial Eigenvalues**			**Extraction Sums of Squared Loadings**			**Rotation Sums of Squared Loadings[a]**
	Total	**% of Variance**	**Cumulative %**	**Total**	**% of Variance**	**Cumulative %**	**Total**
1	**14.620**	15.229	15.229	**14.034**	14.619	14.619	11.392
2	**7.176**	7.474	22.703	**6.608**	6.884	21.502	6.799
3	**3.100**	3.229	25.933	**2.490**	2.594	24.096	5.541
4	**2.122**	2.211	28.143	**1.492**	1.554	25.650	5.409
5	**1.882**	1.961	30.104	**1.277**	1.331	26.980	3.520
6	**1.642**	1.711	31.815	**1.021**	1.063	28.044	5.363
7	**1.493**	1.555	33.370	.901	.938	28.982	5.892
8	**1.451**	1.511	34.881	.831	.866	29.848	6.865
9	**1.404**	1.463	36.343	.773	.805	30.653	4.576
10	**1.322**	1.378	37.721	.702	.731	31.384	2.600
11	**1.286**	1.340	39.061	.660	.688	32.072	3.625
12	**1.276**	1.329	40.390	.649	.676	32.748	3.051
13	**1.229**	1.281	41.671	.621	.647	33.395	1.416
14	**1.197**	1.247	42.918	.587	.612	34.007	3.823
15	**1.186**	1.236	44.154	.552	.575	34.582	2.612
16	**1.152**	1.200	45.353	.538	.561	35.143	1.931
17	**1.144**	1.192	46.545	.528	.549	35.692	2.771
18	**1.133**	1.180	47.725	.509	.530	36.223	2.313
19	**1.105**	1.151	48.876	.477	.497	36.720	1.071
20	**1.061**	1.105	49.981	.435	.454	37.173	1.949
21	**1.047**	1.091	51.072	.420	.438	37.611	1.729
22	**1.024**	1.067	52.139	.395	.411	38.023	.673
23	**1.012**	1.054	53.193	.392	.408	38.431	2.956
24	**1.002**	1.043	54.236	.377	.393	38.824	1.529
25	.981	1.022	55.258				
26	.972	1.012	56.270				
27	.955	.995	57.265				
28	.939	.978	58.243				
29	.930	.969	59.211				
30	.914	.952	60.163				

Extraction Method: Principal Axis Factoring.
a.When factors are correlated, sums of squared loadings cannot be added to obtain a total variance

Table 4.1, which is the table of total variance explained, displays the initial eigenvalues and extraction sums of squared loadings of student mathematics engagement items.Looking at Table 4.1, it shows that the initial eigenvalue that are larger than one are24; meaning that 24 factors were extracted, while the Extraction Sums of Squared Loadings extracted six factors. The two extractions were not the same; there was confusion in the number of factor to keep. So, the scree plot was examined to verify the retained factors.

Scree plot displays the pictorial representation of criteria of Eigenvalue greater than one.

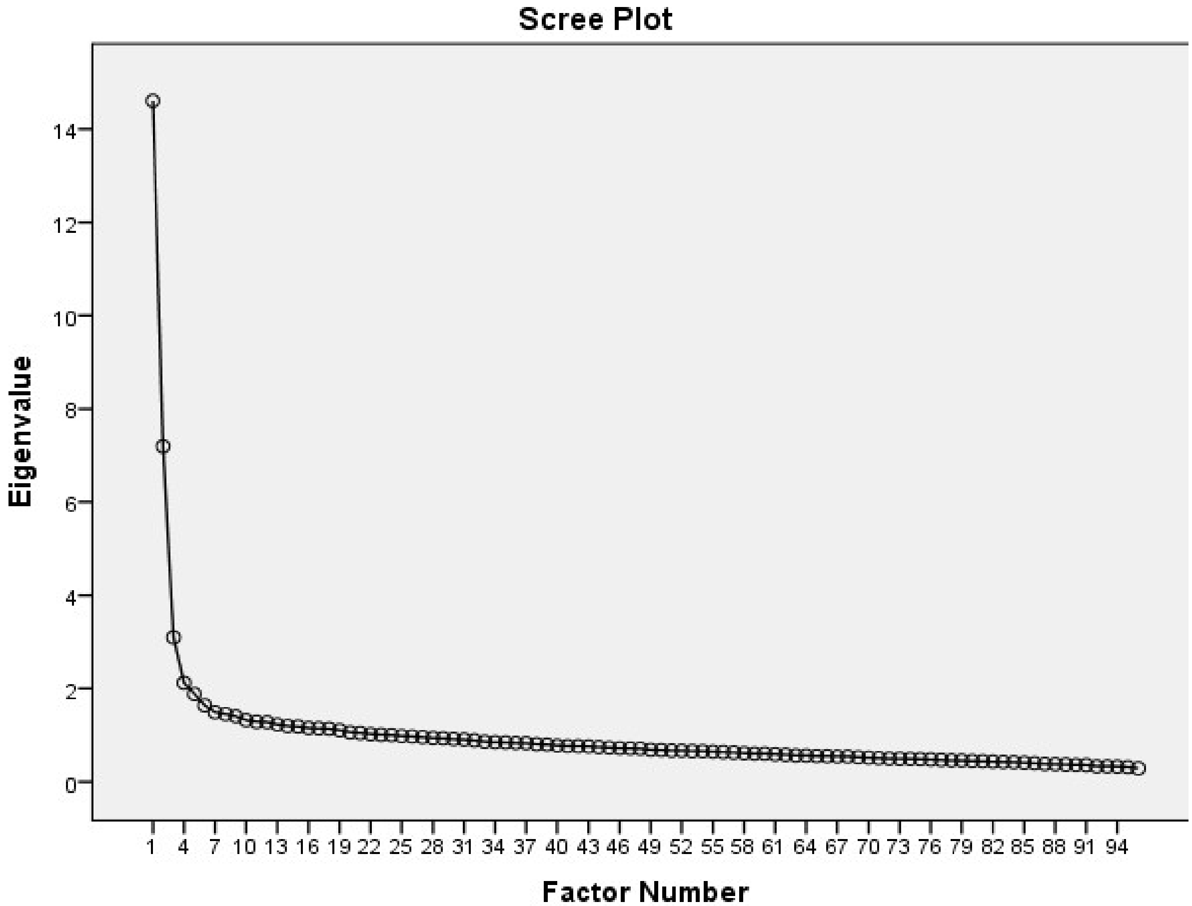

Figure 4.1: Scree Plot of the Extracted Factors

Figure 4.1 shows the scree plot. The scree plot has a sharp decline between 4 and 7 which suggested that the number of factors to retain should be between 4 and 7 factors. Also, the result of the scree plot did not show clearly the number of factor to retain. Therefore, the pattern matrix was further examined for the number of factor to retain.

During exploratory factor analysis; the axis was rotated using promax rotationto produce the pattern matrix of Table 4.2 which showed the factor loading of student mathematics engagement scale.

Table 4.2:Factor loading of Student Mathematics Engagement items after rotation

Table 4.2Pattern Matrix[a]

	Factor															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	. . . .	24
a87	.717															
a91	.664															
a90	.615															
a67	.614															
a89	.610															
a82	.579															
a76	.560															
a88	.548															
a92	.542															
a77	.532															
a86	.522															
a65	.516															
a79	.510															
a83	.502															
a73	.479															
a68	.427															
a93	.376															
a66	.360															
a47		.704														
a46		.643														
a43		.486														
a42		.445														
a39		.430														
a54		.417														
a45		.398														
a53		.384														
a49		.376														
a48																
a20			.814													
a19			.670													
a28			.613													
a36			.551													
a30			.501													
a37			.480													
a7			.427													
a6			.302													
a1				.631												
a4				.592												

a3	.591												
a13		.598											
a17		.597											
a12		.458											
a9		.447											
a61			.534										
a60			.501										
a71			.375										
a15				.638									
a22				.391									
a23				.365									
a11				.344									
a21				.313									
a55					.600								
a57					.473								
a50					.411								
a56					.379								
a80						.594							
a78						.554							
a81						.411							
a75						.360							
a31							**.521**						
a32							**.502**						
a35								.659					
a33								.498					
a18								.373					
a63									.473				
a24									.450				
a69									.391				
a85										**.633**			
a84										**.459**			
a70											**.539**		
a58											**.525**		
a40												**....**	
.												**....**	
.													
a94													**.476**

Extraction Method: Principal Axis Factoring.

Table 4.2 shows the loadings of each item onto their corresponding factor. Based on Tabachnick and Fidell (2007) criteria of factor loading of 0.3 and above and recommendation of Hair et al. (2006) to take 0.30 as minimal loading, the loading of an item to their corresponding factor that was less than 0.3 were discarded. Oblique rotation along with Promax method was employed since the extracted factors were not independent of one another; they are expected to be correlated (Schmitt and Sass, 2011; Pajares and Miller, 1994).

Table 4.2 also shows that 24 factors were extracted after rotation, butonly 11 factors out of these 24 factors have a minimum of three items which were loaded on each of them.The 11 factors have a total of 64items. However, Henson and Roberts (2006) recommend that a minimum of threeitems must load on a factor for a proper interpretation of the construct. Therefore, these 11 factors were identified as the retained factors through Principal Axis Factoring extraction and Promaxrotation method.

Discussion

Exploratory factor analysis was used for the extraction of student mathematics engagement items using principal axis factoring and rotation of axis using promax rotation to show the factor loading. The three output of EFA (Eigenvalue greater than one criteria of Table 4.1, scree plot diagram of Figure 4.1 and pattern matrix of table 4.2) show that none of these resultsgive the exact number of factor to keep. The three outputs show different result instead of the three to give the same result. These results are in agreement with the submission of Garrido, Abad and Ponsoda, 2012; Ruscio, and Roche, 2012; Henson and Roberts, 2006; Hayton et al., 2004 and Courtney, 2013 that says thatparallel analysis is the accurate and robust method which works better than the commonly used Kaiser's rule, screeplot testand Simple Configuration Measure.

However, due to the inability of the output of exploratory factor analysis (that is, the Table 4.1 of total variance explain of eigenvalue larger than one, the scree plot and the Table 4.2 of pattern matrix) to give the same and accurate number of factor to keep, and also due to over estimation of the number of factors using eigenvalue greater than 1 method (Matsumoto 2017; Atari and Jamail, 2016; Cesar and Marisol, 2013 and Velicer, Eaton, and Fava, 2000), the data were subjected to parallel analysis for decision regarding the amount of latent variablesto keep.

Result of Parallel analysis with Eigenvalue of the original data

4.2 Research Question 1b

1b. What are the appropriate numbers of factors to retain in student Mathematics Engagement Scale?

Table 4.3: The eigen value of parallel data.

	Number of variables: 96 Number of subjects: 1008 Number of replications:1000		
Factor	Original Eigenvalue	Random Eigenvalue	Standard Deviation
1	**14.620**	**1.6779**	.0252
2	**7.176**	**1.6342**	.0193
3	**3.100**	**1.6025**	.0172
4	**2.122**	**1.5751**	.0156
5	**1.882**	**1.5506**	.0143
6	**1.642**	**1.5273**	.0128
7	1.493	1.5067	.0121
8	1.451	1.4869	.0116
9	1.404	1.4675	.0113
10	1.322	1.4494	.0108
11	1.286	1.4323	.0107
12	1.276	1.4156	.0105
13	1.229	1.3995	.0101
14	1.197	1.3831	.0100
15	1.186	1.3674	.0097
16	1.152	1.3529	.0100
17	1.144	1.3378	.0094
18	1.133	1.3231	.0091
19	1.105	1.3091	.0091
20	1.061	1.2954	.0087
21	1.047	1.2816	.0088
22	1.024	1.2678	.0085
23	1.012	1.2550	.0080
24	1.002	1.2420	.0081
25	.981	1.2288	.0081
26	.972	1.2163	.0080
27	.955	1.2039	.0079
28	.939	1.1914	.0079
29	.930	1.1795	.0077
30	.914	1.1678	.0078

Table 4.3 shows the Eigenvalue of the original and random data.So, in order to agreeon the amount of factor to keep, the eigenvalue of the original data and eigenvalue of the parallel data

were compared. That is, the first actual eigenvalue was linked to the first arbitraryeigenvalue; also, the second real eigenvalue waslinked to the second arbitrary eigenvalueand so on. Such a contrast can be simplydetermined by inspecting the value of the eigenvalue of the original and random data. After the examination of the two Eigenvalues, the point at which the eigenvalue of the factor of original data is lesser than the average of eigenvalue of parallel data was employed and those factors that fall under these categorieswere discarded. Only factors of the original data with the eigenvalue greater than the eigenvalue of the parallel data were retained (Hayton et al., 2004). Based on this criterion, the eigenvalues of factors 1 to 6 of the original data were greater than the average of eigenvalue of factors 1 to 6 of the parallel data. With this information, factors 1 to 6 of the original data were retained. Table 4.4 shows the loading of the retained factors with their corresponding items.

Table 4.4:Loading of the retained factors and their corresponding items from the result of parallel analysis.

Pattern Matrix[a]

	Factor					
	1	2	3	4	5	6
a87	.717					
a91	.664					
a90	.615					
a67	.614					
a89	.610					
a82	.579					
a76	.560					
a88	.548					
a92	.542					
a77	.532					
a86	.522					
a65	.516					
a79	.510					
a83	.502					
a73	.479					
a68	.427					
a93	.376					
a66	.360					
a64						

a47	.704				
a46	.643				
a43	.486				
a42	.445				
a39	.430				
a54	.417				
a45	.398				
a53	.384				
a49	.376				
a51					
a14					
a48					
a20		.814			
a19		.670			
a28		.613			
a36		.551			
a30		.501			
a37		.480			
a7		.427			
a6		.302			
a1			.631		
a4			.592		
a3			.591		
a5					
a13				.598	
a17				.597	
a12				.458	
a9				.447	
a61					.534
a60					.501
a71					.375

Rotation Method: Promax with Kaiser Normalization.

Table 4.4presents the factor loadings of each item onto their corresponding factor. From the Table, the factors with the loadings that are less than 0.3 were discarded. These variables are:irrelevant variables (items with low loadings: $r < 0.3$);redundant variables (items which are highly correlated: $0.9 \geq r \geq 0.8$) and unclear variable(items that are cross loading).

Six factors appeared after the Oblique rotation. The Pattern Matrix shows the factor loading on Table 4.4. (The Items that failed to load on the identified factors were also discarded). Also, Table 4.4.1 in appendix V shows the structure matrix, which indicate that the factors arenot independent of their own (they are correlated). Not only that, Kaiser Mayer Olkin measure of sampling adequacy and determinant's matrix was verified for thisdata through Bartlett's test of sphericity(see Table 4.4.2 in appendix VI).45 students Engagement statements were identified from the six factors. The factors have been named on pages 90 to 93 along with the test items that loaded on each factor. These items were tested on a different sample of 1600 for confirmatory factor analysis.

Discussion

Table 4.3 and Table4.4 show the result of Parallel analysis (PA). Table 4.3 shows that only six factors should be retained while table 4.4 shows the factor loading of each construct which is contrary to the criteria of Eigenvalue greater than one which extracted 24 factors. This result is in support of Cesar and Marisol (2013) where they concluded that PA extracted three factors against seven factors extracted by eigenvalue greater than one criteria due to over extraction of factors. Also, the result of PA supported thework ofAtari and Jamail (2016) where PA and CFA retained five factors against six factors retained by eigenvalue greater than one criteria of the result of exploratory factor analysis.

4.4.1 Group Name and Description of EFA Factor

Subsequently,researcher looked at the items that load on the same factor to give a common name.

PERSONAL AGENCY ENGAGEMENT

(a87, c1) I tell my teacher what I normally do to understand Mathematics during Mathematics class so that others can learn.

(a91, c2) I suggest different formula to my teacher during Mathematics class to help me learn.

(a90, c3) If I noticed that my teacher has not explained any topic clearly in Mathematics class, I tell my teacher to give more explanations on the topic so that others can learn.

(a67, c4) I bring any question that is not clear to me in my text book for my teacher to solve in the class.

(a89, c5) I ask my teacher to teach us any question I cannot solve in my Mathematics text book during class.

(a82, c6) any time I need more explanations on a topic in Mathematics class, I tell my teacher.

(a76, c7) I tell my teacher to use different method to solve Mathematics problems in the class so that I can understand better.

(a88, c8) I tell other students how Mathematics questions can be solved so that I can learn more.

(a92, c9) I ask my teacher to give more explanation on a Mathematics topic any time I don't understand in the class.

(a77, c10) I offer suggestions to my teacher on how to solve difficult topics in Mathematics.

(a86, c11) I ask my teacher to give me extra work on Mathematics to help me learn.

(a65, c12) I solve problems on topics that have not been taught by my teacher in Mathematics and bring it for my teacher to mark.

(a79, c13) During Mathematics class, I ask my teacher questions for clarity.

(a83, c14) I let my teacher know whatever thought that comes to my mind as per what the teacher is teaching us in Mathematics class.

(a73, c15) I ask my teacher to let me do the correction of assignment given to us on the chalk board for other students.

(a68, c16) When I come across a new topic in Mathematics I study it until it clear to me.

(a93, c17) When I have solution to any problem in Mathematics, I ask my teacher the same question to know whether he or she knows it.

(a66, c18) I pass Mathematics test or Examination because I have the ability to solve Mathematics problems.

POSITIVE AFFECTIVE ENGAGEMENT

(a47, d1) I solve Mathematics problems because I need it for my future carrier.

(a46, d2) Any time my family/guardian(s) provide me with the material I need to pass Mathematics I feel happy.

(a43, d3) Learning during Mathematics class is important to me.

(a42, d4) I enjoy staying in Mathematics class.

(a39, d5) I love to do well in Mathematics class.

(a54, d6) I have interest to learn more during Mathematics class.

(a45, d7) any time my teacher uses teaching aid to teach me during Mathematics class I feel happy.

(a53, d8) I have a goal to make a good grade in Mathematics.

(a49, d9) before a quiz or examination in Mathematics, I work hard for me to pass.

NEGATIVE AFFECTIVE ENGAGEMENT

(a20, e1) I don't understand and follow directions during Mathematics class.

(a19, e2) I disrupt the class during Mathematics lesson because I don't understand Mathematics.

(a28, e3) I don't have interest in what my teacher is teaching me during Mathematics class.

(a36, e4) I don't participate in Mathematics class because I feel that my teacher will embarrass me.

(a30, e5) I get disturbed and unhappy whenever my Mathematics teacher entered the class to teach Mathematics.

(a37, e6) I don't have interest in many of the topics that our teacher teaches us in mathematics class.

(a7, e7) I don't come to Mathematics class at all.

(a6, e8) I concentrate on other things during Mathematics class because I don't understand what my teacher is teaching me

POSITIVE BEHAVIOURAL ENGAGEMENT

(a1, f1) I pay attention to what my teacher is teaching me during Mathematics class.

(a4, f2) I do all the Mathematics assignment given to me by my teacher.

(a3, f3) I do Mathematics homework after school.

NEGATIVE BEHAVIOURALENGAGEMENT

(a13, g1) I do not respond at all to the questions asked by my teacher during Mathematics class.

(a17, g2) I don't ask questions during Mathematics class.

(a12, g3) I don’t go over to study what my teacher teaches me during Mathematics class.

(a9, g4) I don’t study my Mathematics notebook before coming to Mathematics class.

COGNITIVE ENGAGEMENT

a61, (h1) I keep trying my Mathematics homework, until when am able to do it.

a60, (h2) I solve my homework problems in Mathematics in a separate book before writing it in my note.

a71, (h3) I am willing to go ahead of what my teacher is teaching me in Mathematics class.

Note: Item 1 under Cognitive Engagement is name as a61 in EFA data and also named as h1 in CFA data.

4.5 Research question 2a: Do the retained six factors of students Mathematics Engagement scale show good model fit indices?

The following model fits were tested for this research question:

i. Root Mean Square Error of Approximation (RMSEA) should be≤ 0.06,which signify suitable model fit (Tabachnick B, Fidell(2007).
ii. Root mean square residual (RMR) and standardized root mean square residual (SRMR) should be ≤0.08,which signify suitable model fit (Tabachnick B, Fidell(2007).
iii. Values for eithernon-normed fit index (NNFI) or normed fit index (NFI)should have alimit of 0.95 or larger,which specify a good fit to the model.
iv. A comparative fit index (CFI) of 0.90 or larger indicates acceptable model fit. (Tabachnick B, Fidell(2007).
v. Incremental fit index (IFI) of 0.95 or larger indicates acceptable model fit. (James B. Schreiber,; Frances K Stage,; Jamie King,; Amaury Nora and Elizabeth A. Barlow (2006).
vi. An acceptable value of Chi-square probability should be ≥ 0.05 (Suhr, 2006).

Result of Confirmatory factor analysis

Table 4.5: Model fit indices of the retained factors of students Mathematics Engagement scale

Model fit	ACCEPTED MODEL FIT	DIM1	DIM2	DIM3	DIM4	DIM5	DIM6
RMSEA	≤.06	0.12	0.076	0.073	0.072	0.046	0.044
RMR	≤.08	0.070	0.066	0.055	0.056	0.041	0.046
NFI	≥.95	0.89	0.93	0.94	0.93	0.96	0.96
NNFI	≥.95	0.89	0.93	0.94	0.94	0.97	0.97
IFI	≥.95	0.90	0.94	0.95	0.94	0.97	0.97
CFI	≥.95	0.90	0.94	0.95	0.94	0.97	0.97
RMSEA 90%	< 0.06;	0.12;	0.075;	0.072;	0.071;	0.045;	0.042;
CI	< 0.08	0.12	0.078	0.075	0.073	0.048	0.045
$\chi 2$	> 0.05	10089.46	6607.04	5007.38	6038.65	3028.80	3367.89
COMMENT		**Not Fit**	**Not Fit**	**Not Fit**	**Not Fit**	**Fit**	**Fit**

DIM = Dimension

Based on the results obtained from the EFA and PA, CFA was carried out on a sample of 1600 which is different from the initial 1008 samples used for EFA. The extracted 45 items from the result of EFA and PA was used for the analysis. Thesesubsequent six models were verified on the 45 items: Unifactorial model D1 (student mathematics engagement scale), bifactorial model D2 (Personal agency engagement and affective engagement), three correlated factors model D3 (Personal agencyengagement, affective engagement and congnitive engagement), four correlated factors model D4 (Personal agencyengagement, affectiveengagement, behaviouralengagement and congnitive engagement), five correlated factors model D5 (Personal agency, engagement, positive affectiveengagement, negative affectiveengagement, behaviouralengagement and negative behavioural), six correlated factor model D6 (Personal agency, engagement,positive affectiveengagement, negative affectiveengagement, positive behavioural engagement, negative behaviouralengagement and congnitive engagementengagement). These six models were estimated with lisrel.

Table 4.5 shows the model fit of the six factors model. From the table, the models with the best fit are D5 and D6, but D5 fitted after the remover of Cognitive items. With these results, D6 was chosen. Looking at model 6, the RMSEA, RMR, SRMR,NFI,NNFI,CFI and IFI were adequate, so the model was selected for item calibration.

Discussion:

Table 4.5 gives the detailinformation of the fit indices of student mathematics engagement scale. The table shows that student mathematics engagement scale is not one dimensional nor two or three, up to four-dimensional scale as the fit indices of the engagement scale was not significant under one to four dimensions. Even one cannot say that the scale shows good model fit under five dimensions as the model only fitted after the removal of cognitive engagement. So the results of table 4.5 clearly show that student mathematics engagement scale have six dimensions as all the fit indices under the 6-dimensional model satisfied the model fit criteria. Not only that, the result of model fit confirmed the result of Parallel analysis which retained six factors. However, this result supported the work of Atari and Jamail, (2016), where PA retained the same number of factor as the number of factors confirmed by CFA during the validation of their survey instrument.

4.6 Research Question 2b:Do the student Mathematics Engagement items show convergent validity?

To answer this research question,proper examination of standardized regression weight was done and computation of composite reliability was carried out. The result is shown on the table below.

Table 4.6: Validity Index of students Mathematics Engagement items:

Item		Factor(F)	LD	LD^2	1-(LD)^2	X=∑1-(LD)^2	∑FLD	Y=(∑FLD)^2	Z=∑ X&Y	CR=$\frac{Y}{Z}$
c17	<--	PERANG	**0.571**	0.32604	0.673959	6.985419	7.745	59.98503	66.97044	**0.895694**
c9	<--	PERANG	**0.643**	0.41345	0.586551					
c2	<--	PERANG	**0.685**	0.46923	0.530775					
c3	<--	PERANG	**0.665**	0.44223	0.557775					
c5	<--	PERANG	**0.675**	0.45563	0.544375					
c8	<--	PERANG	**0.588**	0.34574	0.654256					
c11	<--	PERANG	**0.668**	0.44622	0.553776					
c14	<--	PERANG	**0.686**	0.4706	0.529404					
c6	<--	PERANG	**0.673**	0.45293	0.547071					
c10	<--	PERANG	**0.639**	0.40832	0.591679					
c7	<--	PERANG	**0.641**	0.41088	0.589119					
c15	<--	PERANG	**0.611**	0.37332	0.626679					
d8	<--	POSAFF	**0.596**	0.35522	0.644784	3.832686	3.604	12.98882	16.8215	**0.772156**
d9	<--	POSAFF	**0.601**	0.3612	0.638799					
d7	<--	POSAFF	**0.636**	0.4045	0.595504					
d1	<--	POSAFF	**0.576**	0.33178	0.668224					
d2	<--	POSAFF	**0.615**	0.37823	0.621775					
d3	<--	POSAFF	**0.58**	0.3364	0.6636					
e6	<--	NEGAFF	**0.606**	0.36724	0.632764	5.185461	4.715	22.23123	27.41669	**0.810865**
e4	<--	NEGAFF	**0.632**	0.39942	0.600576					
e5	<--	NEGAFF	**0.656**	0.43034	0.569664					
e3	<--	NEGAFF	**0.657**	0.43165	0.568351					
e1	<--	NEGAFF	**0.598**	0.3576	0.642396					
e2	<--	NEGAFF	**0.607**	0.36845	0.631551					
e7	<--	NEGAFF	**0.48**	0.2304	0.7696					
e8	<--	NEGAFF	**0.479**	0.22944	0.770559					
f2	<--	POSBEH	**0.633**	0.40069	0.599311	1.814759	1.885	3.553225	5.367984	**0.661929**
f3	<--	POSBEH	**0.606**	0.36724	0.632764					
f1	<--	POSBEH	**0.646**	0.41732	0.582684					
g2	<--	NEGBEH	**0.537**	0.28837	0.711631	2.69714	2.268	5.143824	7.840964	**0.656019**
g1	<--	NEGBEH	**0.635**	0.40323	0.596775					
g3	<--	NEGBEH	**0.621**	0.38564	0.614359					
g4	<--	NEGBEH	**0.475**	0.22563	0.774375					
c16	<--	COGNIT	**0.69**	0.4761	0.5239	2.442775	2.487	6.185169	8.627944	**0.716876**
c12	<--	COGNIT	**0.624**	0.38938	0.610624					
h1	<--	COGNIT	**0.63**	0.3969	0.6031					
h2	<--	COGNIT	**0.543**	0.29485	0.705151					

Kimberlin and Winterstein (2008) note that, for a researcher to establish convergent validity, the relevant correlations between the measured variables and their latent construct must be considerably differ from zero and sufficiently large. Based on convergent validity principle, the loadings of factor that are less than 0.40 are feeble and the loadings of factor that are greater than 0.40 are robust for good convergent validity (Garson, 2010). Based on this criteria, the loading between the measured variables (items) and latent variable (factor) that are less than .4 were discarded, which resulted into 37 items that loaded on 6 factors. Also, composite reliabilities of the entire constructs ranged from 0.66 to 0.90.

Discussion:

Table 4.6 displays the standardized regression weight of each item of students mathematics engagement scale to their corresponding latent variable. All the loadings are greater than 0.4 (bolded under the column labeled 'LD'), which showed a good level of convergent validity. This result support the submission of Garson, (2010) who said that, the loadings of factor that are greater than 0.40 are robust for good convergent validity. Not only that, the composite reliability (CR) of each of the construct of student mathematics engagement scale was computed using the formula in appendix VII. The values of CR ranged from 0.66 to 0.90. These results supported the work of Ahmad, Zulkurnain and Khairushalimi (2016) and Hamid, Sami and Sidek (2017). They said that; for a good convergent validity, the value of CR for each construct should be between 0.60 to 0.90.

4.7 Research Question 3a: Are items of each of the dimensions of Student Mathematics Engagement scale unidimensional?

Unidimensionality of each of the dimension of student mathematics engagement scale was carried out to show the pattern of relationship between each construct and their corresponding items

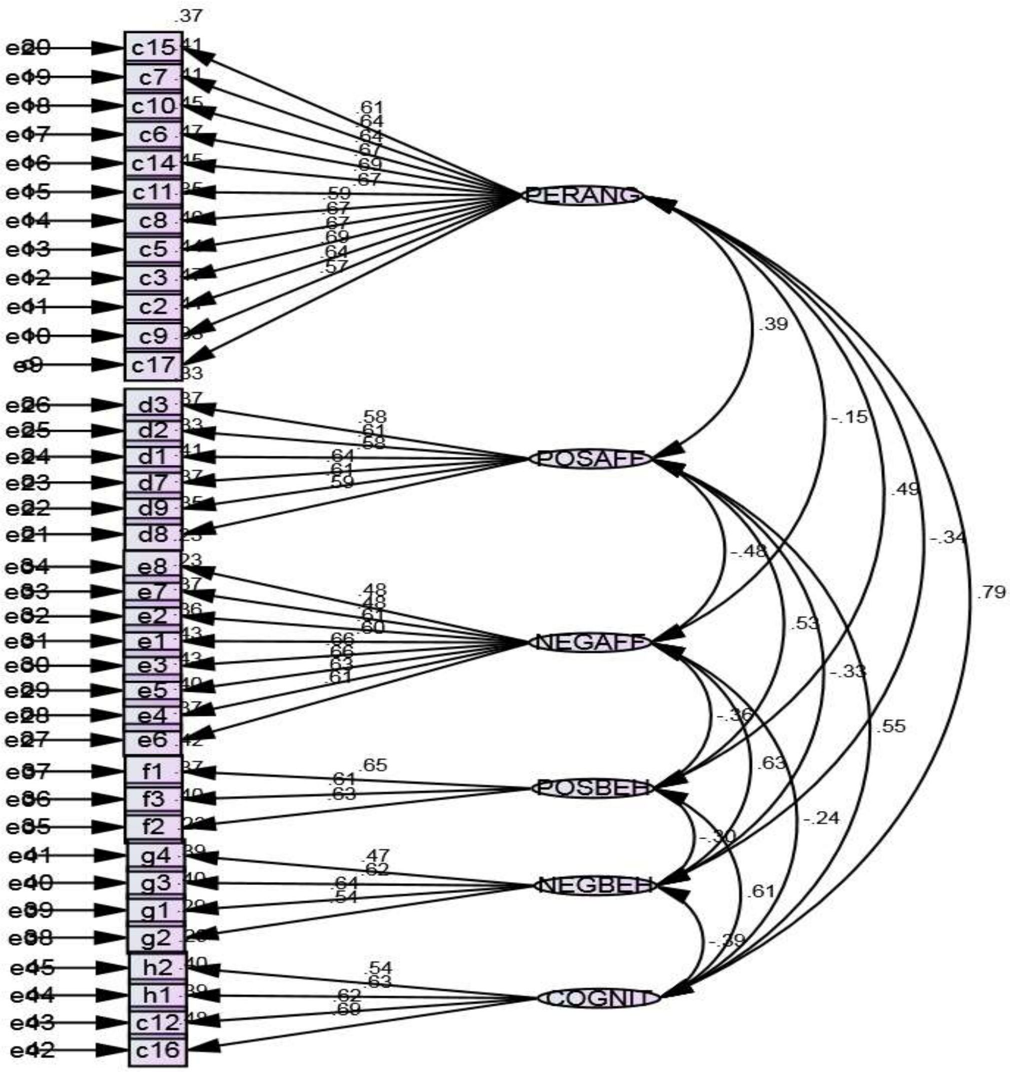

Figure 4.2: The unidimensionality of Student Mathematics Engagement items andtheir corresponding factors.

Table 4.8: unidimensionality of 37 items of Student Mathematics Engagement scale

Item			Estimate
c17	<---	PERANG	.571
c9	<---	PERANG	.643
c2	<---	PERANG	.685
c3	<---	PERANG	.665
c5	<---	PERANG	.675
c8	<---	PERANG	.588
c11	<---	PERANG	.668
c14	<---	PERANG	.686
c6	<---	PERANG	.673
c10	<---	PERANG	.639
c7	<---	PERANG	.641
c15	<---	PERANG	.611
d8	<---	POSAFF	.596
d9	<---	POSAFF	.601
d7	<---	POSAFF	.636
d1	<---	POSAFF	.576
d2	<---	POSAFF	.615
d3	<---	POSAFF	.580
e6	<---	NEGAFF	.606
e4	<---	NEGAFF	.632
e5	<---	NEGAFF	.656
e3	<---	NEGAFF	.657
e1	<---	NEGAFF	.598
e2	<---	NEGAFF	.607
e7	<---	NEGAFF	.480
e8	<---	NEGAFF	.479
f2	<---	POSBEH	.633
f3	<---	POSBEH	.606
f1	<---	POSBEH	.646
g2	<---	NEGBEH	.537
g1	<---	NEGBEH	.635
g3	<---	NEGBEH	.621
g4	<---	NEGBEH	.475
c16	<---	COGNIT	.690
c12	<---	COGNIT	.624
h1	<---	COGNIT	.630
h2	<---	COGNIT	.543

To answer this research question, the 37 items that show convergent validity were verified to ascertain the unidimensionality of the items to their respective factors through confirmatory factor analysis. After this verification, the result reviewed that each of the item identified with a factor without cross loading of the item onto two or more factors. Figure 4.2 shows the item that unidimensionally loaded onto their respective factor. Furthermore, Table 4.8also confirmed the standardised regression weight (loading) of individual item to their corresponding factor.The standardised regression weight displayed the level of relationship between the individual item and their corresponding factor. The table has also showed that each item unidimensionally loaded onto their respective factor.

Discussion

The result of Figure 4.2 and Table 4.8 show that all the item under each of the construct of students mathematics engagement scale have strong relationship with their underline latent variable**.** No cross loading of items onto two or more factors. Each of the item unidimensionally loaded onto its corresponding factor.The result also shows that student mathematics engagement scale have six dimensions as confirmed by Parallel analysis.

4.8 Research Question 3b: To what extent are the Student Mathematics engagement items locally independent of one another using graded response mode of IRT framework?

Pearson moment correlation, polychoric test of model and polyserial test of close fit were carried out to check whether the items of students mathematics engagement scale are locally independent of their own.

Table 4.9:Correlation and Test of statistics for 37-SME Items

PE=Pearson Product Moment Model				PC=Polychoric Test of		PS=Polyserial Test of Close Fit	
Variable	**VS**	**Variable**	**Correlation**	χ^2**(LD)**	**P-Value**	**RMSEA**	**P-Value**
1	VS	2	0.456	12.888	0.116	0.020	1.000
1	VS	3	0.528	42.123	0.000	0.052	1.000
1	VS	4	-0.288	52.217	0.000	0.059	1.000
2	VS	3	0.509	49.902	0.000	0.057	1.000
2	VS	4	-0.154	36.175	0.000	0.047	1.000
3	VS	4	-0.215	30.164	0.000	0.042	1.000
+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+
5	VS	4	0.489	21.849	0.005	0.003	1.000
6	VS	1	-0.152	40.661	0.000	0.051	1.000
6	VS	2	-0.136	45.582	0.000	0.054	1.000
6	VS	3	-0.157	32.206	0.000	0.043	1.000
6	VS	4	0.222	36.870	0.000	0.047	1.000
6	VS	5	0.210	7.691	0.464	0.000	1.000
+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+
37	VS	34	0.465	42.687	0.000	0.052	1.000
37	VS	35	0.523	37.014	0.000	0.048	1.000
37	VS	36	0.532	64.333	0.000	0.066	1.000

Table 4.9 shows the summary of Pearson Product Moment Correlation and Test of statistics among the retained items of SMES. The retained 37-SMES resulted to 648 Correlations which explained the magnitude of the relationship between each pairs of items. From the table, the correlation between item 1 and 2 was 0.456, item 1 and 3 was 0.456, item 1 and 4 was -0.288, item 2 and 3 was 0.509. This correlation process goes on through all the 37 items. Not only that, Table 4.9 also shows test of statistics of the items. The test of statistics assesses the overall model-data fit of ordered polytomous graded Model and Standardized local dependence (LD) $\chi 2$ of everycouple of variables. (LD) $\chi 2$ assesses the local dependence amongtwo items that are present in a scale.

To evaluate Standardized local dependence (LD) $\chi 2$ of each pair of items, Tay et al. (2015) posited that for items to be locally independent,the value of (LD) $\chi 2$ must be larger than 3. For this work, all the 648 pairs of items in SMES have their values greater than 3 (3.092 to 95.671) except item 37 VS item 16, item 18 VS item 10, item 26 VS item 10with values 2.723, 1.516 and 2.305 respectively, which are very minute compare to the remaining 645 pairs. These results show that items in SMES are locally independent.More so, Tay et al. (2015) also suggested that non-significant p-values of over 0.05 and RMSEA that are close to zero show good models fit. For this work, all the values of P are greater than 0.05 (0.969 to 1.000) and all the values of RMSEA are near zero, i.e from 0.000 to 0.070 which indicate a good fit. These results show that SMES is a measure of students' engagement in Mathematics.

Discussion

The result of table 4.9 shows that the 37-items of SMES have 648 pairs of correlations that range from 0.001 to 0.606. The result shows that all the items are meanifully related. Meaning that, they are all measuring students' engagement in Mathematics. Not only that, the value of Standardized local dependence (LD) $\chi 2$ of all the 648 pairs of items in SMES have their values greater than 3 (3.092 to 95.671) except item 37 VS item 16, item 18 VS item 10, item 26 VS item 10with values 2.723, 1.516 and 2.305 respectively, which are very minute compare to the remaining 645 pairs. This result shows that all theitems of students mathematics engagement scale are locally independent. This work support the work of Tay et al. (2015) who posited that for items to be locally independent, the value of (LD) $\chi 2$ must be larger than 3.

4.9 Research Question 3c: How many items were selected as good items during calibration process?

Polytomous Graded Response Model of IRT framework was carried out to assess this research question.

Table 4.10:Item Parameter Estimates of Graded model of four-category SME scale.

Items	Label	a_i	s.e	b_i	s.e	Items	Label	a_i	s.e	b_i	s.e
1	a1	**0.15***	0.07	2.06	0.08	20	c2	1.61	0.11	-1.36	0.16
2	a2	0.79	0.00	0.32	0.00	21	c3	1.32	0.09	**-3.78***	0.15
3	a3	0.69	0.07	1.88	0.09	22	c4	1.52	0.10	-0.89	0.20
4	a4	0.75	0.08	1.33	0.10	23	c5	1.86	0.14	-2.36	0.22
5	a5	1.21	0.10	0.43	0.09	24	c6	1.52	0.10	-0.52	0.15
6	a6	0.79	0.07	0.86	0.14	25	c7	1.53	0.13	-2.19	0.18
7	a7	1.04	0,08	0.56	0.08	26	c8	1.18	0.08	-1.87	0.12
8	a8	0.64	0.06	1.17	0.10	27	d1	0.53	0.05	1.37	0.06
9	a9	0.54	0.07	1.42	0.07	28	d2	0.74	0.09	0.86	0.07
10	a10	0.66	0.06	1.71	0.08	29	d3	0.50	0.12	0.91	0.06
11	a11	0.62	0.07	0.12	0.07	30	e1	1.55	0.58	-1.65	0.16
12	a12	0.59	0.06	1.42	0.08	31	e2	1.26	0.09	-1.45	0.13
13	b1	1.12	0.12	2.44	0.14	32	e3	1.07	0.07	-1.09	0.10
14	b2	1.64	0.14	2.69	0.17	33	e4	0.76	0.07	-1.28	0.13
15	b3	1.10	0.13	1.17	0.10	34	f1	0.62	0.09	1.45	0.12
16	b4	1.06	0.11	2.29	0.13	35	f2	0.54	0.07	1.21	0.07
17	b5	0.66	0.10	0.99	0.07	36	f3	0.50	0.06	2.86	0.08
18	b6	1.06	0.08	1.10	0.08	37	f4	0.96	0.07	1.07	0.06
19	c1	2.19	0.14	-2.15	0.28						

To further confirm the retained factors with their corresponding measured variables after CFA, the retained items were subjected to analysis of graded response model (GRM, Samejima 1969, 2010) of IRT framework after the verification of unidimensionality and local independence of student mathematics engagement items. The results revealed that the slope parameters (a_i discrimination parameters) and Threshold parameters (b_i difficulty parameters) for the graded response model fit to the 35-item four-category of SME scale out of 37items confirmed by Confirmatory factor analysis. According to the classification of Baker, 2001; De Ayala, 2009 and Toland, 2013, the range of model fit for the slope parameter (discrimination parameters) and threshold parameters (difficulty indices) for ordered polytomous graded response IRT models, are 0.5 to 3 and -3 to 3 respectively.

Discussion:

Table 4.10 summarizes the item calibration. From the table, all the discrimination parameters (a_i) were adequate with the exception of items 1 (a1) which have a value of **0.15** and item 21 (c3) with the value of **-3.78** for threshold parameter. These two Items were removed. However, the range of a_i includes: 0.54 to 1.21 for factor 1(a2 to a12), then 0.66 to 1.64 for factor 2 (b1 to b6), 1.32 to 2.19 for factor 3 (c1, c2, c4 to c8), 0.50 to 0.74 for factor 4 (d1 to d3), 0.76 to 1.55 for factor 5 (e1 to e4), and 0.50 to 0.96 for factor 6 (f1 to f4). The values of standard errors (s.e) of (ai) parameters were very small, which range from 0.03 to 0.14. Also, the thresholds parameter of the retained items ranges from -2.36 to 2.86 and the s.e of (b_i) parameters were also very small.It ranges from 0.06 to 0.28. This result shows that the remaining Thirty-five items of student mathematics engagement scale represent good measure of students' level of engagement in Mathematics.

4.10 Research Question 4: What are the discriminant validity indices of the identified factors of students Mathematics Engagement scale?

Average variance extracted was carried out using the formula in appendix VII to check this research question out.

Table 4.11: Discriminant validity indices of the identified factors

	PERANG	POSAFF	NEGAFF	POSBEH	NEGBEH	GOGNIT
PERANG	**0.6464**	0.0.336	0.116	0.377	0.269	0.636
POSAFF		**0.6010**	0.373	0.372	0.229	0.425
NEGAFF			**0.5931**	0.258	0.459	0.168
POSBEH				**0.6286**	0.193	0.416
NEGBEH					**0.5707**	0.269
GOGNIT						**0.6239**

Table 4.11 assesses the discriminant validity of student mathematics engagement scale (SMES). Discriminant validity is established when the value of Average Variance Extracted (AVE) of one latent variable in a model is larger than the maximum squared correlation between that construct and other constructs in that model or when the square root of AVE is greater than

the maximum correlation between a construct and other constructs in a model (Hair *et al.,* 2014), (when within construct variance is greater than the shared variance).

Discussion:

In table 4.11, the square root of the AVE of the six sub scale of SMES are arranged on the diagonal with bolded values, while the correlation of each of the construct with other constructs are arranged off diagonal row by column. Personal Agency Engagement has 0.6464as the value of its square root of AVE which is larger than its correlation with other constructs (‘r’ ranges from 0.116 to 0.636). Also, Positive Affective Engagement has 0.6010as the value of its square root of AVE which is larger than its correlation with other constructs (‘r’ ranges from 0.229 to 0.425). Furthermore,Negative Affective Engagement has 0.5931 as the value of its square root of AVE which is larger than its correlation with other constructs (‘r’ ranges from 0.116 to 0.459).

Moreover, Positive behavioural Engagement has 0.6286as the value of its square root of AVE which is larger than its correlation with other constructs (‘r’ ranges from 0.193to 0.416). Then, Negative Behavioural Engagement has 0.5707as the value of its square root of AVE which is larger than its correlation with other constructs (‘r’ ranges from 0.193 to 0.459), except Cognitive Engagementwhich has 0.6239as the value of its square root of AVE larger than its correlation with all other constructs, except Personal agency Engagement with value of 0.636. Correlation of Cognitive Engagementwith other constructs ranges from 0.168 to 0.425.From this result,it is clear that Cognitive Engagement has the value of the square root of its AVE lesser than its correlation with just Personal Agency Engagementalone. In view of this, one can boldly say that all the sub-scales of SMES have discriminant validity indices.

4.11 Research Question 5:Is there any relationship between the identified factors of Students Mathematics Engagement scale?

Sample correlation coefficient analysis was use to check this out.

Table 4.12: Sample correlation coefficient between all pairs of factors

Sample Correlation			**Estimate**
PERANG	<-->	POSAFF	.336
PERANG	<-->	NEGAFF	-.116
PERANG	<-->	POSBEH	.377
PERANG	<-->	NEGBEH	-.269
PERANG	<-->	COGNIT	.636
POSAFF	<-->	NEGAFF	-.373
POSAFF	<-->	POSBEH	.372
POSAFF	<-->	NEGBEH	-.229
POSAFF	<-->	COGNIT	.425
NEGAFF	<-->	POSBEH	-.258
NEGAFF	<-->	NEGBEH	.459
NEGAFF	<-->	COGNIT	-.168
POSBEH	<-->	NEGBEH	-.193
POSBEH	<-->	COGNIT	.416
NEGBEH	<-->	COGNIT	-.269

Sample correlation coefficient between all pairs of factors was done to find the pattern of relationships between the factors. The result of table 4.12 and path diagram in figure 4.4 showed that the sample correlation between the factors ranges between -.373 to 0.636, meaning that the factors correlate well, and there exist connection among the latent variables. In other words, the items in the extracted factors are likely to be measuring the same trait.

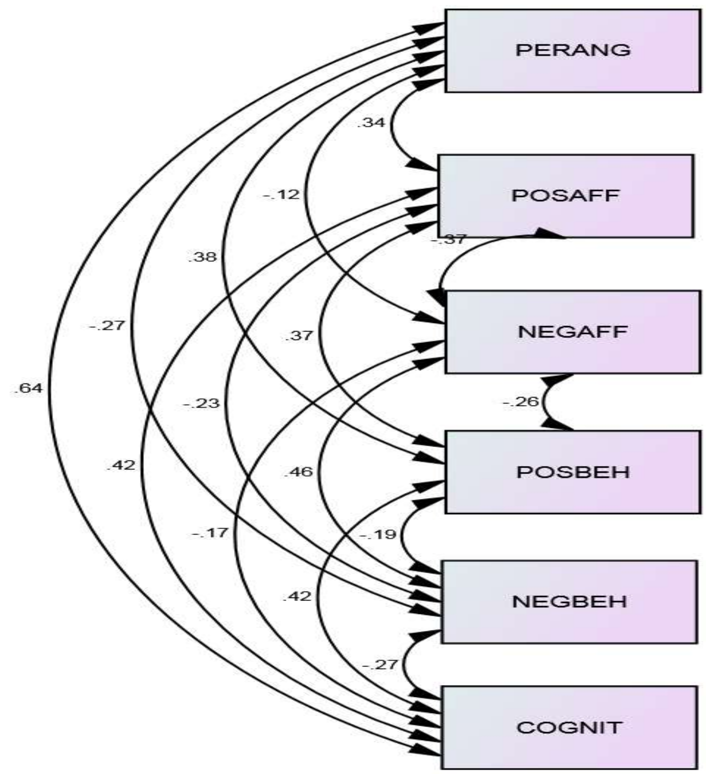

Figure 4.3:Path diagram of sample correlation between the identified factors of studentMathematics Engagement scale

Discussion:

The path diagram of (Figure 4.3) represents the pictorial representation of table 4.11 which shows the correlation between the identified factors of student mathematics engagement scale. The table and the path diagram clearly show that all the sub-scales of student mathematics engagement scale are measuring the level of students' engagement in Mathematics.

4.12: Research Question 6a:Is the student Mathematics Engagement scale reliable?

Ordinal alpha coefficient analysis was use to carry this out.

Table 4.13a: Reliability coefficient Statistics of the extracted items

Ordinal Alpha	**Cronbach's Alpha**	**Cronbach's Alpha Based on Standardized Items**	**No of Items**
0.90	0.814	0.806	35

Table 4.13ashowsthe Ordinal alpha coefficient,which revealed the reliability coefficient of the entire scale of student mathematics engagement scale. The value of 0.90 shows that the entire scale of studentmathematics engagementscale is highly reliable.

Discussion:

Ordinal alpha coefficient analysis was used to carry out the internal consistency of student mathematics engagement scale. The choice of Ordinal alpha rather than Cronbach's alpha was due to the inability of Cronbach's alpha to accurately estimate the true relationshipof items with ordinal data when a scale contains intercorrelatedfactors(Anne, Gadermann, Guhn and Zumbo, 2012), and also, it cannot be said that Cronbach's alpha measure internal consistency or unidimensionality of items with ordinal data (Sijtsma, 2009).

Table 4.13b showing the stability of Student Mathematics Engagement Scale

Table 4.13b: internal consistency of Students Mathematics Engagement Scale

	Internal consistency	Ordinal Alpha	Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	Mean	Standard deviation	No of Items
First Testing	Students mathematics engagement items	**0.89**	0.813	0.787	88.75	11.858	35
Second Testing	Studentsmathematics engagement items	**0.92**	0.817	0.808	87.05	11.553	35
Third Testing	Students mathematics engagement items	**0.90**	0.814	0.806	87.24	11.468	35

Table 4.13b showed the Reliability statistics of the first, second and third testing of student mathematics engagement scale.

Discussion:

The scale was tested on the same sample of 600 subjects at two different periods and also tested on a different sample of 1008 subjects to measurethe internal consistency of the scale. TheOrdinal Alpha coefficient reliabilities of the scale were0.89, 0.92, 0.90, respectively,while the Cronbach's Alpha reliabilities were 0.813, 0.817 and 0.819,respectively. However, the high and the similarity in the value of reliability coefficientat different periods of testing showed that the scale possessed good internal consistency over time and also, highly reliable. Not only that, the value of mean (88.75, 87.05 and 87.24) and the standard deviation (11.858,11.553 and 11.468)of the scale at three differenttesting are very similar, which also showed the consistency of the scale.

4.13: Research Question 6b: How reliable are each of the sub-scale of students Mathematics Engagement scale?

Ordinal alpha coefficient analysis was used to carry this out.

Table 4.14:Reliability of all the 6 sub-scale of Students Mathematics Engagement scale?

Factor	Name	Ordinal Alpha	Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	No of Items
Factor 1	Personal Agency Engagement	0.87	.846	.846	11
Factor 2	Positive Affective Engagement	0.69	.716	.726	6
Factor 3	Negative Affective Engagement	0.73	.843	.847	7
Factor 4	Positive Behavioural Engagement	0.68	.676	.687	3
Factor 5	Negative Behavioural Engagement	0.73	.724	.727	4
Factor 6	Cognitive Engagement	0.77	.691	.695	4

The reliability of all the sub-scale of students mathematics engagement scale ranges from 0.68 to 0.87. This shows that all the sub-scales of student mathematics engagement scale are reliable.

4.14 Research Question 7: Which of the sub-scales of Students Mathematics Engagement scale is the best predictor of Mathematics achievement?

Table 4.15a:Level of prediction of regression model

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.198[a]	.039	.034	6.879

a. Independent variable: Congnit_Eng, Neg_Aff_Eng, Pos_Beh_Eng, Per_Ang Eng, Pos_Aff_Eng, Neg_Beh_Eng

b. Dependent Variable: MATHS TEST

Table 4.15a provides the *R* and R^2.These wereused to definethe level of fitness at which regression model predict dependent variable. The value of Rdenotes the value of multiple correlation coefficients. In this case, Rrepresents the measures of quality of the prediction of Mathematicsachievement test. Here, the value of .198shows the level of prediction ascribed to Mathematics achievement test. The "R Square" shows the squaredmultiple correlation coefficients.This refers to the amount of variance in the dependent variable that can be accounted for by the independent variables. From the table, all the latent variables explain 39% of the variation in the Mathematicsachievement. This result indicates a good level of prediction which shows the fitness of the regression model.

Table 4.15b:The regression model fits

ANOVA[a]

Model		**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
1	Regression	1936.507	6	322.751	6.821	.000[b]
	Residual	47364.350	1001	47.317		
	Total	49300.857	1007			

a. Dependent Variable: MATHS TEST

b. Predictors: (Constant), Congnit_Eng, Neg_Aff_Eng, Pos_Beh_Eng, Per_Ang Eng, Pos_Aff_Eng, Neg_Beh_Eng

Table 4.15b shows the F column. The Fexamined how well the regression model fit the data. The table displays that the measured variables significantly predict the mathematics achievement, F (6, 1001) = 6.821, *p*<.05 (That is, the regression model have a good data fit model.).

Table 4.15c: Statistical significance of sub-scale of Student Mathematics Engagement

Model		**Unstandardized Coefficients**		**Standardized Coefficients**			**95.0% Confidence Interval for B**	
		B	**Std. Error**	**Beta**	**t**	**Sig.**	**Lower Bound**	**Upper Bound**
1	(Constant)	22.561	2.057		10.968	**.000**	18.525	26.598
	Per_Ang Eng	-.050	.039	-.048	-1.287	**.198**	-.126	.026
	Pos_Aff_Eng	.130	.090	.056	1.446	**.148**	-.046	.307
	Neg_Aff_Eng	-.113	.073	-.064	-1.541	**.124**	-.257	.031
	Pos_Beh_Eng	.162	.126	.047	1.284	**.200**	-.086	.410
	Neg_Beh_Eng	-.302	.102	**-.115**	-2.952	**.003**	-.502	-.101
	Congnit_Eng	-.082	.096	-.033	-.859	**.390**	-.270	.106

MATHS TEST

Unstandardized coefficients specify the extent to which Mathematics achievement varies with the measured variable when all other measured variables are held constant. Table 4.15c examined whether the value of unstandardized/standardized coefficient is equal to 0 in the distribution. If *p* is less than .05, this suggests that the coefficients are significantly different from 0. The *p*-value isfound in the "**Sig.**" columns. The result of table 4.15c shows that Negative Behavioural Engagement is statistically significantly different from 0 (zero), which means that it has a unique contribution to the level of students' performance in Mathematics with the value of $P = 0.003 < 0.05$. Not only that, figure 4.5gives the pictorial representation of regression model of students' mathematics achievement and the six dimensions of student mathematics engagement. However, with the result of table 4.15c and the graph in figure 4.5, Negative BehaviouraL Engagement appeared the best predictor out of the six sub-scales of student mathematics engagement scale with the value of Bêta = -.115.

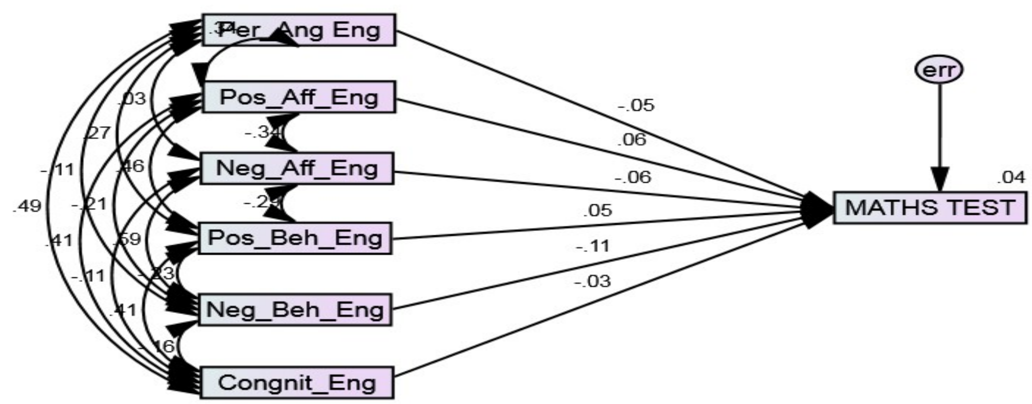

Figure 4.4: Pictorial representation of students' mathematics achievement and the six-dimensions of students mathematics engagement scale.

Discussion:

After the regression analysis of each of the sub-scale of student mathematics engagement scale was conducted on Mathematics achievement, an F-value of table 4.15b was showed to be statistically significant;F (6, 1001) = 6.821, P =0.000<.05. This implied that at least one of the model (sub-scale of student mathematics engagement scale) explains a significant amount of variance in the outcome variable (Mathematics achievement). That is, at least one of the sub-scales of student mathematics engagement scale has a unique contribution to the level of student performance in Mathematics. With this information, table 4.15a was inspected to know the amount of contribution. Looking at table 4.15a, the R^2 statistics, which is the coefficient of determination, showed that 39% of variance in the outcome variable (Mathematics achievement) was explained by a set of predictor variables (sub-scales of student mathematics engagement scale).

Not only that, table 4.15c was examined for the value of regression beta coefficients and t-value. The beta coefficient is the degree of change in the outcome variable for every 1-unit of change in the predictor variable (Wang, Tang and Tan(2011). In this wise, the beta coefficients can be negative or positive, and always have a t-value and significance of the t-valuethat are associated with each. Here, thet-testwas used to assess whether the beta coefficient is significantly different from zero. If the beta coefficient is not statistically significant (i.e., the t-value is not significant), it means that the variables (sub-scales of student mathematics engagement scale) does not significantly predict the outcome (Mathematics achievement).

In this regression output, since the beta coefficient is significant, the sign of beta was examined. If the beta coefficient is positive, it implies that for every 1-unit increase in the predictor variable, the outcome variable will increase by the beta coefficient value. If the beta coefficient is negative, it implies that for every 1-unit increase in the predictor variable, the outcome variable will decrease by the beta coefficient value. So, for this work, beta coefficient has negative values of -0.115 for Negative Behavioural Engagement, $P = 0.003 < 0.05$, which means that only Negative Behavioural Engagement predicted Students achievement in mathematics. The value of beta which is -0.115, implies that, this research work have been able to discover that, for every 1-unit increase in student Negative Behavioural engagement in Mathematics, there will be 12% decrease in the students' performance in Mathematics. So, the equation becomes; $y= 0.12x + c$, where y is the outcome variable (Mathematics Achievement), x is the predictor variable (Student Mathematics Engagement), 0.12 is the beta coefficient, and c is the constant.

CHAPTER FIVE

SUMMARY OF FINDINGS, IMPLICATIONS, LIMITATION, CONCLUSIONS, RECOMMENDATIONS AND SUGGESTION FOR FURTHER STUDIES

This chapter discusses the summary of findings, implications, limitation of the study, conclusions, recommendations, and suggestion for further studies.

5.1. Summary of Findings

The mainoutcomes of this research are as follows:

After the Exploratory factor analysis, Parallel analysis retained six (6) factors instead of eleven (11) factors retained by eigenvalue greater than one and over estimation of numbers of factors by scree test methods which justified the claim of past researchers who said that eigenvalue larger than one and scree test methods tend to over factor.

The Constructed student mathematics engagement items fitted to six Models instead of 3 or 4 as mentioned by some student engagement in School scale developers.

The Constructed student mathematics engagement items were unidimensional. This implies that the test measures only one latent trait (Students Mathematics Engagement).

The Constructed student mathematics engagement test items were locally independent of one another. This implies that each item in the test did not give a clue to the examinee in answering another item.

Polytomous graded response model of IRT framework were used during calibration process after the confirmatory factor analysis which removed two (2) items. This implies that there is need for items calibration for further confirmation of the retained items.

After the items calibration, 35 items were distinctly loaded on 6 sub-scales namely: Personal Agency Engagement, Positive Affective Engagement, Negative Affective Engagement, Positive Behavioural Engagement, Negative Behavioural Engagement and Cognitive Engagement.

The validity index of studentmathematics engagement items ranges from 0.475to 0.690. The value clearly indicated that the items of the scale were meaningfully related and contributed to the construct being measured.

The reliability coefficient of the whole items of studentmathematics engagement scalewhich have 35 items was found to be 0.90, and the reliability coefficient of each of sub-scale of studentmathematics engagement scale ranges between 0.68 and 0.87 which shows that both the entire scale and the sub-scale of student mathematics engagement scale were highly reliable.

The six sub-scales were utilizedto evaluate the level of student engagement in mathematics. The result showed that Negative behaviour had the largest influence on the achievement of students in Mathematics.

5.2Implications

This study has implications for Engagement items developers, other test developers, school management, teachers and students. The use of sophisticated statistical tool, that is, EFA, PA, CFA and polytomous graded response model of the IRT framework during the development and validation of a survey instrument will produce a solid and robust instrument. Thus, the polytomous graded response model of the IRT framework is more effective in the calibration of survey items with three or more response mode, as it enhances selection of items that best measure students' practices and also gives adequate information concerning the behaviour of an item as well as the examinees. Another implication of the findings is that the usage of student mathematics engagement scale has shown the best predictor of students' Mathematics achievement among the sub-scale of student mathematics engagement scale.

5.3 Limitation

The researcher would have loved to sample the entire senior secondary school Mathematics students in Ekiti State, but the cost of printing test booklets and answer sheets was high thus, this sample was used.

5.4 Conclusions

A valid and reliable instrument was constructed by gathering information from a total sample of 3,616 senior secondary school 2 students, from both public and private schools in Ekiti State by survey design.The sample was selected through a multistage cluster sampling procedure. The initial 100 items of student mathematics engagement scale were generated from three sources viz: the item pool of students statement about their engagement in Mathematics which was collected by the researcher through an open-ended questionnaire from the representative of the target population; from other secondary schools Mathematics teachers based on their experiences and from the researcher, based on her experiences as a secondary school Mathematics teacher.

The analysis of the results were in threephases viz: the exploratory factor analysis, which showed the loading of identified item to their corresponding factor; parallel analysis, which showed the amount of latent variable to keep;CFA, which tells how well the identified latent variable fit the hypothesized data and the unidimensionality of the retained items; IRT frame work, which examined whether the retained items are locallydependentor locally independent as well as the selection of the final items. Finally, the usage of the scale showed the best predictor of students' achievement in Mathematics out of the six sub-scales of student mathematics engagement. At the first analysis, 24 factors were extracted, but only 11 factors that had a minimum of three items loaded on each of themwere retained for further verification. These 11factors contain64 items. At the second analysis, the 64 items were reduced by the parallel analysis to 45 items. These 45 items were further subjected to Confirmatory factor analysis which latter reduced to 37 items. These 37 items were subjected to Polytomous graded response model of IRT frame work for item calibration. The calibration process reduced the items to 35. Finally, these 35 items unidimensionally loaded on 6 factors (sub-scales) of student mathematics engagement scale.

These factors are;Personal Agency Engagement, Positive Affective Engagement, Negative Affective Engagement, Positive Behavioural Engagement, Negative Behavioural Engagement and Cognitive Engagement. The correlation between the factors ranged between -.373 and 0.636, which shows that they areexpected to be measuring student Mathematics engagement. The reliability of the entire scale was 0.90 meaning that the items in the student mathematics engagement scale are highly reliable. Also, each of sub-scale of student mathematics engagement are highly reliable with α ranges from 0.68 to 0.87 and the validity index of the retained items ranges from 0.475 to 0.690. The values clearly show that the variables of student mathematics engagement scale were meaningfully connected and contributed to the construct being measured. Hence, the finding in this work offersconcrete support for the adequacy of student mathematics engagement scale as a measure of student engagement in Mathematics.

5.6 Recommendations

Based on the results of this work, the following suggestions were made:

Educators and researchers who may be willing to develop and validate survey instruments should make use of sophisticated statistical tool, that is, exploratory factor analysis, Parallel analysis, confirmatory factor analysis and polytomous graded response model of the IRT framework during the development and validation of their instruments, as this will produce a solid and robust instrument.

The validated scale can be used to develop a way for schools to comply with learning assessment standards and not to depend on the use of standardised achievement tests alone.

The validated scale can be used by stakeholder to investigate the attitudes, perceptions, and beliefs of students about Mathematics and how an adjustment can be made to improve the teaching and learning of Mathematics in Nigeria.

The scale can also help to motivate teachers to ask more open-ended questions for classroom discussions to improve student engagement in critical thinking during Mathematics class and also strengthen the connection between teachers and students in Mathematics class.

Furthermore, the validated scale can be used by the researcher, the ministry of education officials and other stakeholders in educational measurement and evaluation who may be interested in measuring students'engagement in Mathematics in secondary school.

The study will also add to the array of literature on scale construction and validation in Nigeria.

5.7 Suggestion for Further Studies

Studies of this nature can be carried out in other parts of Nigeria on other subjects.

Studies of this nature can be carried out in other parts of Nigeria using other constructs rather than student engagement. For example, student's attitude.

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APPENDIX I

INTERNATIONAL CENTRE FOR EDUCATIONAL EVALUATION
UNIVERSITY OF IBADAN, IBADAN.
ITEMS POOLS OF STUDENT MATHEMATICS ENGAGEMENT SCALE

LGA: ____
SHOOL CODE:____________________________________
CLASS: ____________ AGE: ____________ SEX: MALE ☐ FEMALE ☐

The following statements are meant to find out your current engagement with Mathematics. For each statement, choose the response that is closest to your current practices in

the appropriate column. Try and be as frank and truthful as possible. Given the correct information will assist the researcher to help you better. Please tick ( ) in front of each statement. NOTE: Do not write your name and the name of your school, so that you will be able to give the correct information about your engagement with Mathematics. Also note that five periods is the Standard periods that is allotted to Mathematics in a week. With this information;
All the time: means 5 times in a week. **Most of the time:** means 3 to 4 times in a week. **Sometimes:** means 1 to 2 times in a week. **Almost never:** means below 1 time in a week.

NO	STATEMENT	RESPONSES			
		All the time	**Most of the time**	**Sometimes**	**Almost never**
1	I pay attention to what my teacher teaches me during Mathematics class.				
2	I have natural skills to solve Mathematics problems.				
3	I do Mathematics homework after school				
4	I do all the Mathematics assignment given to me by my teacher				
5	I answered questions posed to me by my teacher during Mathematics class				
6	I concentrate on other things during Mathematics class because I don't understand what my teacher is teaching me				
7	I don't come to Mathematics class at all.				
8	I ask my friends to reteach me what my teacher is teaching during mathematics class so that I can learn better.				
9	I don't study my Mathematics notebook before coming to Mathematics class				
10	I don't interact during Mathematics class				
11	I do Mathematics homework where there is no distraction so that I can concentrate on whatI am doing				
12	I don't go over to study what my teacher teaches me during Mathematics lesson				
		All the time	**Most of the time**	**Sometimes**	**Almost never**
13	I do not respond at all to the questions asked by my teacher during Mathematics class.				
14	I follow rules and regulation during Mathematics class.				
15	Myself and my friends solve Mathematics problems together.				
16	I attend Mathematics extra lesson each week for more than one hour.				

17	I don't ask questions during Mathematics class.				
18	I cram my Mathematics note any time I have test or Exam.				
19	I disrupt the class during Mathematics lesson because what my teacher is teaching me in Mathematics is not clear to me.				
20	I do not understand and follow directions during Mathematics class				
21	I complete Mathematics classwork during Mathematics class				
22	Any time I teach my friend a topic in Mathematics I understand the topic better.				
23	I work on Mathematics each day for more than one hour.				
24	I get to Mathematics class on time				
25	I respond promptly to questions during Mathematics class				
26	I like my teacher because he/she uses different method to teach me a topic in Mathematics.				
27	If I did not get solution to the problems I am solving in Mathematics at the first attempt I get discouraged				
28	I don't have interest in what my teacher is teaching me during Mathematics class.				
29	I like to solve Mathematics problems during Mathematics class.				
30	I get disturbed and unhappy whenever my Mathematics teacher entered the class to teach Mathematics.				
31	I feel happy to do Mathematics when am encouraged verbally by somebody.				
32	When my friend solve Mathematics question during Mathematics class, I feel I can solve the question also				
33	When my peers perform better than meconsistently in Mathematics class, I feel discourage.				
34	When I am in good mood I do better in Mathematics.				
35	I don't have interest in school because of Mathematics				
36	I don't participate in Mathematics class because I feel that my teacher will embarrass me				
37	I don't have interest in most of the topics we learn during mathematics lesson.				
		All the time	**Most of the time**	**Sometimes**	**Almost never**
38	Any time my teacher gives me different questions to solve on a Mathematics topic I feel happy				
39	I love to do well in Mathematics class				
40	I feel that Mathematics is very difficult because of what people are saying about Mathematics.				
41	I solve Mathematics problems because of the pressure I received from my Mathematics teacher				
42	I enjoy staying in Mathematics class				

43	Learning during Mathematics class is important to me				
44	I choose easy problems to do in Mathematics.				
45	Any time my teacher uses teaching aid to teach me during Mathematics class I feel happy				
46	Any timemy family/guardian(s) provide me with the material I need to pass Mathematics I feel happy.				
47	I solve Mathematics problems because I need it for my future carrier				
48	If I don't get any question in Mathematics, I try to work on it until am able to solve it				
49	before a quiz or examination in Mathematics, I work hard for me to pass				
50	I solve several problems on the topic my teacher is teaching me in my text book for me to understand the topics better.				
51	I study previously solved problems for me to pass a Mathematics test or Examination				
52	I am willing to solve a difficult problem in Mathematics class				
53	My desire is to have a good mark in Mathematics				
54	I have desire to learn during mathematics lesson.				
55	I list out all the topics inside my text book so that I can learn more in Mathematics				
56	I solve problems in Mathematics to be sure whether I know some of the topic in mathematics.				
57	I work on my Mathematics text book for me to go ahead of my teacher in the class room.				
58	I am hopeful about my future because of Mathematics				
59	I solve any Mathematics problems that is related to what my teacher is teaching me on my own				
60	I solve my homework problems in Mathematics in a separate book before writing it in my note.				
61	I try as much as possible to solve my mathematics homework, even when I found it difficult to understand.				
62	I try to understand what my teacher is teaching me in mathematics class, even though the topic seems difficult to me.				
		All the time	**Most of the time**	**Sometimes**	**Almost never**
63	I can do almost everything during Mathematics class if I keep trying.				
64	I seek for different method on a topic in Mathematics for me to learn different way of solving mathematics problems				
65	I solve problems on topics that have not been taught by my teacher in Mathematics and bring it for my teacher to mark				
66	I pass Mathematics test or Examination because I have the				

	ability to solve Mathematics problems				
67	I bring any question that is not clear to me in my text book for my teacher to solve in the class.				
68	When I come across a new topic in Mathematics I study it until it clear to me.				
69	If I put in more effort in mathematics, I know I can do better.				
70	My scores in Mathematics tests or examinations reflect my ability in Mathematics				
71	I am willing to go ahead of what my teacher is teaching me in Mathematics class.				
72	I ask my teacher that I want to sit at the front during Mathematics class so that I can hear the teacher clearly				
73	I ask my teacher to let me do the correction of assignment given to us on the chalk board for other students				
74	I ask my teacher to allow me to sit at the front so as to see the board clearly in Mathematics class				
75	I ask my teacher not to give us assignment so that I will have time to play				
76	I tell my teacher to use different method to solve Mathematics problems in the class so that I can understand better				
77	I offer suggestions to my teacher on how to solve difficult topics in Mathematics.				
78	I contributed to class discussions during Mathematics class				
79	I ask questions during Mathematics class, from my teacher so that I learn more.				
80	During Mathematics class, I ask questions in order to confuse the teacher				
81	During Mathematics class, I ask irrelevant questions that does not relate to the topic				
82	Any time I need more explanations on a topic in Mathematics class, I tell my teacher				
83	I let the teacher know whatever thought that comes to my mind as per what the teacher is teaching us in Mathematics lesson				
		All the time	**Most of the time**	**Sometimes**	**Almost never**
84	I made my teacher to realize thosetopic that I have interest in Mathematics class				
85	I made my teacher to know that I am interested in solving only simple questions during Mathematics class				
86	I ask my teacher to give me extra work on Mathematics for me tostudy				
87	I discuss with my teacher those things I normally do to				

	understand Mathematics during Mathematics class.				
88	I tell other students how Mathematics questions can be solved so that I can learn more.				
89	I ask my teacher to teach us any question I cannot solve in my Mathematics text book during class				
90	If I noticed that my teacher has not explainedany topic clearly in Mathematics class, I tell my teacher to give more explanations on the topic so thatothers can learn.				
91	I suggest different formula to my teacher during Mathematics class to help me learn				
92	I ask my teacher to give more explanation on a Mathematics topic any time I don't understand in the class.				
93	When I have solution to any problem in Mathematics, I ask my teacher the same question to know whether he or she knows it.				
94	I appreciate the teacher when he/she make the class interesting during Mathematics class.				
95	I tell my teacher that I need to be motivated before I can learn Mathematics.				
96	I ensure that my Mathematics teacher does not punish me so that I will not run away from Mathematics class.				

APPENDIX II
MATHEMATICS ACHIEVEMENT TEST

OBJECTIVES QUESTION IN MATHEMATICS (SS 2) Time: $1\frac{1}{2}$ hours

Instruction: Answer all questions. Pick the correct answer from Option A- D

1. Express 302.10495 correct to five significant figures: A. 302.10 B. 302.11 C. 302.105 D. 302.1049.
2. Convert 35_{10} to number in base 2: A. 1011 B. 100011 C. 100011 D. 11001
3. The sum of 11011_2, 11111_2, 10000_2, is 10m10n0. What is the values of m and n.:
   A. m=0, n= 0 B. m=1, n= 0 C. m=0, n= 1 D. m=1, n= 1

4. A trader bought an engine for \$15,000.00 outside Nigeria. If the exchange rate is \$0.075 to N1.00, how much did the engine cost in naira?

A. N250, 000.00 B. N200, 000.00 C. N150, 000.00 D. N100, 000.00

5. Adding 42 to a given positive number yields the same result as squaring the number. what the number? A. 14 B. 13 C. 7 D. 6

6. Simplify $\frac{\log\sqrt{27}}{\log\sqrt{81}}$ A. 3 B. 2 C. $\frac{3}{2}$ D. $\frac{3}{4}$

7. If $9^{(2-x)} = 3$, find x. A. 1 B. $\frac{3}{2}$ C. 2 D. $\frac{5}{2}$

8. A sales boy gave a change of N68 instead of N72. Calculate his percentage error:

A. 4% B. $5\frac{5}{9}\%$ C. $5\frac{15}{17}\%$ D. 7%

9. If $23_x = 32_5$, find the value of x.: A. 7 B. 6 C. 5 D. 4

10. Simplify $\frac{1\frac{7}{8}\times 2\frac{2}{5}}{6\frac{3}{4}\div\frac{3}{4}}$ A. 9 B. $4\frac{1}{2}$ C. 2 D. $\frac{1}{2}$

11. Express $\frac{2}{x+3}-\frac{1}{x-2}$ as a simple fraction:

A. $\frac{x-7}{x^2+x-6}$ B. $\frac{x-1}{x^2+x-6}$ C. $\frac{x-2}{x^2+x-6}$ D. $\frac{x+7}{x^2+x-6}$

12. Simplify: $10\frac{2}{5}-6\frac{2}{3}+3$ A. $6\frac{4}{15}$ B. $6\frac{11}{15}$ C. $7\frac{4}{15}$ D $7\frac{11}{15}$

13. If X = { 0, 2, 4, 6 }, Y = { 1, 2, 3, 4 }, and Z = { 1, 3 } are subsets of U = {x: $0 \le x \le 6$}, find X ∩ (Y U Z). A. {0, 2, 6} B. {1, 3} C. {0, 6} D. {}

14. Make U the subject of the formula, $E = \frac{m}{2g}(v^2 - u^2)$.

A. $u = \sqrt{v^2 - \frac{2Eg}{m}}$ B. $u = \sqrt{\frac{v^2}{m} - \frac{2Eg}{4}}$ C. $u = \sqrt{v - \frac{2Eg}{m}}$ D. $u = \sqrt{\frac{2v^2Eg}{m}}$

15. If y varies inversely as y and y varies directly as z, what is the relationship between x and z ?A. x α z B.x α $\frac{1}{z}$ C. x α z^2 D. x α $z^{½}$

16. What is the number of terms in the Arithmetic Progression (A.P.) 2, -9, -20, ……, -141

A. 11 B. 12 C. 13 D. 14

17. Given that x > y and 3 < y, which of the following is/are true

l. y > 3 ll. X < 3 lll. x > y > 3

A. l only B.l and ll only C. l and lll D. l, ll and lll

18. A farmer uses $\frac{2}{5}$ of his land to grow cassava, $\frac{1}{3}$ of the remainder for yam and the rest for maize. Find the part used for maize. A. $\frac{2}{15}$ B. $\frac{2}{5}$ C. $\frac{2}{3}$ D. $\frac{4}{5}$

19. Solve the equations: $3x - 2y = 11$, $x + 2y = -3$
A. $x = 1, y = -2$ B. $x = 1, y = 3$ C. $x = 2, y = -1$ D. $x = 4, y = -3$

20. One of the factors of $(mn - nq - n^2 + mq)$ is $(m - n)$. The other factor is:

A. $(n - q)$ B. $(q - n)$ C. $(n + q)$ D. $(q - m)$

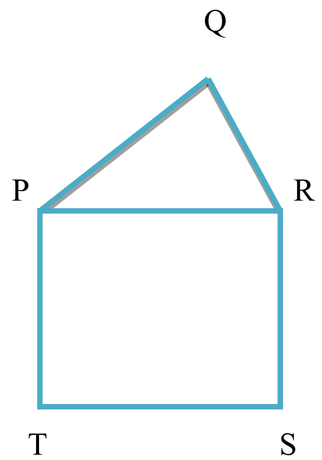
21. Form the equation whose roots are $x = \frac{1}{2}$ and $-\frac{2}{3}$.
A. $6x^2 - x + 2 = 0$ B. $6x^2 - x - 2 = 0$ C. $6x^2 + x + 2 = 0$ D. $6x^2 + x + - 2 = 0$

22. Solve for x in the equation $\frac{3}{5}(2x - 1) = \frac{1}{4}(5x - 3)$. A. 0 B. 1 C. 2 D. 3

23. What must be added to $(2x – 3y)$ to get $(x – 2y)$? A. $5y – x$ B. $y – x$ C. $x - 5x$ D. $x – y$

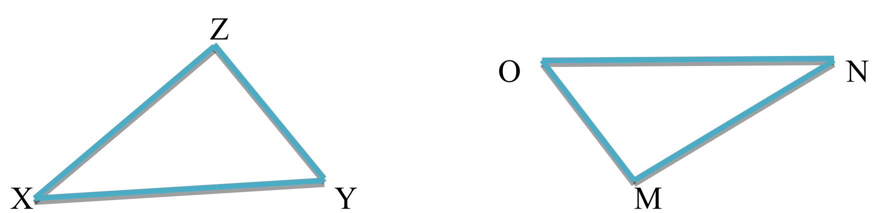
24. If $x + y = 2y – x + 1 = 5$, find the value of x. A. 3 B. 2 C. 1 D. -1

25.



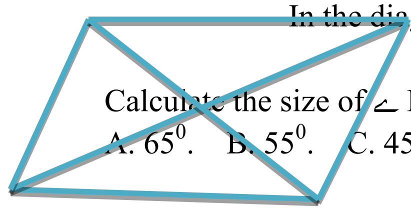
In the diagram, PRST is a square. If $|PR| = 24cm$, $|QR| = 10cm$ and $\angle PQR = 90^0$; find the perimeter of Polygon PQRST. A. 112cm B. 98cm C. 86cm D. 84

26.



In the diagrams, $|ZX| = |MN|$, $|ZY| = |MO|$ and $|XY| = |NO|$. Which of the following statement is true? A. $\Delta ZYX \equiv \Delta OMN$ B. $\Delta YZX \equiv \Delta NOM$.
C. $\Delta ZXY \equiv \Delta MON$. D. $\Delta XYZ \equiv \Delta NOM$.

In the diagram, PQRS is a rhombus and ∠ PSQ =35^0

27. 

Calculate the size of ∠ PRQ.

A. 65^0. B. 55^0. C. 45^0. D. 35^0

28. A pyramid has a rectangular base with dimensions 12m by 8m. If its height is 14 m, evaluate the capacity. A. $344m^3$ B.$448m^3$ C.$632m^3$ D. $840m^3$

29. The slant height of a cone is 5cm and the radius of its base is 3cm. Find, correct to the near whole number, the capacity of the cone. [Take $\pi = \frac{22}{7}$]

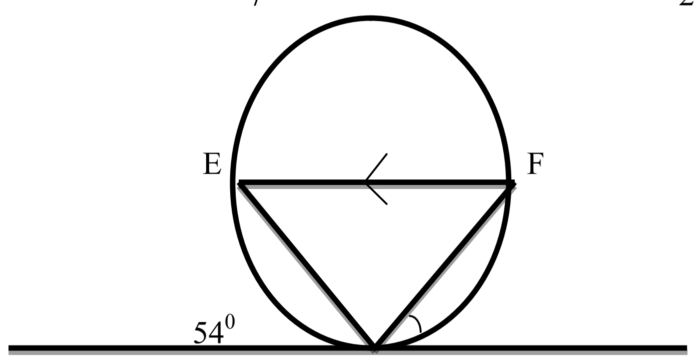
A. 48 cm^3 B. 47 cm^3 C. 38 cm^3 D. 13 cm^3

30. In what number base is the addition 465 + 24 + 225 = 1050?

A. ten B. nine C. eight D. seven

31. The dimensions of a four-sided tank are 2m by 7m by 11m. If its capacity is equivalent to that of a cylinder-shaped tank of height 4cm, calculate the base radius of the cylinder-shaped tank. [Take $\pi = \frac{22}{7}$] A. 14m B. 7m C. $3\frac{1}{2}$ m D. $1\frac{3}{4}$ m

32.



In the diagram, GL is a tangent to the circle at H. If EFL//GL, calculate the size of <EH

A. 126^0 B. 72^0 C. 54^0 D. 28^0

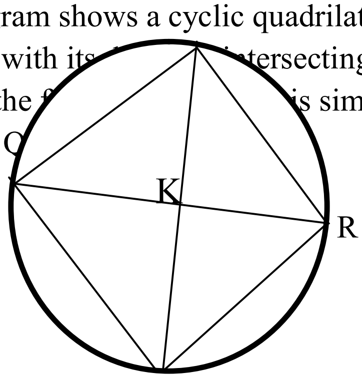
QThe diagram shows a cyclic quadrilateral

33. PQRS with its ... intersecting at K.

Which of the ... is similar

to triangle Q

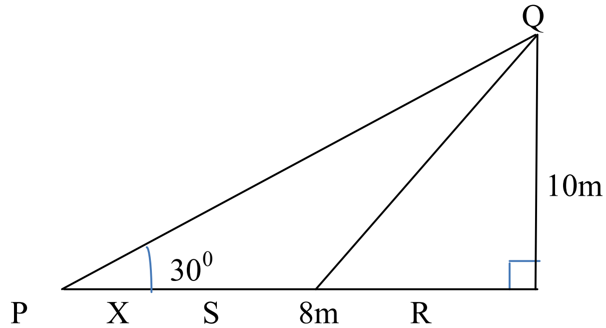
P



A.ΔPQK B.ΔPKS C.ΔSKR D. ΔPSR

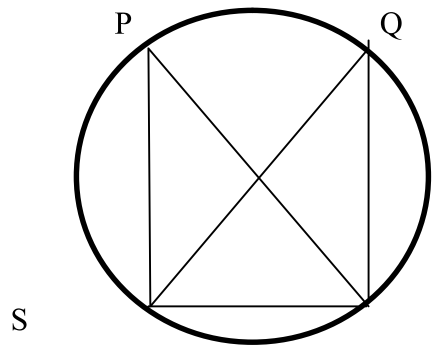
S

34.



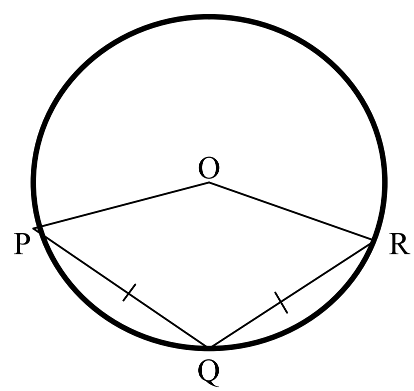
In the diagram, |QR| =10m, |SR| = 8m, ∠QPS=30^0,< QRP= 90^0 and |PS| = X. Find X
A. 1.32m B.6.32m C.9.32m D. 17.32m

35.

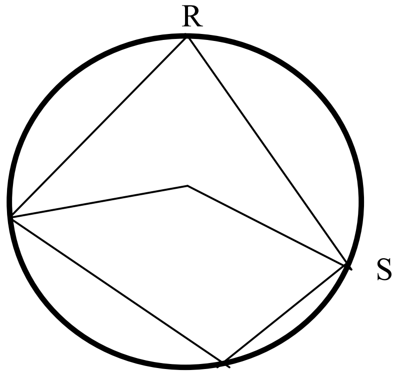


In the diagram, 0 is the center of the circle, < SQR = 60^0, < SPR = y and <SOR = 3X. What isthe value of (x+ y). A. 110^0 B. 100^0 C. 80^0 D. 7

36



In the diagram, OP and OR are radii, $|PQ| = |QR$ and reflex $< POR$ is $240^{0.}$ Calculate the value of X. A. 60^0 B 55^0 C. 50^0 D. 45^0

37. 

The diagram is a circle with Centre 0. PRST are points on the circle. Find the value $< PRS$.
A. 144^0 B 72^0 C. 40^0 D. 36^0

T

38. A kite flies on a taut string of length 50m inclined at an angle of 54^0 to the horizontal ground. The height of the kite above the ground is
A. $50 \tan 36^0$ B. $50 \sin 54^0$ C. $50 \tan 54^0$ D. $50 \sin 36^0$

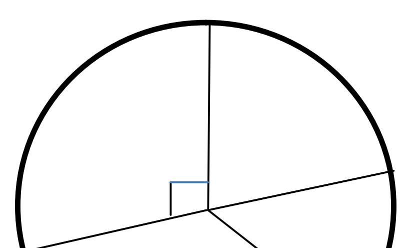
39. If $\sin x = \frac{5}{13}$ and $0^0 \leq x \geq 90^0$ find the value of ($\cos x - \tan x$.)
A. $\frac{7}{13}$ B $\frac{12}{13}$ C. $\frac{79}{156}$ D. $\frac{209}{156}$

40. The bearing of Y and X is 060^0 and the bearing of Z from Y = 060^0. Find the bearing of X from Z. A. 300^0 B 240^0 C. 180^0 D. 120^0

41. The probability of an event P happening is $\frac{1}{5}$ and that of event Q is $\frac{1}{4}$. If the events are independent, Find the probability that neither of them happens?
A. $\frac{4}{5}$ B. $\frac{3}{4}$ C. $\frac{3}{5}$ D. $\frac{1}{20}$

42. The probabilities that Kebba, Ebou and Omar will hit a target are $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ respectively. Find the probability that only Kebba will hit the target.
A. $\frac{2}{5}$ B. $\frac{7}{60}$ C. $\frac{1}{30}$ D. $\frac{1}{60}$

The pie chart shows the distribution of 600



Mathematics students test books for Arts, Business, Science and Technical classes. Use it to answer questions 43 and 44

43. How many Textbooks are for the Technical class?
A. 100 B. 150 C. 200 D. 250

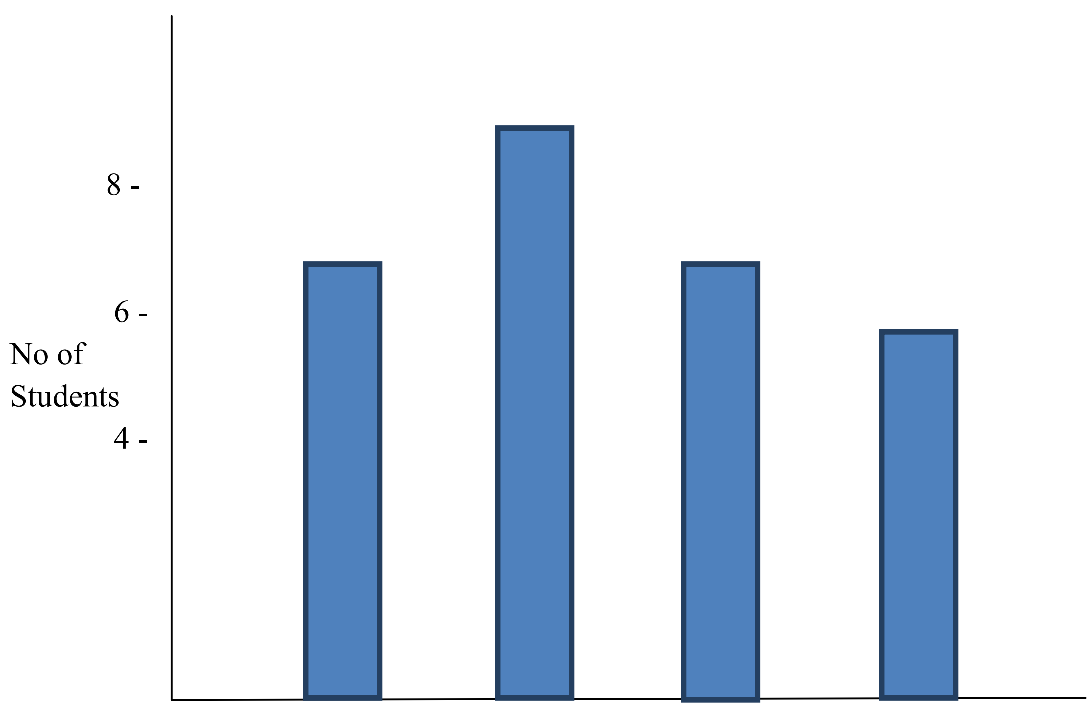
44. What percentage of the total number of text book belongs to science?
A. $12\frac{1}{2}\%$ B. $20\frac{5}{6}\%$ C. 25% D. $41\frac{2}{3}\%$

45. The population of student in a school is 810. If this is represented on a pie chart, calculate the sectorial angle for a class of 72 students. A. 32^0 B. 45^0 C. 60^0 D. 75^0

46.

Scores	0 – 4	5 - 9	10 - 14
Frequency	2	1	2

The table shows the distribution of the scores of some students in a test. Calculate the mean score. A. 5.6 B. 6.2 C. 6.6 D. 7



2 -

0 –

1 2 3 4

Marks

The bar chart shows the frequency distribution of marks scored by students in a class test. Use the bar chart to answer questions 47 to 49

47. How many students are in the class?

A. 10 B. 24 C. 25 D. 30

48. Calculate the mean of the distribution

A. 6.0 B. 3.0 C. 2.4 D. 1.8

49. What is the median of the distribution?

A. 2 B. 4 C. 6 D. 8

50. The scores of twenty students in a test are as follows: 44, 47, 48, 49, 50, 51, 52, 53, 53, 54, 58, 59, 60, 61, 63, 65, 67, 70, 73, and 75. Find the third quartile.

A. 62 B. 63 C. 64 D. 65

APPENDIX III

ANSWERS TO THE OBJECTIVE QUESTIONS

1	A	**11**	A	**21**	D	**31**	C	**41**	C
2	C	**12**	B	**22**	D	**32**	B	**42**	C
3	C	**13**	B	**23**	B	**33**	C	**43**	D
4	B	**14**	A	**24**	B	**34**	C	**44**	A

5	C	**15**	B	**25**	A	**35**	B	**45**	A
6	D	**16**	D	**26**	D	**36**	A	**46**	C
7	B	**17**	C	**27**	D	**37**	D	**47**	C
8	C	**18**	A	**28**	B	**38**	B	**48**	D
9	A	**19**	C	**29**	B	**39**	C	**49**	A
10	D	**20**	C	**30**	D	**40**	B	**50**	B

APPENDIX IV

Table 4.13

INTERNATIONAL CENTRE FOR EDUCATIONAL EVALUATION
UNIVERSITY OF IBADAN, IBADAN.
FINAL SCALE OF STUDENTS MATHEMATICS ENGAGEMENT SCALE

LGA: ____
SHOOL CODE:____
CLASS: __________________ AGE: _________________ SEX: MALE ☐ FEMALE ☐

The following statements are meant to find out your current engagement in Mathematics. For each statement, choose the response that is closest to your current practices in the appropriate column. Try and be as frank and truthful as possible. Please tick ( ) in front of each statement.

NOTE: Note that five periods is the Standard periods that are allotted to Mathematics in a week. With this information; **All the time:** means 5 times in a week. **Most of the time:** means 3 to 4 times in a week. **Sometimes:** means 1 to 2 times in a week. **Almost never:** means below 1 time in a week.

NO	**STATEMENT**	**RESPONSES**			
	PERSONAL AGENCY ENGAGEMENT	**All the time**	**Most of the time**	**Sometimes**	**Almost never**
1	If I noticed that my teacher has not explained any topic clearly in Mathematics class, I tell my teacher to give more explanations on the topic so that others can learn				
2	I let the teacher know whatever thought that comes to my mind as per what the teacher is teaching us in Mathematics class				
3	I suggest different formula to my teacher during Mathematics class to help me learn				
4	Any time I need more explanations on a topic in Mathematics class, I tell my teacher				
5	I ask my teacher to give me extra work on Mathematics to help me learn				
6	I ask my teacher to give more explanation on a Mathematics topic any time I don’t understand in the class.				
7	I offer suggestions to my teacher on how to solve difficult topics in Mathematics				
8	When I have solution to any problem in Mathematics, I ask my teacher the same question to know whether he or she knows it.				
9	I tell my teacher to use different method to solve Mathematics problems in the class so that I can understand better				

10	I tell other students how Mathematics questions can be solved so that I can learn more				
11	I ask my teacher to let me do the correction of assignment given to us on the chalk board for other students				
	POSITIVE AFFECTIVE ENGAGEMENT	**All the time**	**Most of the time**	**Sometimes**	**Almost never**
12	Any time my family/guardian(s) provide me with the material I need to pass Mathematics I feel happy.				
13	I have a goal to make a good grade in Mathematics				
14	Any time my teacher uses teaching aid to teach me during Mathematics class I feel happy				
15	Learning during Mathematics class is important to me				
16	I solve Mathematics problems because I need it for my future carrier.				
17	before a quiz or examination in Mathematics, I work hard for me to pass.				
	NEGATIVEAFFECTIVE ENGAGEMENT				
18	I don't have interest in what my teacher is teaching me during Mathematics class.				
19	I don't participate in Mathematics class because I feel that my teacher will embarrass me.				
20	I get disturbed and unhappy whenever my Mathematics teacher entered the class to teach Mathematics.				
21	I disrupt the class during Mathematics lesson since I do not understand Mathematics.				
22	I don't understand and follow directions during Mathematics class.				
23	I don't come to Mathematics class at all.				
24	I concentrate on other things during Mathematics class because I don't understand what my teacher is teaching me				
	POSITIVE BEHAVIOURAL ENGAGEMENT				
25	I do all the Mathematics assignment given to me by my teacher.				
26	I pay attention to what my teacher is teaching me during Mathematics class.				
27	I do Mathematics homework after school.				
	NEGATIVE BEHAVIOURAL ENGAGEMENT				
28	I do not respond at all to the questions asked by my teacher during Mathematics class.				
29	I don't go over to study what my teacher teaches me during Mathematics class.				
30	I do not ask questions during Mathematics class.				
31	I donot study my Mathematics notebook before coming to Mathematics class				
	CONGNITIVE ENGAGEMENT	**All the**	**Most of**	**Sometimes**	**Almost**

		time	the time		never
32	I solve problems on topics that have not been taught by my teacher in Mathematics and bring it for my teacher to mark				
33	If I cannot understand my Mathematics homework, I keep trying until I do it				
34	When I come across a new topic in Mathematics I study it until itclear to me.				
35	I solve my homework problems in Mathematics in a separate book before writing it in my note.				

MANUAL FOR SORING OF FINAL SCALE OF STUDENTS MATHEMATICS ENGAGEMENT ITEMS

SORING OF FINAL SCALE OF STUDENTS MATHEMATICS ENGAGEMENT TEST ITEMS

SUB-SCALE	All the time	Most of the time	Sometimes	Almost never
PERSONAL AGENCY ENGAGEMENT	4	3	2	1
POSITIVE AFFECTIVE ENGAGEMENT	4	3	2	1
NEGATIVEAFFECTIVE ENGAGEMENT	1	2	3	4
POSITIVE BEHAVIOURAL ENGAGEMENT	4	3	2	1
NEGATIVE BEHAVIOURAL ENGAGEMENT	1	2	3	4
CONGNITIVE ENGAGEMENT	4	3	2	1

APPENDIX V

Table 4.4.1: Structure Matrix of Students Mathematics Engagement Scale

	Factors					
Items	**Factor 1**	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**	**Factor 6**
a87	0.656					
a91	0.644					
a67	0.638					0.372
a88	0.614					
a86	0.610					
a77	0.602					
a89	0.593					
a73	0.582					0.337
a65	0.576					
a90	0.557					
a68	0.553					0.452
a83	0.546					
a76	0.545					
a82	0.542					
a66	0.525			0.374		0.355
a79	0.523					
a23	0.503			0.321		

a92	0.483					
a93	0.462					
a64	0.457					0.447
a84	0.418					
a42		0.600		0.481		
a47		0.598				
a46		0.583				
a43		0.532		0.303		
a54		0.492				0.322
a39		0.456				
a45		0.454				0.339
a49	0.358	0.449				0.309
a53		0.430				
a14		0.395		0.309		
a51		0.385				
a34						
a28			0.637		0.340	
a19			0.632		0.349	
a20			0.632			
a36			0.626		0.365	
a37			0.588		0.374	
a30			0.577			
a7			0.517		0.360	
a6			0.468		0.369	
a75			0.459		0.311	
a3	0.318			0.570		0.325
a1		0.302		0.567		
a4				0.522		
a2	0.405			0.466		
a5	0.313			0.337		
a13			0.468		0.683	
a12			0.400		0.519	
a17					0.515	
a9					0.397	
a61	0.391	0.339		0.361		0.541
a60	0.325					0.515
a71	0.303					0.448

Table4.4.1 in appendix 4showed the structure matrix, whichshows that the factors are notindependent of their own (they are correlated).

APPENDIX VI

Sample Size adequacy

Table 4.4.2 Sampling Adequacy ofStudent Mathematics Engagement Scale

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.930
Bartlett's Test of Sphericity	Approx. Chi-Square	27627.071
	Df	4560
	Sig.	.000

Determinant = 4.69×10^{-13}

According to (Field, 2005) KMO and significant levelgives the Bartlett’s test of sphericity as well as Kaiser-Meyer-Olkin measure of sampling adequacy. The value of KMO must be higher than 0.5 if the sample is sufficient. The figures ranges between 0 and 1 and a value of 0 specifies that the sum of partial relationships is bigcomparative to the sum of relationships, which showsdispersion in the arrangement of relationships (therefore, factor analysis may not be appropriate).If the value is close to one, this shows that the arrangements of relationships are moderatelysolid and so factor analysis should producedistinctive and consistent factors. Kaiser (1974) acclaimedthata value greater than 0.5 should be accepted. And if the value is less than 0.5, researcher should collect more data or to think of other variable to add. Furthermore Kaiser said that, values between 0.5 and 0.7 are average, values between 0.7 and 0.8 are good, values between 0.8 and 0.9 are great and values above 0.9 are superb. For these data the value is 0.930 which falls into the range of being superb.See appendix 5

Not only that, Bartlett's test is extremely significant ($p < 0.001$), therefore, factor analysis is suitable. The determinant is $4.69 \times 10^{-13} < 0.00001$ imply that there is a problem of multicollinearity and singularity in the data. Field, (2005) noted that for any data to be free from the problems of multicollinearity and singularity, the determinant must be > 0.00001.To avoid this problem some items need to be discarded. To discard these items, series of validation stages were carried out.

APPENDIX VII

Formulae for calculating AVE and CR

$$\textbf{AVE}(\mathbf{C_F}) = \frac{\sum_i^n LD^2}{\left(\sum_i^n LD^2\right) + \ell}$$

$$\ell = \sum_i^n 1 - LD^2$$

$$\text{CR}(C_F) = \frac{\left(\sum_i^n LD\right)^2}{\left[\left(\sum_i^n LD\right)^2 + \left(\sum_i^n 1 - LD^2\right)\right]}$$

Where:

AVE = Average variance extracted

CR = Composite Reliability of construct F

C_F =Construct F

LD = Factor Loading

ℓ = error variance of the 'n' items (i = 1, 2, . . .,n) of construct F

n = *number* of items in a construct C_F

Fornell-Larcker criterion has been in use since 1981 to calculate Average Variance Extracted (AVE) for discriminant validity of a construct. AVE assesses the degree of variance apprehended by a construct againstthe level owing to measurement error. According to Ahmad, Zulkurnain and Khairushalimi (2016) and Hamid, Sami and Sidek (2017), the value of AVE should be between 0.5 to 0.9.See the formula below. Also, according to Fornell-Larcker criterion, the convergent validity of the measurement construct can be evaluated by computing Composite Reliability (CR) using CR formula. The value of CR should be between 0.60 to 0.90 (Ahmad, Zulkurnain and Khairushalimi (2016) and Hamid, Sami and Sidek (2017).